Introduction to Theory of Mesoscopic Systems

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Introduction

Einstein's Miraculous Year - 1905

Six papers:

1. The light-quantum and the photoelectric effect.
   Completed March 17.

2. A new determination of molecular dimensions.
   Completed April 30. Published in1906
   Ph.D. thesis.

3. Brownian Motion.
   Received by Annalen der Physik May 11.

4,5.The two papers on special relativity.
   Received June 30 and September 27

6. Second paper on Brownian motion.
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Diffusion and Brownian Motion:

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Q: Are these papers indeed important enough to stay in the same line with the relativity and photons. Why ?

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Nobel Prize

By far the largest number of citations

Brownian Motion - history

Robert Brown
(1773-1858)

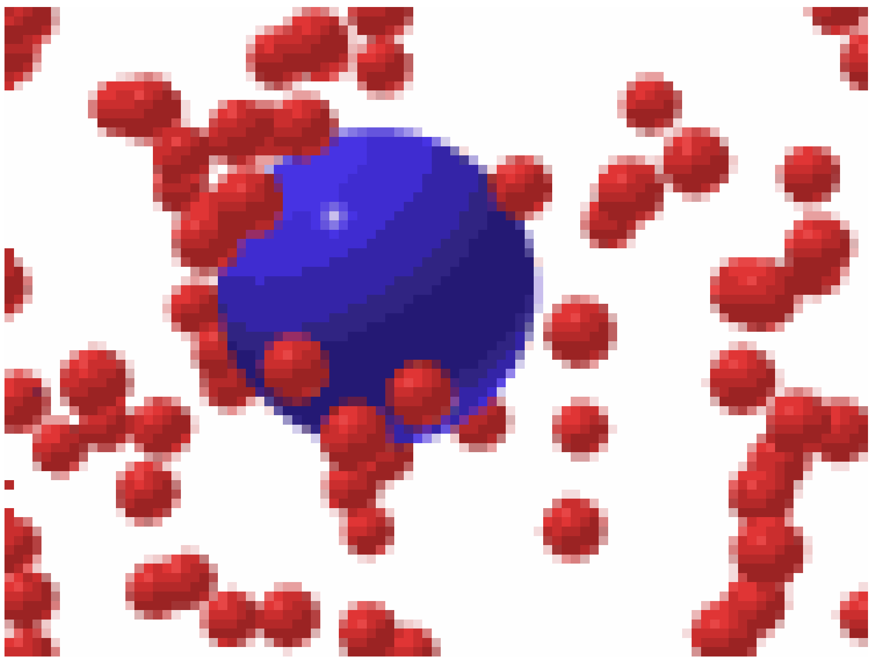
The instrument with which Robert Brown studied Brownian Motion and which he used in his work on identifying the nucleus of the living cell. This instrument is preserved at the Linnean Society in London.

Brownian Motion - history

Robert Brown, *Phil.Mag*. ***4***,*161(1828)*; ***6***,*161(1829)*

Random motion *of particles suspended in water ("dust or soot deposited on all bodies in such quantities, especially in London")*

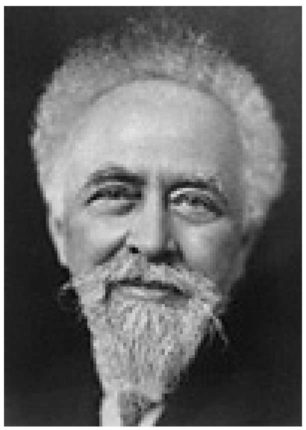
Action of water molecules pushing against the suspended object ?



Giovanni Cantoni (Pavia). ***N.Cimento,*** ***27***,*156(1867)*.

The Nobel Prize in Physics 1926

"for his work on the discontinuous structure of matter, and especially for his discovery of sedimentation equilibrium"



Jean Baptiste Perrin
France

b. 1870
d. 1942

... measurements on the Brownian movement showed that Einstein's theory was in perfect agreement with reality. Through these measurements a new determination of Avogadro's number was obtained.

The Nobel Prize in Physics 1926
From the Presentation Speech by Professor C.W. Oseen, member of the Nobel Committee for Physics of The Royal Swedish Academy of Sciences on December 10, 1926

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Action of water molecules pushing against the suspend object

Problems:

1. Each molecules is too light to change the momentum of the suspended particle.
2. Does Brownian motion violate the second law of thermodynamics ?

Jules Henri Poincaré (1854-1912)

"We see under our eyes now motion transformed into heat by friction, now heat changes inversely into motion. This is contrary to Carnot's principle."

H. Poincare, "The fundamentals of Science", p.305, Scientific Press, NY, 1913

Problems:

1. Each molecules is too light to change the momentum of the suspended particle.
2. Does Brownian motion violate the second law of thermodynamics ?
3. Do molecules exist as real objects and are the laws of mechanics applicable to them?

Kinetic theory

Ludwig Boltzmann
1844 - 1906

$$S = {\color{red}k} \log W + const$$

entropy

probability

Kinetic theory

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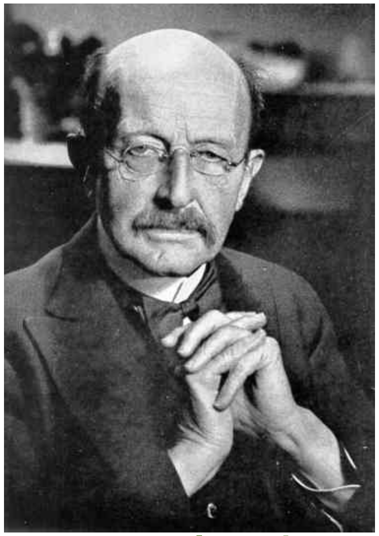
1844 - 1906

$$S = k \log W + const$$

entropy

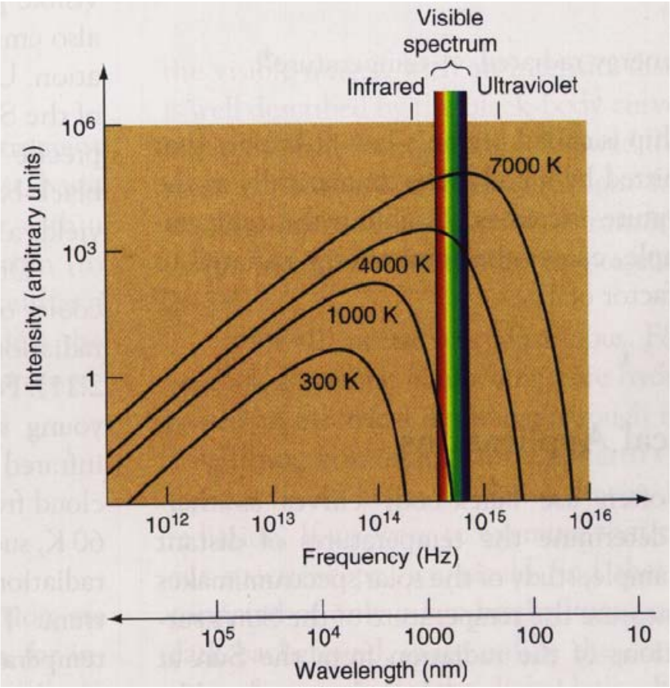
probability

k is Boltzmann constant



Max Planck
1858 - 1947

$$\rho(\nu, T) = \frac{8\pi h \nu^3}{c^3\left[\exp\left(\frac{h\nu}{kT}\right) - 1\right]}$$



From Micro to Macro

$$S = k \log W + const$$

From Macro to Micro

"It is of great importance since it permits exact computation of Avogadro number … . The great significance as a matter of principle is, however … that one sees directly under the microscope part of the heat energy in the form of mechanical energy."

Einstein, 1915

Brownian Motion - history

Einstein was not the first to:

1. Attribute the Brownian motion to the action of water molecules pushing against the suspended object

2. Write down the diffusion equation

3. Saved Second law of Thermodynamics

L. Szilard, Z. Phys, 53, 840(1929)

Brownian Motion - history

Einstein was not the first to:

1. Attribute the Brownian motion to the action of water molecules pushing against the suspended object

2. Write down the diffusion equation

3. Saved Carnot's principle [L. Szilard, Z. Phys, 53, 840(1929)]

Einstein was the first to:

1. Apply the diffusion equation to the probability

2. Derive the diffusion equation from the assumption that the process is markovian (before Markov) and take into account nonmarkovian effects

3. Derived the relation between diffusion const and viscosity (conductivity), i.e., connected fluctuations with dissipation

By studying large molecules in solutions sugar in water or suspended particles Einstein made molecules visible

Diffusion Equation

$$\frac{\partial \rho}{\partial t} - D\nabla^2 \rho = 0$$

Diffusion constant

Einstein-Sutherland Relation for electric conductivity σ

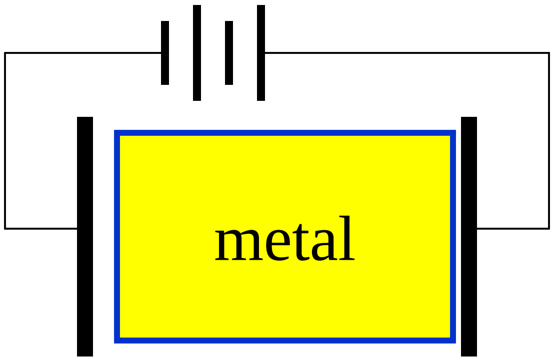
William Sutherland
(1859-1911)

$$\boxed{\sigma = e^2 D \nu \qquad \nu \equiv \frac{dn}{d\mu}}$$

If electrons would be degenerate and form a classical ideal gas

$$\nu = \frac{1}{T n_{tot}}$$

Einstein-Sutherland Relation for electric conductivity σ



$$n = n(\mu)$$

$$\frac{dn}{dx} = \frac{dn}{d\mu}\frac{d\mu}{dx} = eE\frac{dn}{d\mu}$$

Density of electrons

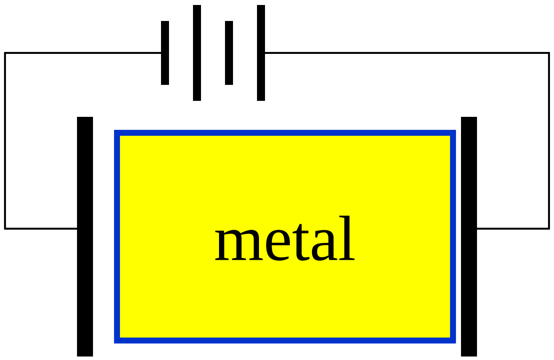
Chemical potential

Electric field

No current

$$eD\frac{dn}{dx} = \sigma E$$

Einstein-Sutherland Relation for electric conductivity σ



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No current

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Conductivity

$$\boxed{\sigma = e^2 D\nu \qquad \nu \equiv \frac{dn}{d\mu}}$$

Density of states

Diffusion Equation

$$\frac{\partial \rho}{\partial t} - D\nabla^2 \rho = 0$$

Lessons from the Einstein's work:

- Universality: the equation is valid as long as the process is marcovian
- Can be applied to the probability and thus describes both fluctuations and dissipation
- There is a universal relation between the diffusion constant and the viscosity
- Studies of the diffusion processes brings information about micro scales.

What is a Mesoscopic System?

- Statistical description
- Can be effected by a microscopic system and the effect can be macroscopically detected

Meso can serve as a microscope to study micro

Brownian particle was the first mesoscopic device in use

Brownian particle was the first mesoscopic device in use

First paper on Quantum Theory of Solid State (Specific heat)
Annalen der Physik, 22, 180, 800 (1907)

First paper on Mesoscopic Physics
Annalen der Physik, 17, 549 (1905)

Finite size quantum physical systems

Atoms

Nuclei

Molecules

.

.

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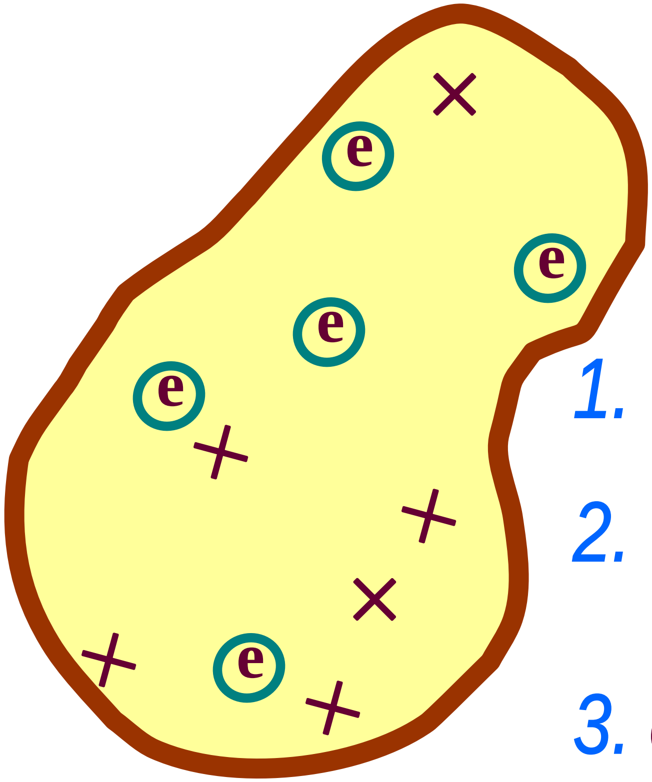
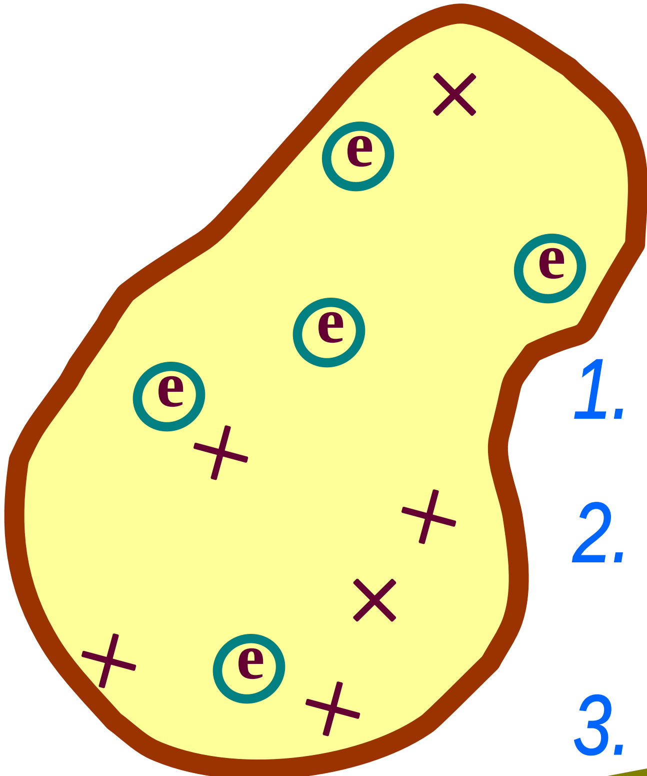
Quantum Dots

Quantum Dot



1. *Disorder (× – impurities)*
2. *Complex geometry*
3. ***e-e*** *interactions*

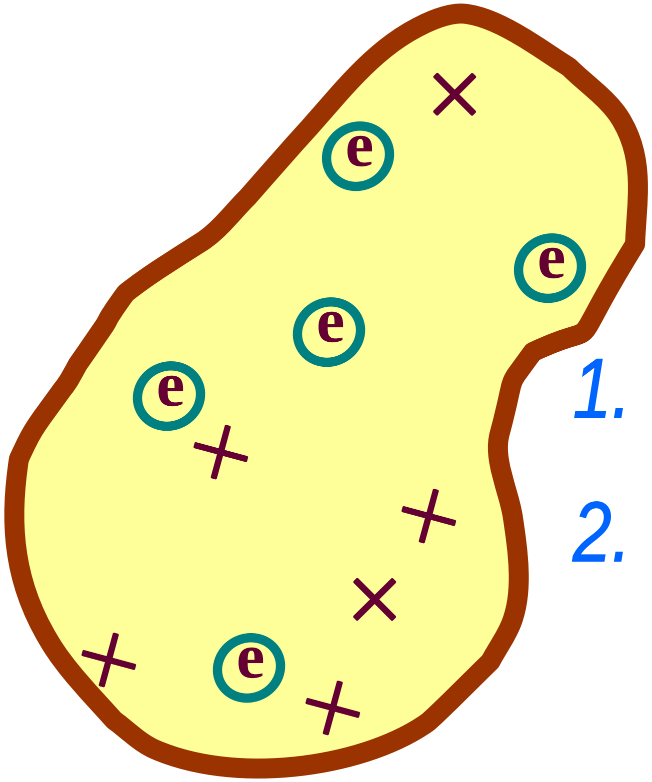


Quantum Dot

1. *Disorder (× – impurities)*
2. *Complex geometry*
3. ~~***e-e** interactions~~ for a while

Realizations:

- Metallic clusters
- Gate determined confinement in 2D gases (e.g. ***GaAs/AlGaAs***)
- Carbon nanotubes
-
-



1. Disorder (× – impurities)

2. Complex geometry

How to deal with disorder?

- ~~Solve the Shrodinger equation exactly~~
- Start with plane waves, introduce the mean free path, and . . .

How to take quantum interference into account

?

Lesson 1:

Beyond Markov chains:

Anderson Localization

and

Magnetoresistance

Т. 18 Журнал экспериментальной и теоретической физики. Вып. 4
1948

ОБ ИЗМЕНЕНИИ ЭЛЕКТРИЧЕСКОГО СОПРОТИВЛЕНИЯ ТЕЛЛУРА В МАГНИТНОМ ПОЛЕ ПРИ НИЗКИХ ТЕМПЕРАТУРАХ

Р. А. Ченцов

R.A. Chentsov "***On the variation of electrical conductivity of tellurium in magnetic field at low temperatures***", *Zh. Exp. Theor. Fiz. v.18, 375-385, (1948).*

Таблица 2

Уменьшение сопротивления теллура в магнитном поле

Образец	Температура (°К)	Максимальное уменьшение сопротивления
Te-1	2,13	$0,7 \cdot 10^{-3}$
Te-2	2,15	$1,0 \cdot 10^{-3}$
Te-4	1,96	$1,1 \cdot 10^{-3}$
Te-5	1,96	$0,5 \cdot 10^{-3}$

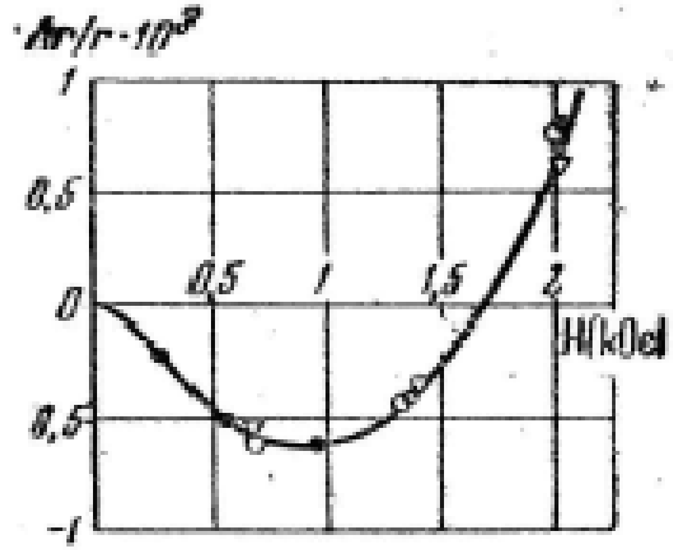


Рис. 2

Quantum particle in random quenched potential

Quantum particle in random quenched potential

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

Quantum particle in random quenched potential

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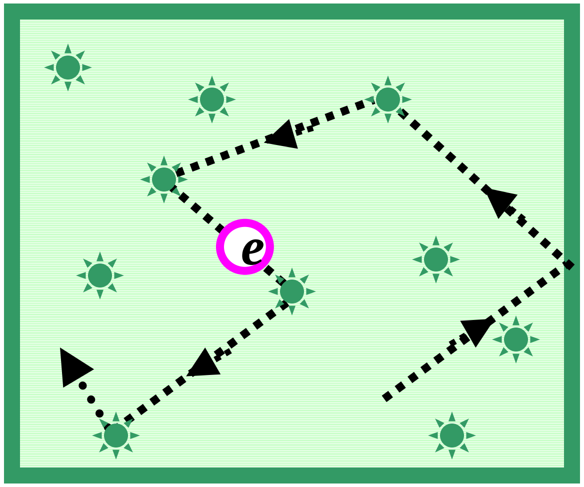
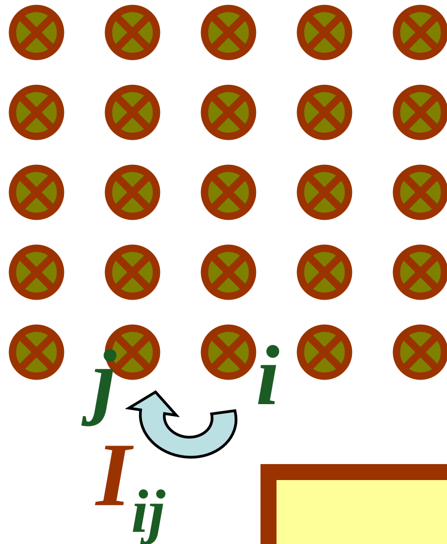


Anderson Model



- *Lattice - tight binding model*
- *Onsite energies* ε_i - *random*
- *Hopping matrix elements* I_{ij}

$-W < \varepsilon_i < W$

uniformly distributed

$$I_{ij} = \begin{cases} I & i \text{ and } j \text{ are nearest neighbors} \\ 0 & \text{otherwise} \end{cases}$$

Anderson Transition

$I < I_c$

Insulator

All eigenstates are localized

Localization length ξ

$I > I_c$

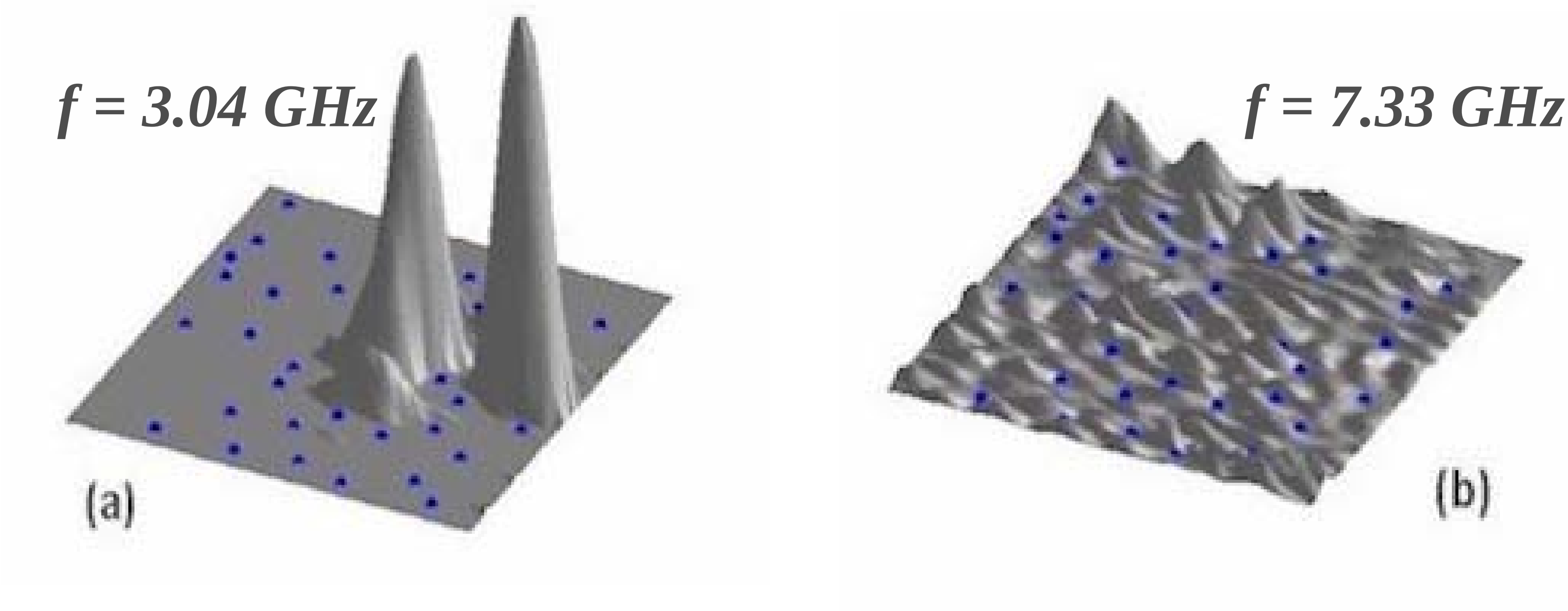
Metal

There appear states extended all over the whole system

VOLUME 85, NUMBER 11 PHYSICAL REVIEW LETTERS 11 SEPTEMBER 2000

Correlations due to Localization in Quantum Eigenfunctions of Disordered Microwave Cavities

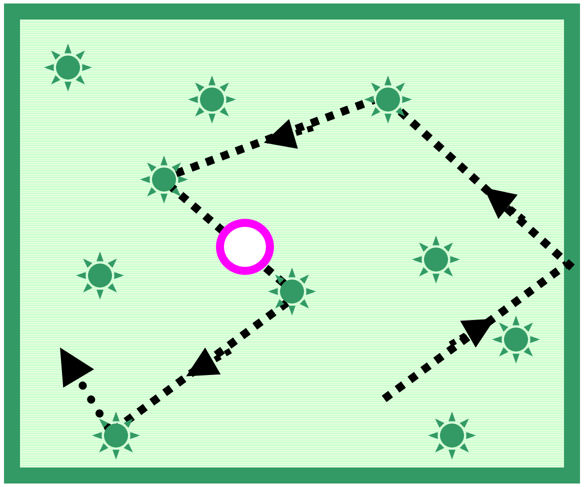
Prabhakar Pradhan and S. Sridhar
Department of Physics, Northeastern University, Boston, Massachusetts 02115
(Received 28 February 2000)



Anderson Insulator ***Anderson Metal***

Classical particle in a random potential

Diffusion



1 particle - random walk

Density of the particles ρ

Density fluctuations $\rho(\mathbf{r},t)$ at a given point in space $\mathbf{r}$ and time t.

$$\frac{\partial\rho}{\partial t} - D\nabla^2\rho = 0$$

Diffusion Equation

D - Diffusion constant

$$D = \frac{l^2}{d\tau}$$

l mean free path

τ mean free time

d # of dimensions

Einstein - Sutherland Relation for electric conductivity σ

$$\sigma = e^2 D \nu \qquad \nu \equiv \frac{dn}{d\mu}$$

$$G = \underbrace{\frac{e^2}{h}\left(\nu L^d\right)\frac{Dh}{L^2}}_{g(L)}$$

Conductance

$$G = \sigma L^{d-2}$$

for a cubic sample of the size L

$$\boxed{g(L) = \frac{hD/L^2}{1/\nu L^d}}$$

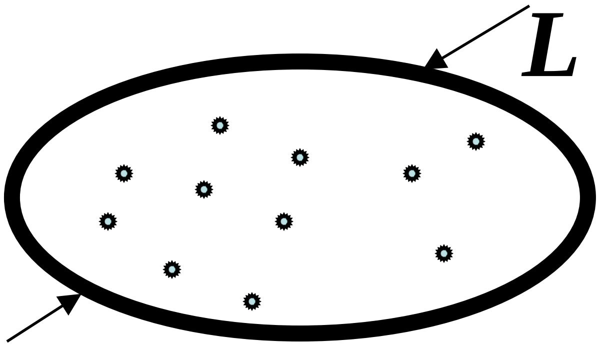
$$= \frac{\text{Thouless energy}}{\text{mean level spacing}}$$

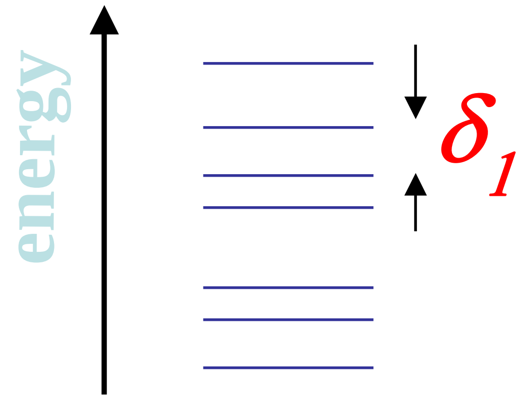
Dimensionless Thouless conductance

Energy scales (*Thouless, 1972*)

1. Mean level spacing

$$\delta_1 = 1/\nu \times L^d$$





L *is the system size;*

d *is the number of dimensions*

2. Thouless energy

$$E_T = hD/L^2$$

D *is the diffusion const*

E_T *has a meaning of the inverse diffusion time of the traveling through the system or the escape rate (for open systems)*

$$g = E_T / \delta_1$$

dimensionless Thouless conductance

$$g = Gh/e^2$$

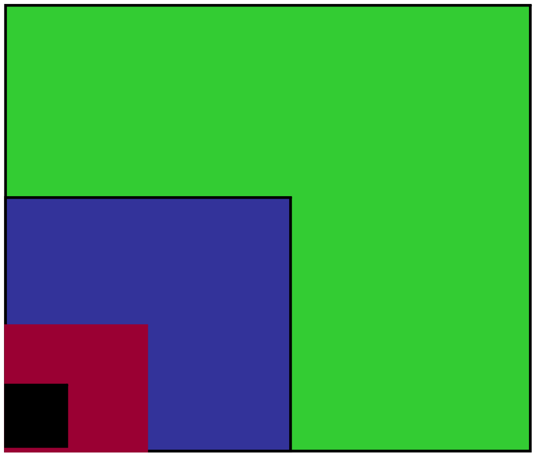
Scaling theory of Localization

(Abrahams, Anderson, Licciardello and Ramakrishnan 1979)

$g = E_T / \delta_1$

Dimensionless Thouless conductance

$g = Gh/e^2$



$L = 2L = 4L = 8L$ ....

without quantum corrections

$E_T \propto L^{-2} \qquad \delta_1 \propto L^{-d}$

$$\frac{E_T}{\delta_1} \longrightarrow \frac{E_T}{\delta_1} \longrightarrow \frac{E_T}{\delta_1} \longrightarrow \frac{E_T}{\delta_1}$$

$$g \longrightarrow g \longrightarrow g \longrightarrow g$$

$$\boxed{\frac{d(\log g)}{d(\log L)} = \beta(g)}$$

$$\boxed{\frac{d(\log g)}{d(\log L)} = \beta(g)}$$

$\beta -$ function is

Universal, i.e., material independent

But

It depends on the global symmetries, e.g., it is different with and without T-invariance (in orthogonal and unitary ensembles)

Limits:

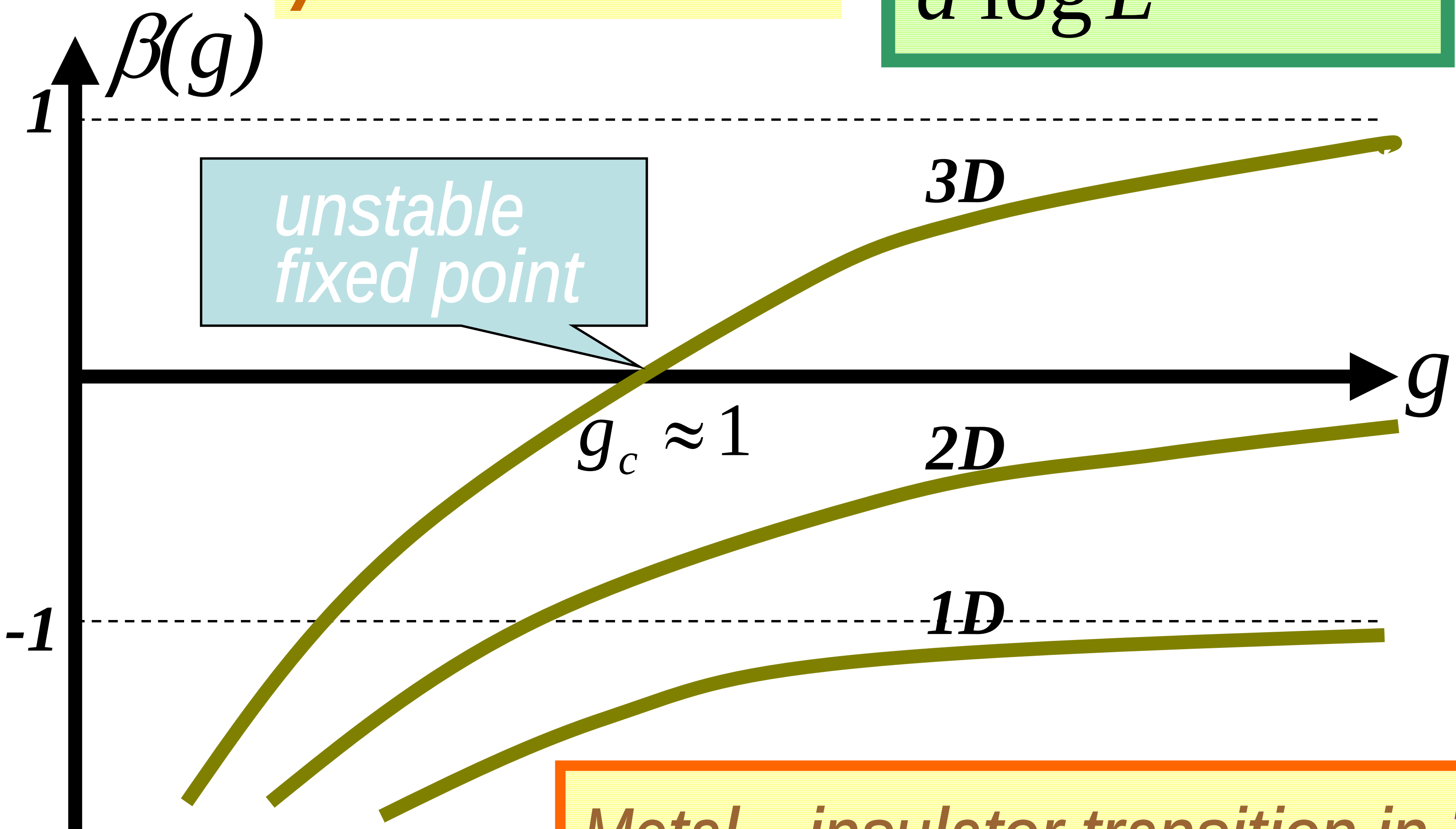
$$g >> 1 \quad g \propto L^{d-2} \quad \beta(g) = (d-2) + O\left(\frac{1}{g}\right)$$

$$\boxed{\begin{array}{ll} >0 & d>2 \\ ?? & d=2 \\ <0 & d<2 \end{array}}$$

$$g << 1 \quad g \propto e^{-L/\xi} \quad \beta(g) \approx \log g < 0$$

β - function

$$\frac{d\log g}{d\log L} = \beta(g)$$



Metal – insulator transition in ***3D***

All states are localized for ***d=1,2***

Questions:

Why

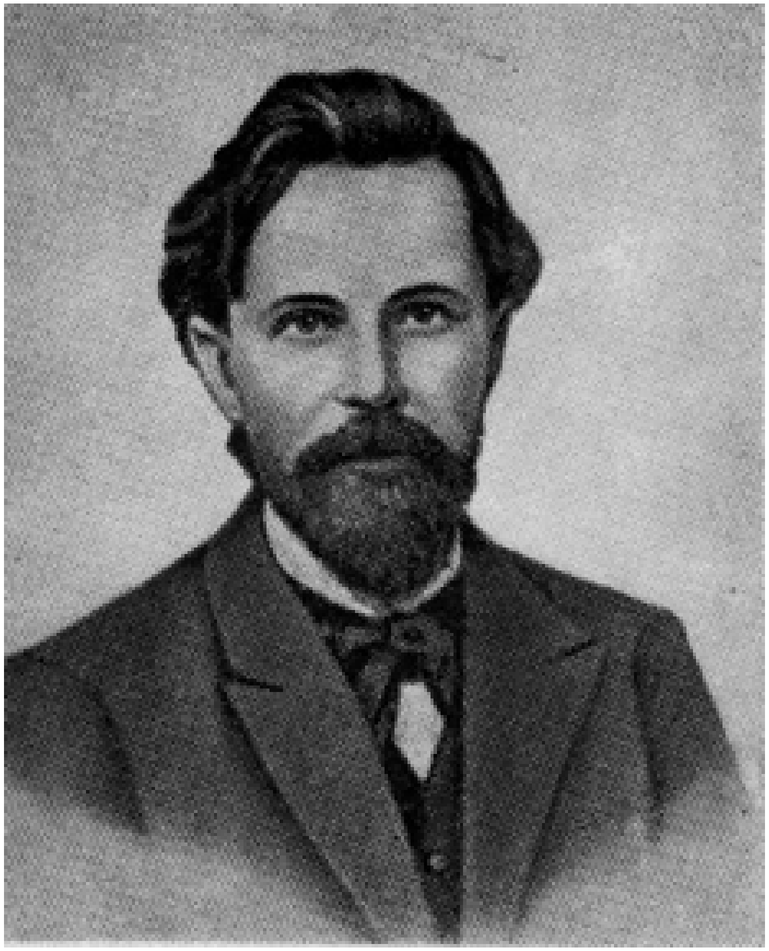
- the scaling theory is correct?
- the corrections of the diffusion constant and conductance are negative?

Why diffusion description fails at large scales ?

Diffusion description fails at large scales

Why?

Einstein: there is no diffusion at too short scales – there is memory, i.e., the process is not marcovian.



Andrei Markov
1856-1922

- A. A. Markov. « Rasprostranenie zakona bol'shih chisel na velichiny, zavisyaschie drug ot druga ». *Izvestiya Fiziko-matematicheskogo obschestva pri Kazanskom universitete*, 2-ya seriya, tom 15, pp 135-156, **1906**.
- A. A. Markov. « Extension of the limit theorems of probability theory to a sum of variables connected in a chain ». reprinted in Appendix B of: R. Howard. *Dynamic Probabilistic Systems, volume 1: Markov Chains*. John Wiley and Sons, 1971.

Diffusion description fails at large scales

Why?

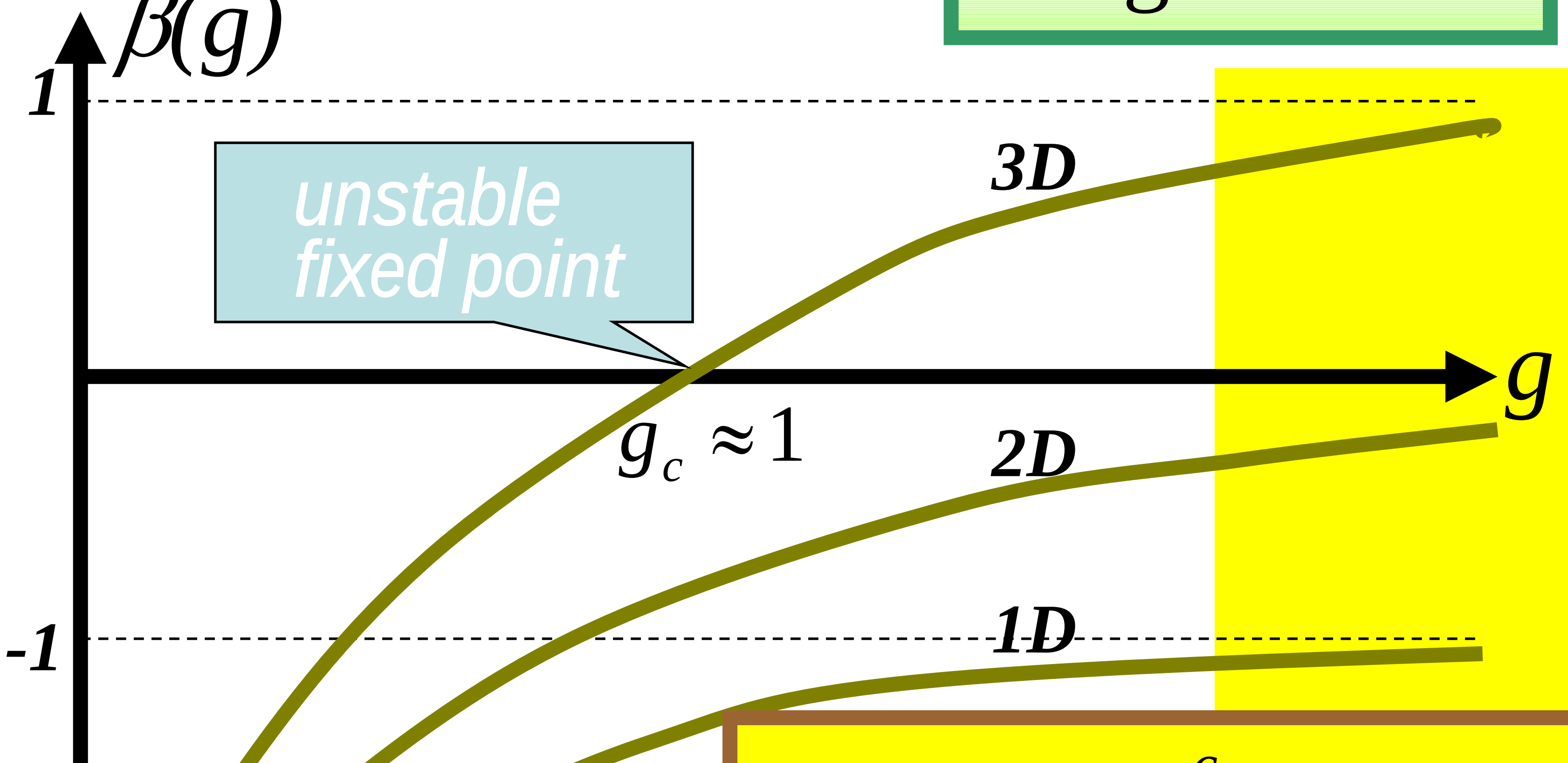
Einstein: there is no diffusion at too short
scales – there is memory, i.e.,
the process is not marcovian.

Why there is memory at large distances
in quantum case ?

Quantum corrections at large Thouless
conductance – weak localization
Universal description

β - function

$$\frac{d\log g}{d\log L} = \beta(g)$$



$$\beta(g) = d - 2 + \frac{c_d}{g}$$

$$c_d = ? \quad \pm ?$$

$$g(L) = \sigma_{cl} L^{d-2} - \begin{cases} \dfrac{c_d}{d-2} & d \neq 2 \\ c_2 \log\left(\dfrac{L}{l}\right) & d = 2 \end{cases}$$