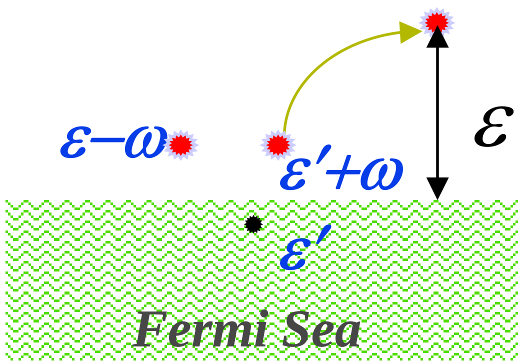
Quasiparticle decay rate at $T = 0$ in a clean Fermi Liquid.



$$\frac{\hbar}{\tau_{e-e}(\varepsilon)} \propto \begin{cases} \varepsilon^2/\varepsilon_F & d=3 \\ \left(\varepsilon^2/\varepsilon_F\right)\log\left(\varepsilon_F/\varepsilon\right) & d=2 \\ \varepsilon & d=1 \end{cases}$$

Conclusions:

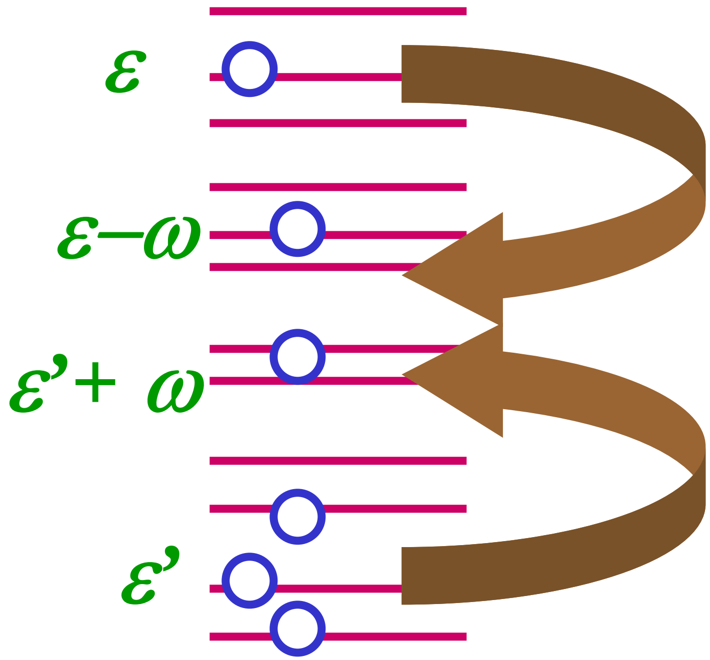
1. For d=3,2 from $\varepsilon << \varepsilon_F$ it follows that $\varepsilon\tau_{e\text{-}e} >> h$, i.e., that the qusiparticles are well determined and the Fermi-liquid approach is applicable.

2. For d=1 $\varepsilon\tau_{e\text{-}e}$ is of the order of h, i.e., that the Fermi-liquid approach is not valid for $1d$ systems of interacting fermions.

Luttinger liquids

Quasiparticle relaxation rate in 0D case

T=0



Offdiagonal matrix element

$$M\left(\omega, \varepsilon, \varepsilon'\right) \sim \frac{\delta_1}{g} << \delta_1$$

Quasiparticle relaxation rate in 0D case

T=0

Fermi Golden Rule

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \sum_{0<\omega<\varepsilon} \sum_{-\omega<\varepsilon'<0} \frac{\left|M(\omega,\varepsilon,\varepsilon')\right|^2}{\delta_1}$$

0D **case:** $L < L_\varepsilon$, i.e., $\varepsilon < E_T$

$$L_\varepsilon = \sqrt{\frac{hD}{\varepsilon}}$$

- $M \propto \delta_1(L)/g$
- Each $\sum$ gives $\approx \dfrac{\varepsilon}{\delta_1}$

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \delta_1 \left(\frac{\varepsilon}{E_T}\right)^2$$

(U.Sivan, Y.Imry & A.Aronov,1994)

Quasiparticle relaxation rate in disordered conductors

$T=0$

Fermi Golden Rule

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \sum_{0<\omega<\varepsilon} \sum_{-\omega<\varepsilon'<0} \frac{\left|M(\omega,\varepsilon,\varepsilon')\right|^2}{\delta_1}$$

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$$L_\varepsilon = \sqrt{\frac{hD}{\varepsilon}}$$

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \delta_1 \left(\frac{\varepsilon}{E_T}\right)^2$$

$d>0$ case: $L > L_\varepsilon$, i.e., $\varepsilon > E_T$

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \delta_1 (L_\varepsilon)$$

At $L \approx L_\varepsilon$ the rate is of the order of the mean level spacing δ_1. It should not change, when we keep increasing the system size, i.e. decreasing the Thouless energy

Quasiparticle relaxation rate in disordered conductors

T=0

Fermi Golden Rule

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \sum_{0<\omega<\varepsilon} \sum_{-\omega<\varepsilon'<0} \frac{\left|M(\omega,\varepsilon,\varepsilon')\right|^2}{\delta_1}$$

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d>0 **case:** $L > L_\varepsilon$, *i.e.*, $\varepsilon > E_T$

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \delta_1 (L_\varepsilon)$$

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \frac{\varepsilon}{g(L_\varepsilon)} << \varepsilon$$

A. Schmid 1973
B.A. & A.Aronov 1979

Matrix elements at large, $\varepsilon,\omega >> E_T$, energies

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \sum_{0<\omega<\varepsilon} \sum_{-\omega<\varepsilon'<0} \frac{\left|M(\omega,\varepsilon,\varepsilon')\right|^2}{\delta_1}$$

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \frac{\varepsilon}{g(L_\varepsilon)} << \varepsilon$$

$$\left|M(\omega,\varepsilon,\varepsilon')\right|^2 \propto \frac{\delta_1(L)^3\,\delta_1(L_\omega)}{\omega^2} \propto \frac{\omega^{-2+d/2}}{D^{d/2}}$$

$$\cancel{\frac{\left|M(\omega,\varepsilon,\varepsilon')\right|^2}{\delta_1(L)} \xrightarrow[\omega\to 0]{} \infty}$$

Quasiparticle relaxation rate in disordered conductors

T>0

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \sum_{0<\omega<\varepsilon} \; \sum_{-\omega<\varepsilon'<0} \frac{\left|M(\omega,\varepsilon,\varepsilon')\right|^2}{\delta_1}$$

T=0

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon,T)} \propto \sum_{\omega} \left(1-n_{\varepsilon-\omega}\right) \sum_{\varepsilon'} n_{\varepsilon'+\omega}\left(1-n_{\varepsilon'+\omega}\right) \frac{\left|M(\omega,\varepsilon,\varepsilon')\right|^2}{\delta_1}$$

T>0

$$n_\varepsilon = \left[\exp\frac{\varepsilon}{T} - 1\right]^{-1}$$

Fermi distribution function

$$\left|M(\omega,\varepsilon,\varepsilon')\right|^2 \propto \frac{\omega^{-2+d/2}}{D^{d/2}}$$

a) *T*=0 *-no problems:* $\sum_{\varepsilon'} \propto \omega$ *and* $\sum_{\omega}$ *converges*

b) *T*>0 *-a problem:* $\sum_{\varepsilon'} \propto T$ *and* $\sum_{\omega}$ *diverges !*

Abrahams, Anderson, Lee & Ramakrishnan 1981

$T>0$ -a problem: $1/\tau_{e\text{-}e}$ diverges

$$\frac{h}{\tau_{\text{e-e}}\left(\varepsilon,T\right)}\propto T\sum_{\omega}\frac{\left(1-n_{\varepsilon-\omega}\right)}{\omega^{2-d/2}D^{d/2}}$$

B.A., A.Aronov & D.E. Khmelnitskii (1983):

- Divergence of is not a catastrophe:
  $1/\tau_{e\text{-}e}$ has no physical meaning

- E.g., for energy relaxation of hot electrons processes with small energy transfer ω are irrelevant.

$$\boxed{\frac{h}{\tau_{\varepsilon}}\propto\frac{\varepsilon}{g\left(L_{\varepsilon}\right)}}$$

Q: Is it the energy relaxation rate that determines the applicability of the Fermi liquid approach ?

$T>0$ - a problem: $1/\tau_{e\text{-}e}$ diverges

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon,T)} \propto T\sum_{\omega}\frac{(1-n_{\varepsilon-\omega})}{\omega^{2-d/2}D^{d/2}}$$

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- Divergence of is not a catastrophe: $1/\tau_{e\text{-}e}$ has no physical meaning
- E.g., for energy relaxation of hot electrons processes with small energy transfer ω are irrelevant. $\Rightarrow$ $\frac{h}{\tau_\varepsilon} \propto \frac{\varepsilon}{g(L_\varepsilon)}$
- Phase relaxation: in a time t after a collision $\delta\varphi \approx (2\pi\,\omega\,t)/h \Rightarrow$ processes with energy transfer ω smaller than $1/\tau_\varphi$ are irrelevant.

$$\frac{h}{\tau_\varphi} \propto ?$$

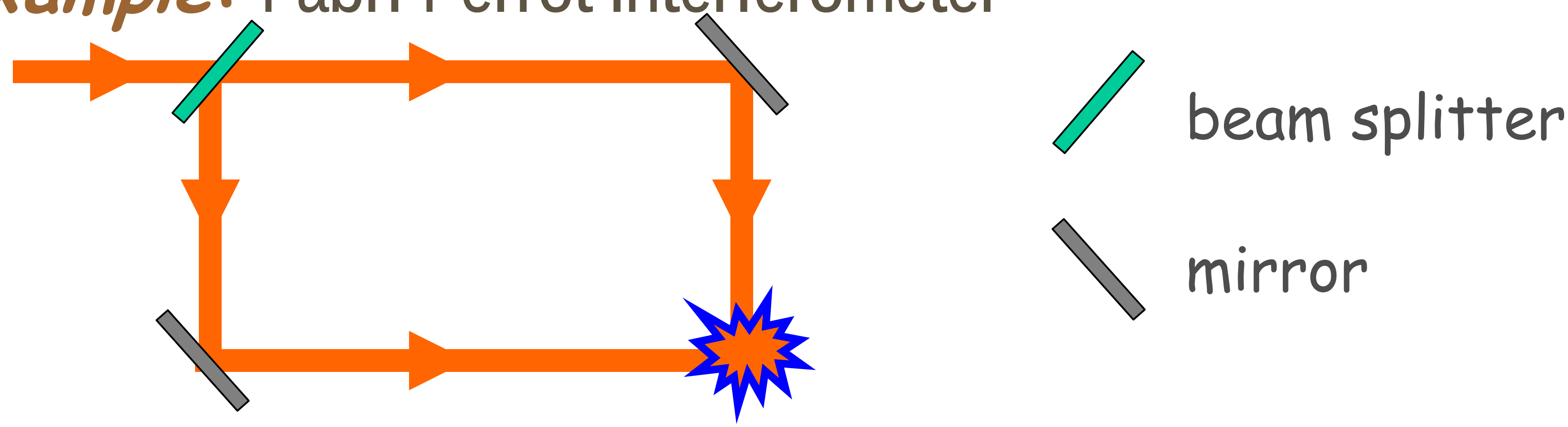
What is Dephasing?

1. Suppose that originally a system(an electron) was in a *pure* quantum state. It means that it could be described by a wave function with a given phase.

2. External perturbations can transfer the system to a different quantum state. Such a transition is characterized by its amplitude, which has a modulus and a phase.

3. The phase of the amplitude can be measured by comparing it with the phase of another amplitude of the same transition.

Example: Fabri-Perrot interferometer



4. Usually we can not control all of the perturbations. As a result, even for fixed initial and final states, the phase of the transition amplitude has a random component.

5. We call this contribution to the phase, $\delta\varphi$, random if it changes from measurement to measurement in an uncontrollable way.

6. It usually also depends on the duration of the experiment, t:

$$\delta\varphi = \delta\varphi(t)$$

7. When the time t is large enough, $\delta\varphi$ exceeds 2π, and interference gets averaged out.

8. Definitions:

$$\boxed{\delta\varphi(\tau_\varphi) \approx 2\pi}$$

τ_φ phase coherence time; $1/\tau_\varphi$ dephasing rate

Why is Dephasing rate important?

Imagine that we need to measure the energy of a quantum system, which interacts with an environment and can exchange energy with it.

Let the typical energy transferred between our system an the environment in time t be $\delta\varepsilon(t)$. The total uncertainty of an ideal measurement is

$$\Delta\varepsilon(t) \approx \delta\varepsilon(t) + \frac{\hbar}{t}$$

environment

quantum uncertainty

$$\delta\varepsilon(t) \xrightarrow[t\to\infty]{} \infty; \quad \frac{\hbar}{t} \xrightarrow[t\to 0]{} \infty$$

There should be an optimal measurement time $t=t^*$, which minimizes $\Delta\varepsilon(t)$:

$$\Delta\varepsilon(t^*) = \Delta\varepsilon_{min}$$

!

$$\delta\varepsilon(t^*) \approx \frac{\hbar}{t^*} \Rightarrow \delta\varphi(t^*) \approx 1 \Rightarrow \begin{array}{c} t^* \approx \tau_\varphi \\ \Delta\varepsilon_{\min} \approx \hbar/\tau_\varphi \end{array}$$

!

Why is Dephasing rate important?

$$\delta\varepsilon(t^*) \approx \frac{\hbar}{t^*} \Rightarrow \delta\phi(t^*) \approx 1 \Rightarrow \begin{matrix} t^* \approx \tau_\varphi \\ \Delta\varepsilon_{\min} \approx \hbar/\tau_\varphi \end{matrix}$$

It is dephasing rate that determines the accuracy at which the energy of the quantum state can be measured in principle.

$T>0$ – a problem: $1/\tau_{e\text{-}e}$ diverges

$$\frac{h}{\tau_{\text{e-e}}(\varepsilon,T)} \propto T\sum_{\omega}\frac{(1-n_{\varepsilon-\omega})}{\omega^{2-d/2}D^{d/2}}$$

B.A., A.Aronov & D.E. Khmelnitskii (1983):

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- Phase relaxation: in a time t after a collision
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  processes with energy transfer ω smaller than $1/\tau_{\varphi}$ are irrelevant.

$$\frac{h}{\tau_{\varphi}(\varepsilon,T)} \propto T\sum_{\omega>h/\tau_{\varphi}}\frac{(1-n_{\varepsilon-\omega})}{\omega^{2-d/2}D^{d/2}}$$

e-e interaction – Electric noise

Fluctuation- dissipation theorem:

Electric noise - randomly time and space - dependent electric field $E^{\alpha}\left(\vec{r},t\right) \Leftrightarrow E^{\alpha}\left(\vec{k},\omega\right)$
Correlation function of this field is completely determined by the conductivity $\sigma\left(\vec{k},\omega\right)$ **:**

$$\left\langle E^{\alpha}E^{\beta}\right\rangle_{\omega,\vec{k}} = \frac{\omega}{\sigma_{\alpha\beta}\left(\omega,\vec{k}\right)}\coth\left(\frac{\omega}{2T}\right)\frac{k_{\alpha}k_{\beta}}{k^{2}} \propto \frac{T}{\sigma_{\alpha\beta}\left(\omega,\vec{k}\right)}$$

Noise intensity increases with the temperature, T, and with resistance

g

$$\left\langle E^{\alpha} E^{\beta} \right\rangle_{\omega,\vec{k}} = \frac{\omega}{\sigma_{\alpha\beta}\left(\omega,\vec{k}\right)} \coth\left(\frac{\omega}{2T}\right) \frac{k_{\alpha} k_{\beta}}{k^{2}} \propto \frac{T}{\sigma_{\alpha\beta}\left(\omega,\vec{k}\right)}$$

$g(L) \equiv \dfrac{h}{e^{2} R(L)}$ *- Thouless conductance – def.*

$R(L)$ *- resistance of the sample with* $\left\{ \begin{array}{c} \textit{length (1d)} \\ \textit{area (2d)} \end{array} \right\}$ L

$$\boxed{\frac{1}{\tau_{\varphi}} \propto \frac{T}{g\left(L_{\varphi}\right)}}$$

$L_{\varphi} \equiv \sqrt{D\tau_{\varphi}}$ *- dephasing length*

D *- diffusion constant of the electrons*

$$\boxed{\frac{1}{\tau_{\varphi}} \propto \frac{T}{g\left(L_{\varphi}\right)}}$$

This is an equation!

$$\boxed{\frac{1}{\tau_\varphi} \propto \frac{T}{g\left(L_\varphi\right)}}$$

This is an equation!

$$g\left(L\right) \propto L^{d-2}$$

$$L_\varphi \propto \sqrt{\tau_\varphi}$$

where is the number of dimensions:

d=1 *for wires;* **d=2** *for films, ...*

$$L_\varphi \propto T^{-1/\left(4-d\right)}$$

$$\boxed{\tau_\varphi \propto T^{-2/\left(4-d\right)} \propto \begin{matrix} T^{-1} & d=2 \\ T^{-2/3} & d=1 \end{matrix}}$$

$$\boxed{\frac{1}{\tau_\varphi} \propto \frac{T}{g\left(L_\varphi\right)} \qquad L_\varphi \equiv \sqrt{D\tau_\varphi}}$$

Fermi liquid is valid (one particle excitations are well defined), provided that

$$T\tau_\varphi\left(T\right) > \hbar$$

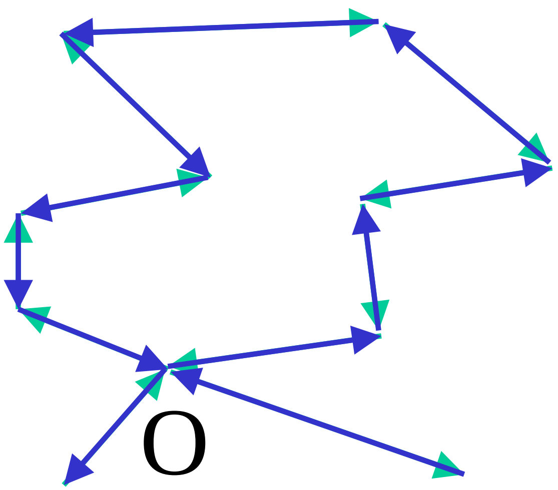
$$\boxed{\frac{1}{\tau_\varphi} \propto \frac{T}{g\left(L_\varphi\right)} \qquad L_\varphi \equiv \sqrt{D\tau_\varphi}}$$

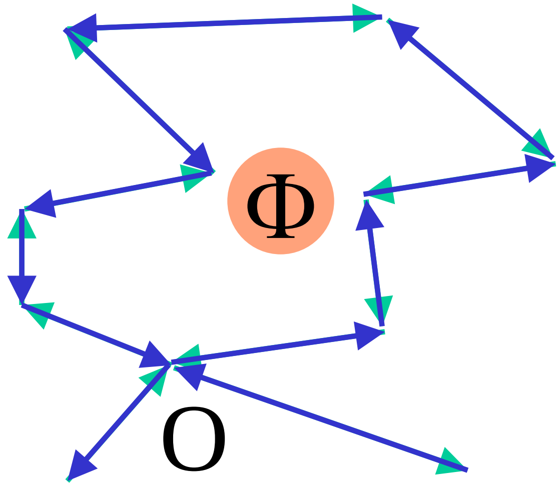
Fermi liquid is valid (one particle excitations are well defined), provided that

$$T\tau_\varphi\left(T\right) > \hbar$$

1. *In a purely* ***1d*** *chain,* $g \leq 1$*, and, therefore, Fermi liquid theory is never valid.*

Magnetoresistance





No magnetic field

$$\varphi_1 = \varphi_2$$

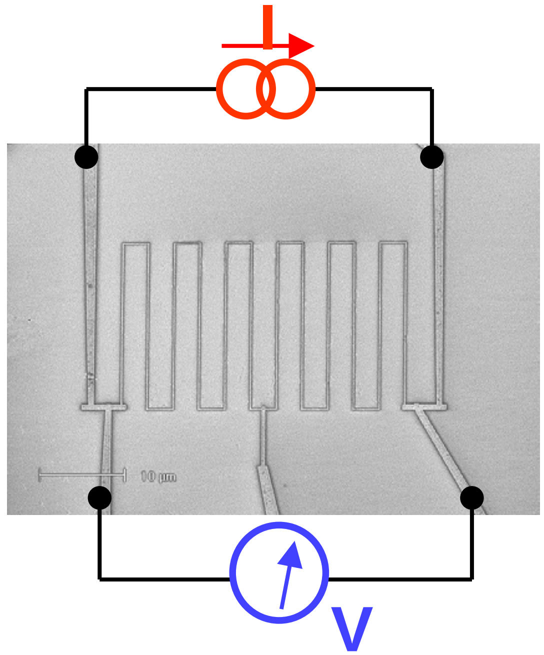
With magnetic field H

$$\varphi_1 - \varphi_2 = 2 * 2\pi\, \Phi / \Phi_0$$

$\Phi = HS$ - ***magnetic flux through the loop***

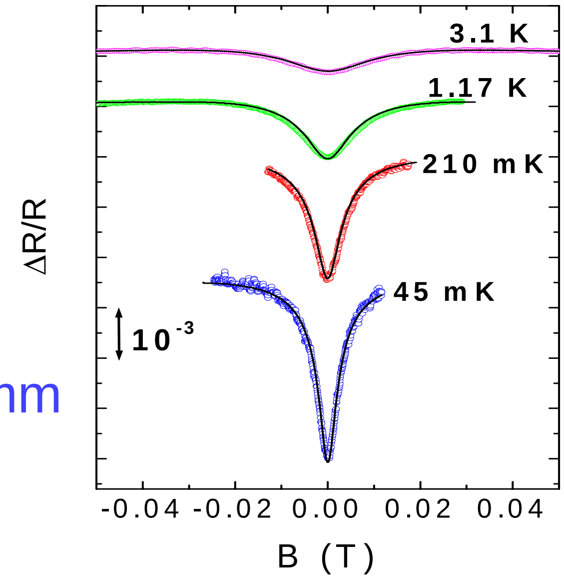
$\Phi_0 = hc/e$ - ***flux quantum***

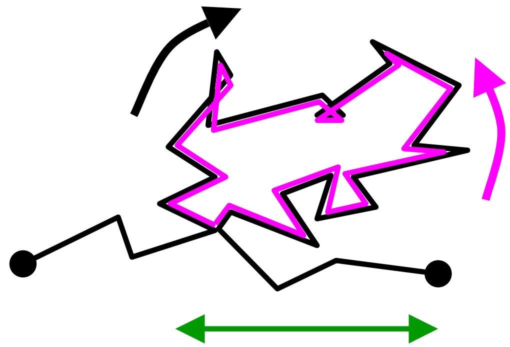
Weak Localization, Magnetoresistance in Metallic Wires



⊙ **B**

L ~ 0.25 mm





$$L_\phi = \sqrt{D\tau_\phi}$$

1d case; strong spin-orbital coupling

$$\frac{\Delta R}{R} = -\frac{h}{e^2 L} R \sqrt{\frac{1}{L_\varphi^{\ 2}} + \frac{1}{3}\left(\frac{A}{L_H}\right)^2}$$

$$L_H = \sqrt{\frac{h}{eH}}$$

A – area of the wire cross-section

Can we always reliably extract the inelastic dephasing rate from the experiment ?

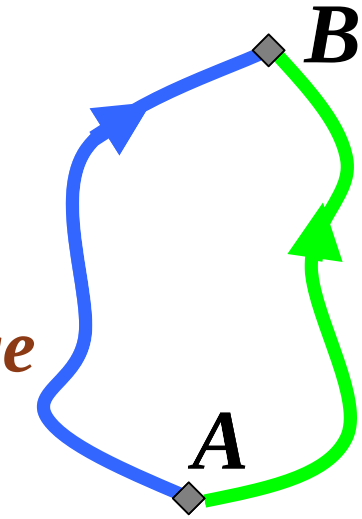
Weak localization:

NO - ***everything that violates T-invariance will destroy the constructive interference***

EXAMPLE: ***random quenched magnetic field***

Mesoscopic fluctuations:

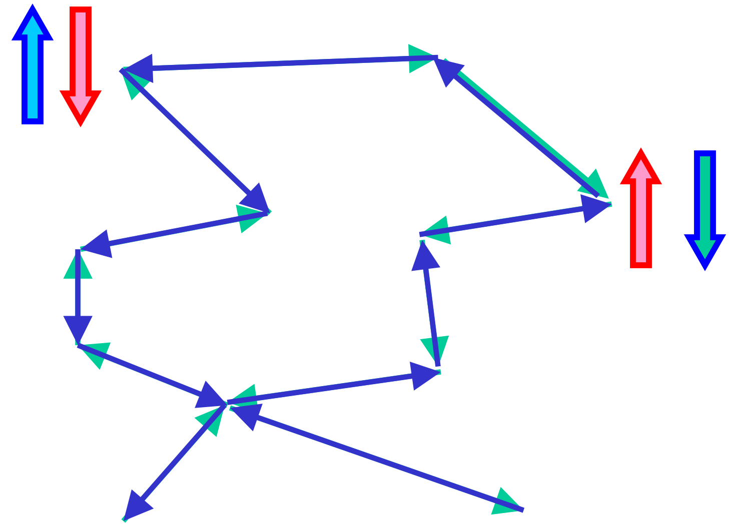
YES - ***Even strong magnetic field will not eliminate these fluctuations. It will only reduce their amplitude by factor 2.***



But

Slow diffusion of the impurities will look as dephasing in mesoscopic fluctuations measurements

Magnetic Impurities



$\Uparrow$ - before $\Uparrow$ - after

T-invariance is clearly violated, therefore we have dephasing

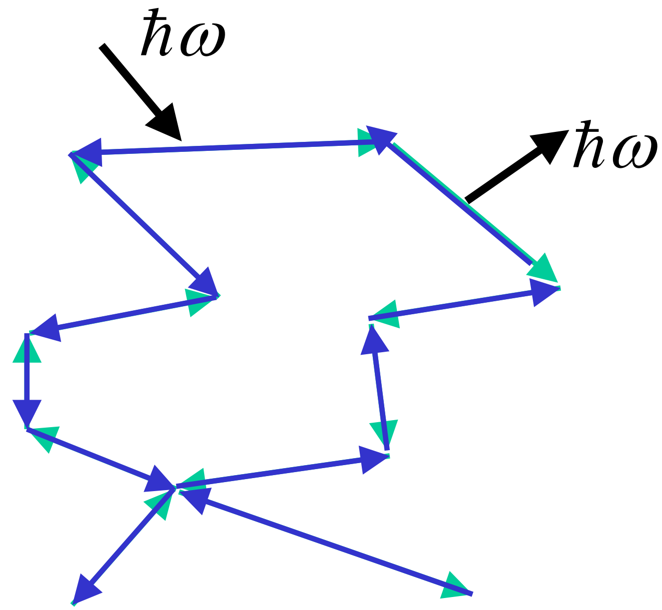
Mesoscopic fluctuations

Magnetic impurities cause dephasing only through effective interaction between the electrons.

$T \to 0$ ***Either Kondo scattering or quenching due to the RKKY exchange.***

In both cases no "elastic dephasing"

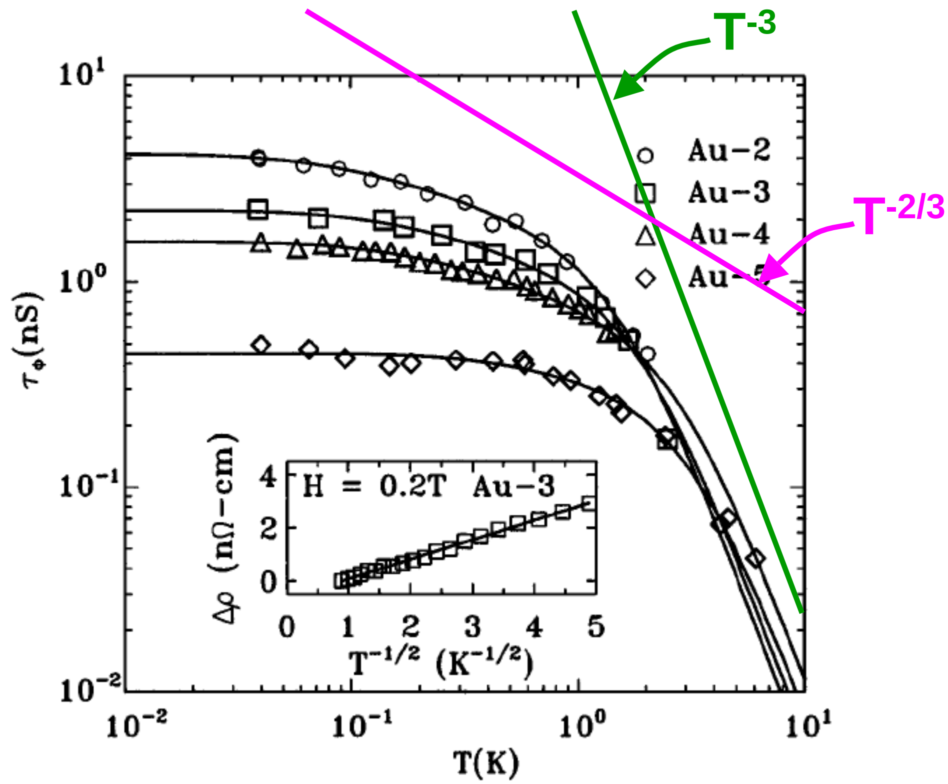
Inelastic dephasing rate $1/\tau_\varphi$ can be separated at least in principle



- other electrons
- phonons
- magnons
- two level systems
-
-

THE EXPERIMENTAL CONTROVERSY

Mohanty, Jariwala and Webb, **PRL 78, 3366 (1997)**

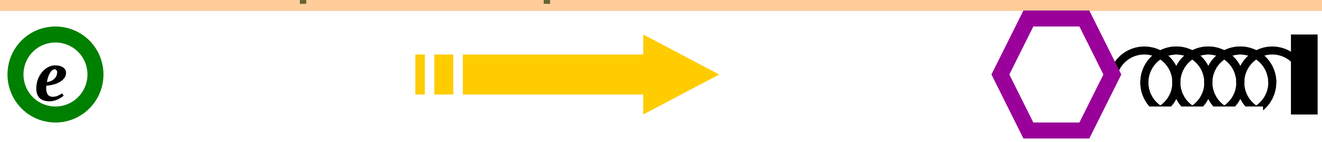


Saturation of τ_φ:

Artifact of measurement ?
Real effect in samples ?

Zero-point Oscillations

Collision between the quantum particle and a harmonic oscillator



ε - energy counted from the Fermi level

$$E_n = \hbar\omega\left(n + \tfrac{1}{2}\right)$$

1. $T > \omega$

$\Downarrow$

$\varepsilon > \omega;\ n > 0$

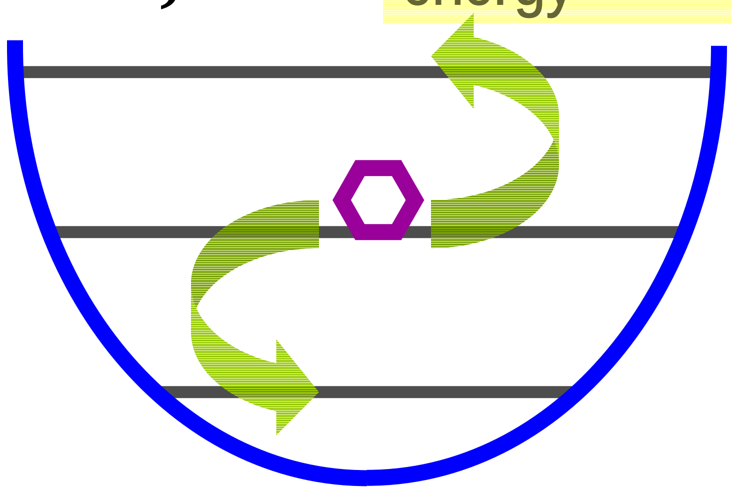
The particle and the oscillator can exchange energy

2. $T << \omega$

$\Downarrow$

$\varepsilon << \omega;\ n = 0$

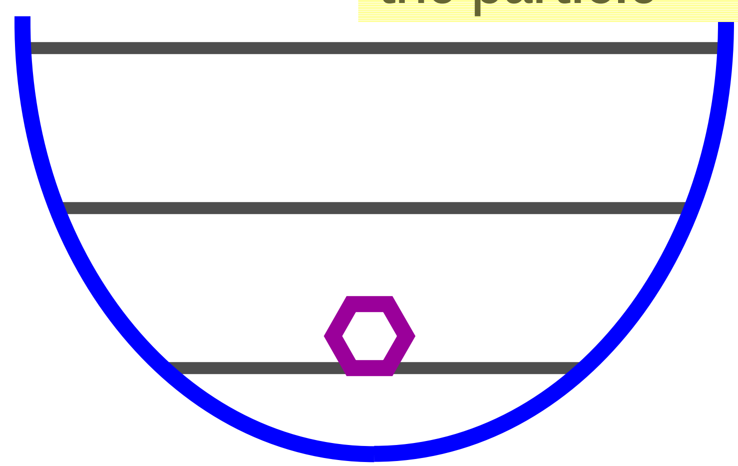
No energy exchange between the oscillator and the particle



n=2

n=1

n=0

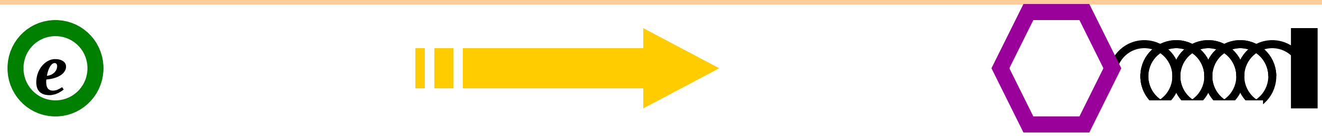


Inelastic scattering → **dephasing**

Pure elastic scattering → **No dephasing**

Zero-point Oscillations

Collision between the quantum particle and a harmonic oscillator



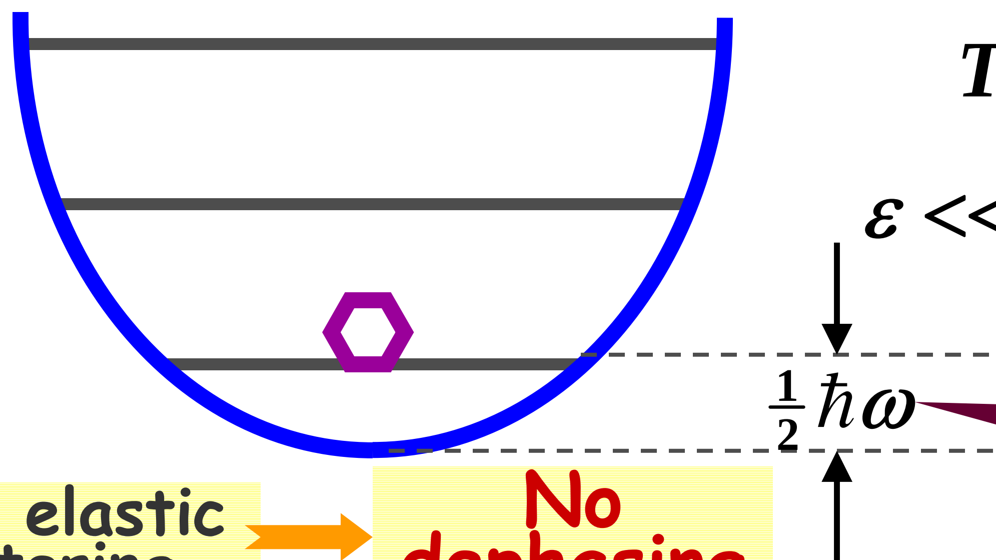
ε - energy counted from the Fermi level

$$E_n = \hbar\omega\left(n + \tfrac{1}{2}\right)$$

$T << \omega$

$\varepsilon << \omega;\ n = 0$

No energy exchange between the oscillator and the particle



Zero-point oscillations

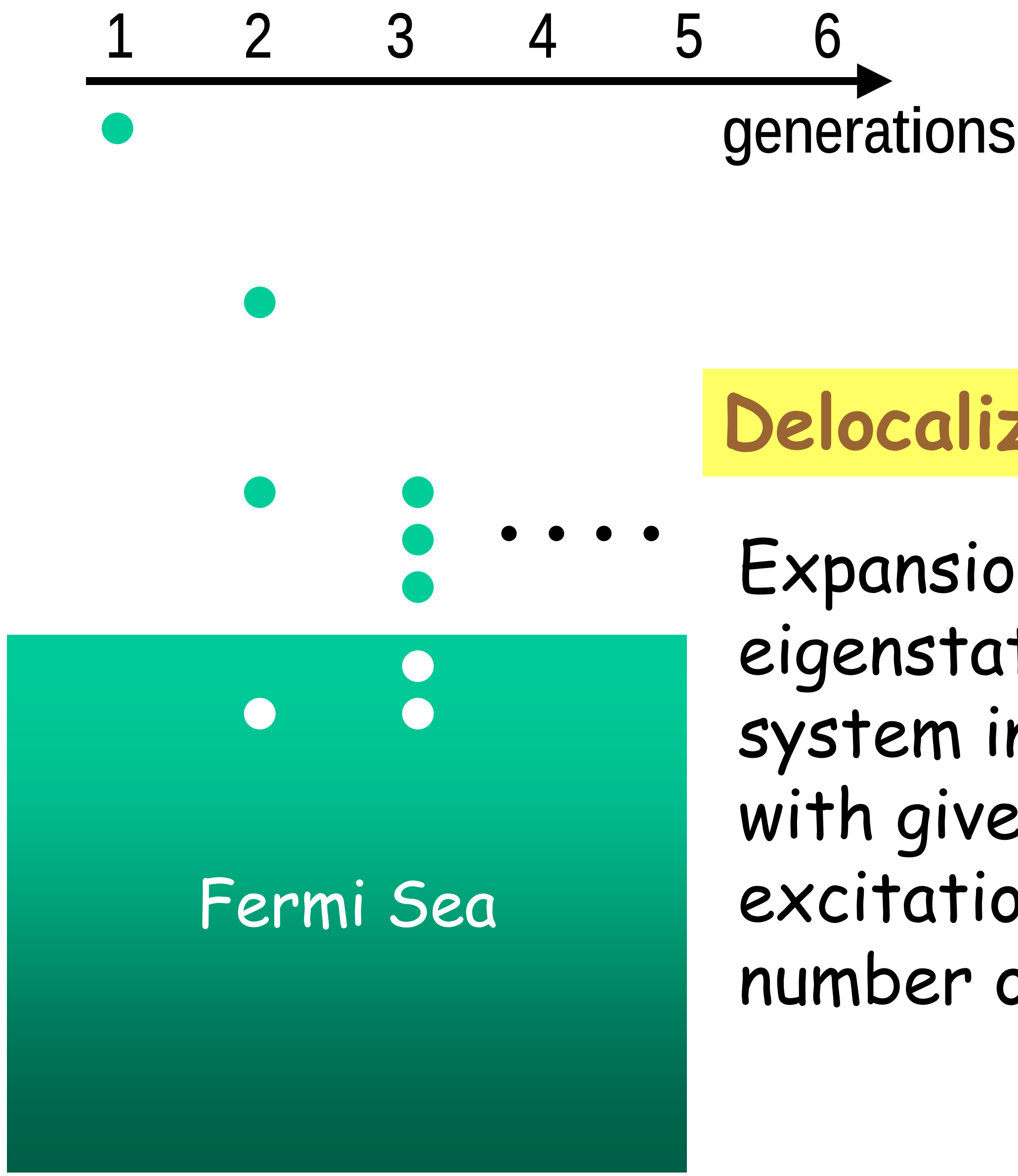
Pure elastic scattering → No dephasing

Chaos in Nuclei – Delocalization

Without interactions between fermions energy of each of the particles is conserved, i.e., there are as many "integrals of motion" as there are excited particles.

For a finite Fermi Gas one should expect Poisson situation for the eigenstates of the whole system.

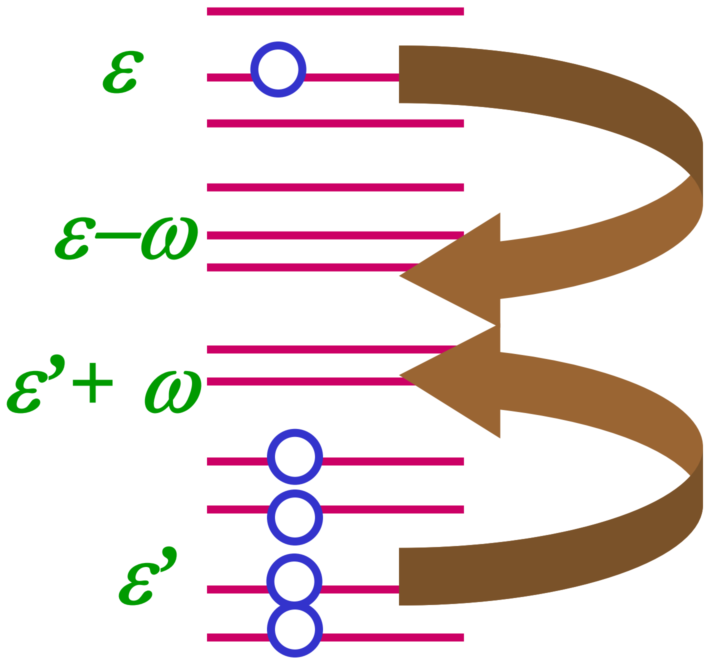
Chaos in Nuclei – Delocalization



Delocalization in Fock space

Expansion of a typical eigenstate of the many-body system in the basis of states with given number of excitations involves a large number of terms

Quasiparticle relaxation rate in 0D case



$$\frac{h}{\tau_{\text{e-e}}(\varepsilon)} \propto \delta_1 \left(\frac{\varepsilon}{E_T}\right)^2$$

(U.Sivan, Y.Imry & A.Aronov,1994)

Becomes incorrect as soon as provided that

$$\varepsilon < \sqrt{E_T \delta_1} = \delta_1 \sqrt{g} \gg \delta_1$$

(B.A, Y.Gefen. A Kamenev & L.Levitov,1994)