Quantum Shot noise

probing interactions and magic

properties of the Fermi sea

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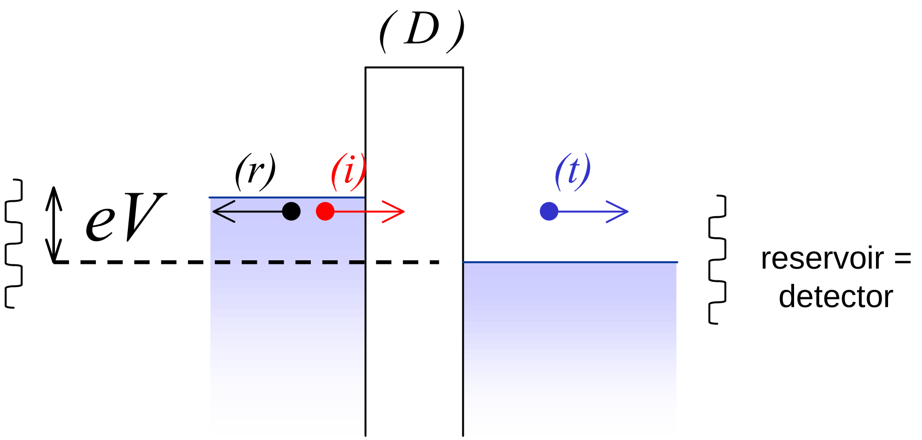
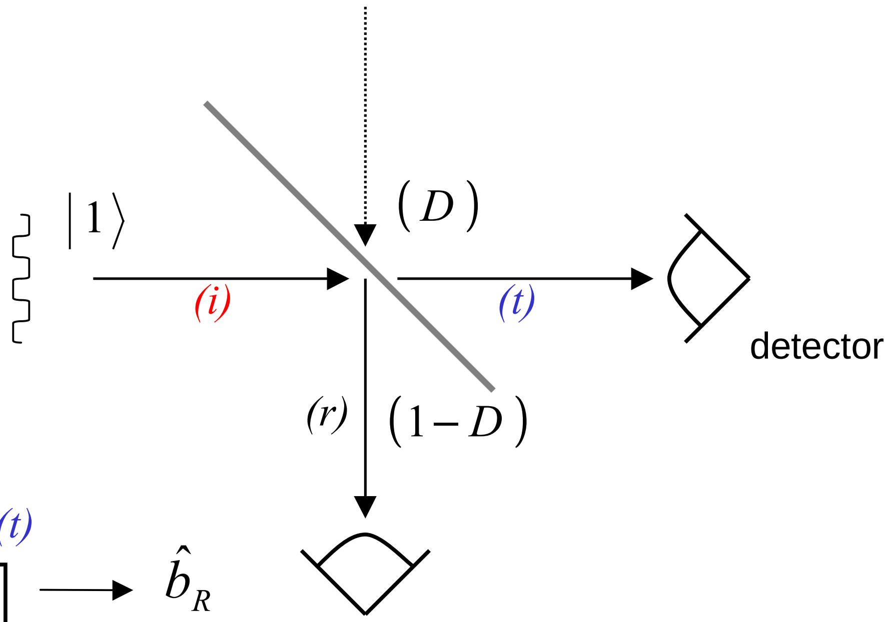
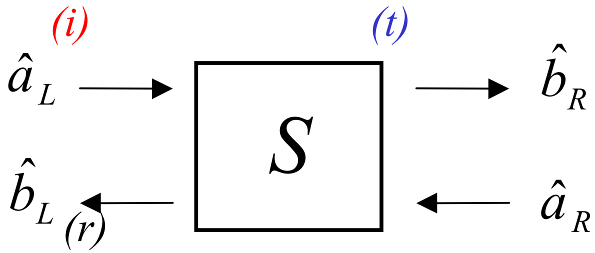
INTRODUCTION

(disclaimer : these are just notes and some references to original works may be missing)

WHY STUDY SHOT NOISE ?







electrons :

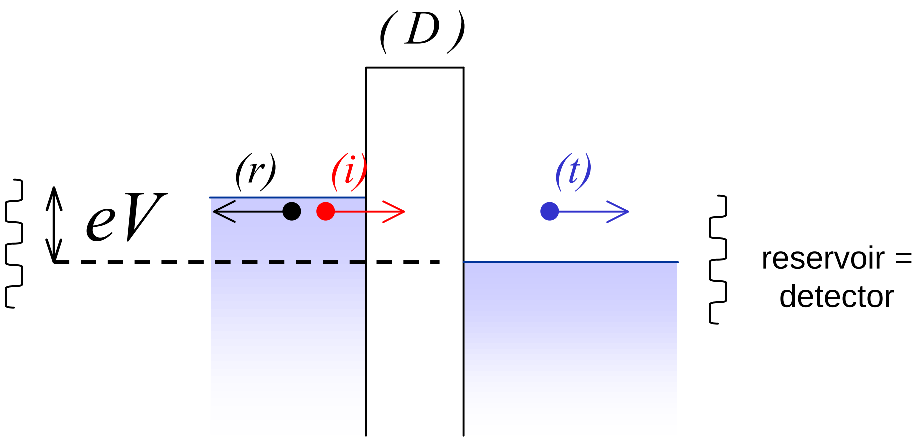
photons:

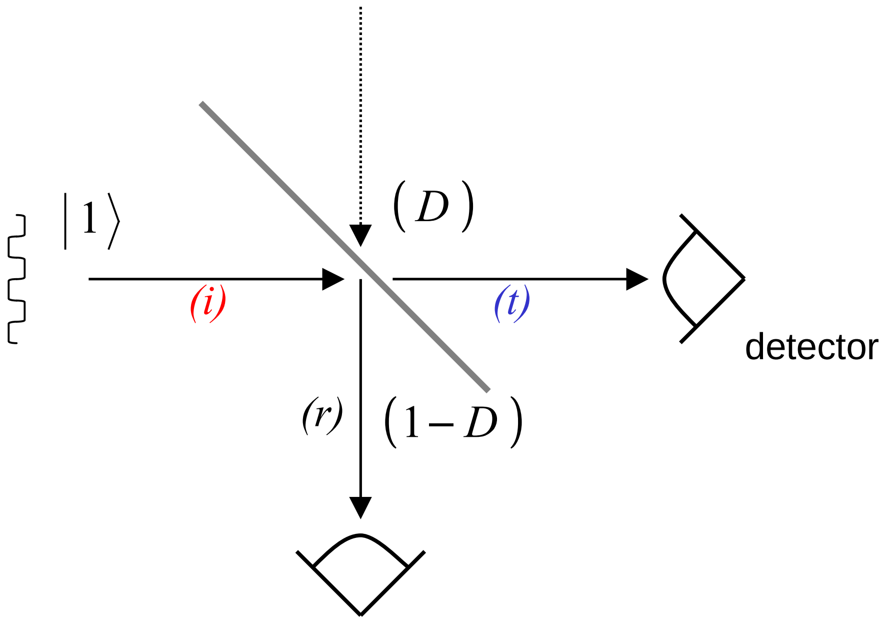
electrons interference

(wave)
scattering description

light interference

from de 70's to 90's (and beyond) mesoscopic physics addressed *single particle* coherence properties via *conductance* measurements





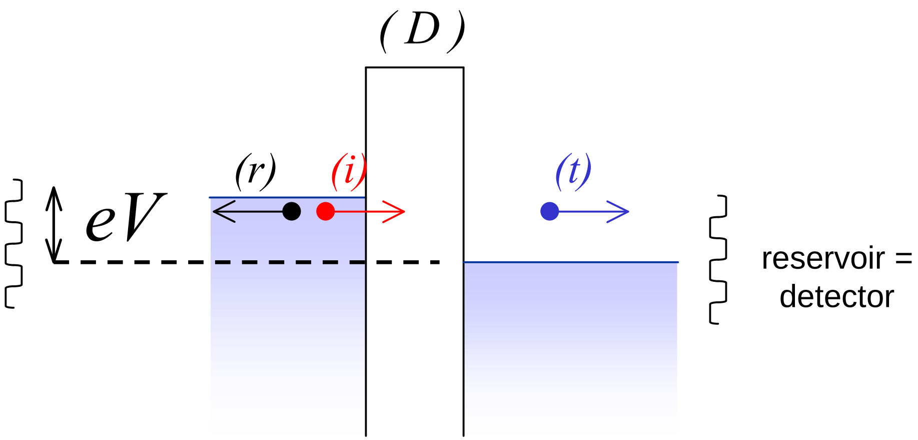
electrons :

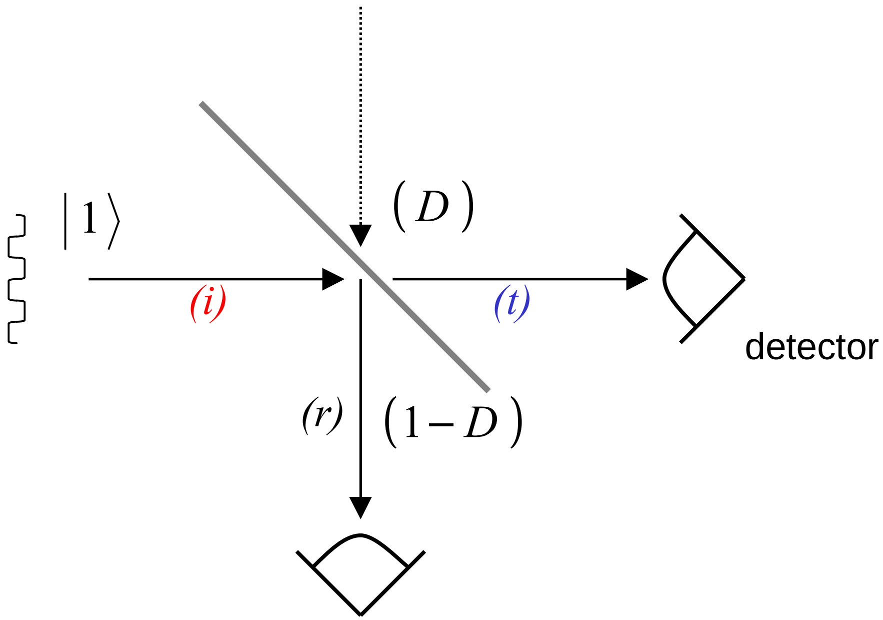
photons:

electrons interference (wave) light interference

shot noise (particle) photon noise

while in the 60's, optics addressed *two* photons properties (Hanbury-Brown Twiss correlations)
this is only beginning of the 90's (mid 90's for experiments) that two-electron correlations where considered via *quantum shot noise*





electrons : *photons:*

electrons interference	(wave)	light interference
shot noise	(particle)	photon noise

different quantum noise results for different quantum statistics (Bose versus Fermi)
Fermi sea gives noiseless electron generation while photons are fundamentally noisy

electronic quantum shot noise studies revealed yet unregarded beautiful properties of the Fermi sea

the magic Fermi sea

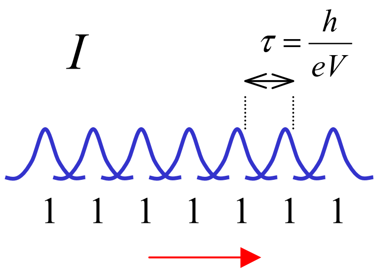
$$dI_{el.} = \frac{e}{h} d\varepsilon \quad \text{or} \quad d\dot{N}_{el.} = \frac{1}{h} d\varepsilon$$

$\Rightarrow$ quantum of conductance : $\dfrac{e^2}{h}$

$$d\dot{N}_{ph.} = \frac{N_{occ.}}{h} d\varepsilon \quad \text{or} \quad dI_{ph.} = (h\nu) N_{occ.} d\nu$$

(no equivalent, no known continuous generation of photon number states)

$$I = e.\frac{eV}{h}$$

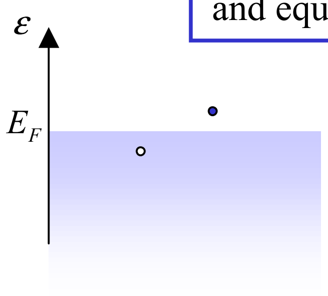


noiseless injection of electrons

noiseless electrons

electron entanglement
using linear ' electron optics '
and equilibrium thermodynamic sources

noiseless electrons may
generate low noise photons



ground state : $|0\rangle$

plus excitations : $c^{+}_{\varepsilon > E_F}\, c_{\varepsilon < E_F} |0\rangle$

Electron-hole pairs are naturally entangled.

Fermi sea rate of entanglement production eV/4h

more with shot noise :

current spectral density :

$$S_I(f) = \left\langle \Delta I^2 \right\rangle / \Delta f$$

proportional to the *charge* of the quasi-particle carrying current (...but only in the Poissonian regime)

$$\boxed{S_I = 2\,q\,I}$$

$q = e$ already in the 20's attempt to determine the electron charge in vacuum diodes (but less accurate that Millikan's experiments, due to space charge effect)

(repulsive interactions reduce shot noise)

$q = e\,/\,3$ in 1997, the Laughlin fractional charge of the Fractional Quantum Hall Effect was unambiguously established via shot noise. The last (but not least) proof definitely establishing the FQH effect.

later :

q = 2e the Cooper pair charge observed at mesoscopic superconducting-normal interfaces.

Future :

q = g e in Luttinger liquids, such as long single wall carbon nanotubes (requires f > THz)

Introduction

I. Electronic scattering (a brief introduction)

1- one ideal mode (+ electron flux/photon flux analogy)
2- one mode scattering
3- multi-mode scattering: Landauer formula
4- multi-terminal Büttiker formula (quantum Kirchhoff law)
5- chiral transport with QHE edge states

II. Quantum Shot noise

1 - Quantum partition noise
- one and two particle partitioning :electrons/ photons
- electronic shot noise

2- scattering derivation of quantum shot noise
a- S(ω) for an ideal one mode conductor
b- quantum shot noise for a single mode
c-zero frequency shot noise and multimode case

3- experimental examples
4- current noise cross-correlations

III Shot Noise and Interactions:

1. Fractional Quantum Hall effect
2.. Superconducting-Normal mesoscopic interfaces
3. Interactions in a QPC : 0.7 structure

IV. Shot noise: ***the*** tool to detect entanglement

V. Shot noise and high frequencies

Introduction

I. Electronic scattering (a brief introduction)

II. Quantum Shot noise

III Shot Noise and Interactions:

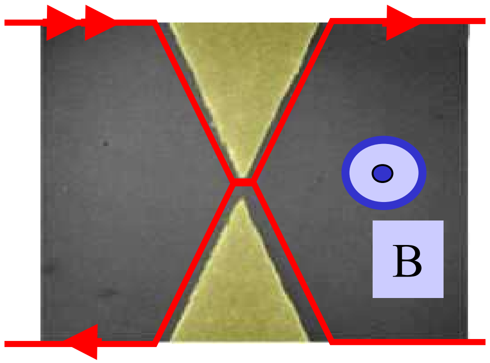
IV. Shot noise: ***the*** tool to detect entanglement

1. Entanglement with the Fermi statistics
2. Coincidence measurements using shot noise correlations

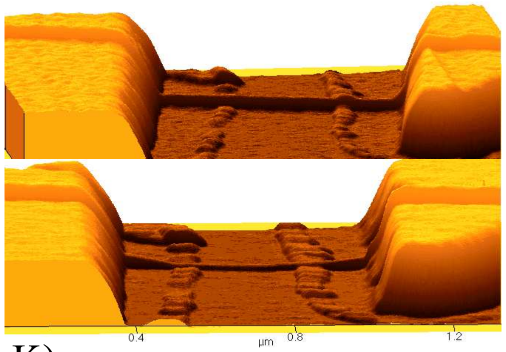
V. Shot noise and high frequencies

1. Photo-assisted Shot Noise
2. High frequency Shot Noise
3. Photon Noise emitted by a Conductor

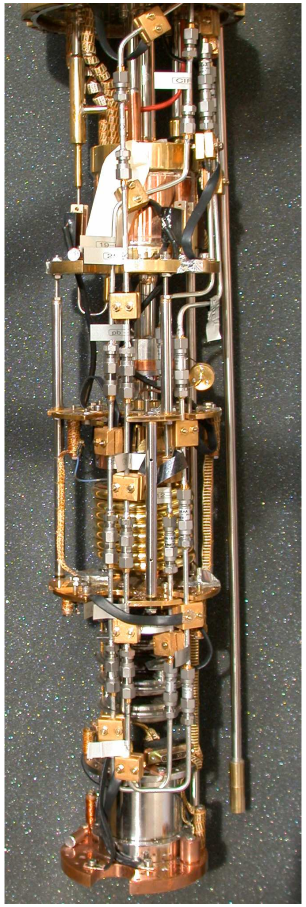
Conclusion



- quantum point contact : shot noise, edge states, co-tunneling of Q.-Dots
- ballistic qubits
- mesoscopic capacitor
- carbone nanotube
- Fractional Quantum Hall effect



- high frequency (40 GHz)
- ultra low noise measurements
- high magnetic field (18T) and low T (20mK)
- lithography
- cryo-electronics



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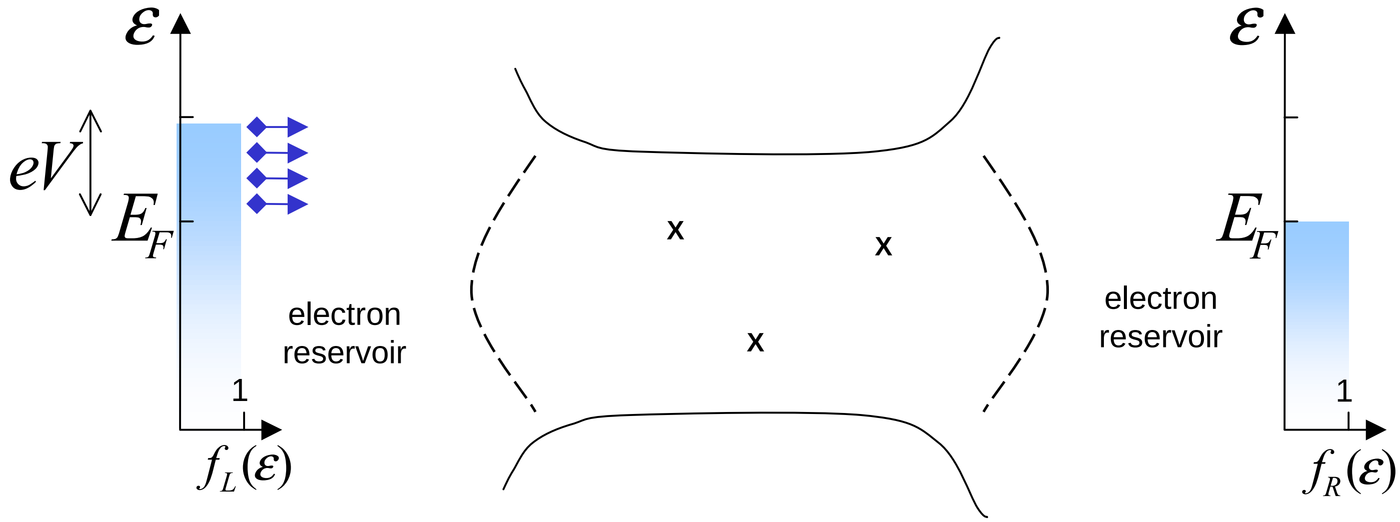
II. Quantum Shot noise

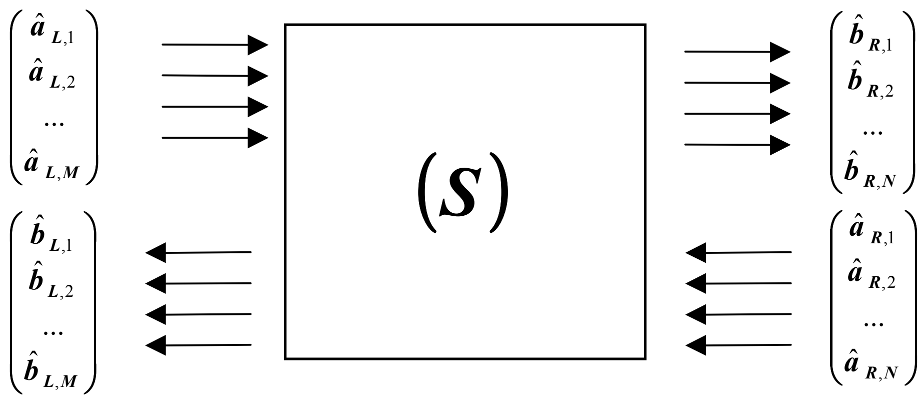
IV. Shot noise: ***the*** tool to detect entanglement

V. Shot noise and high frequencies

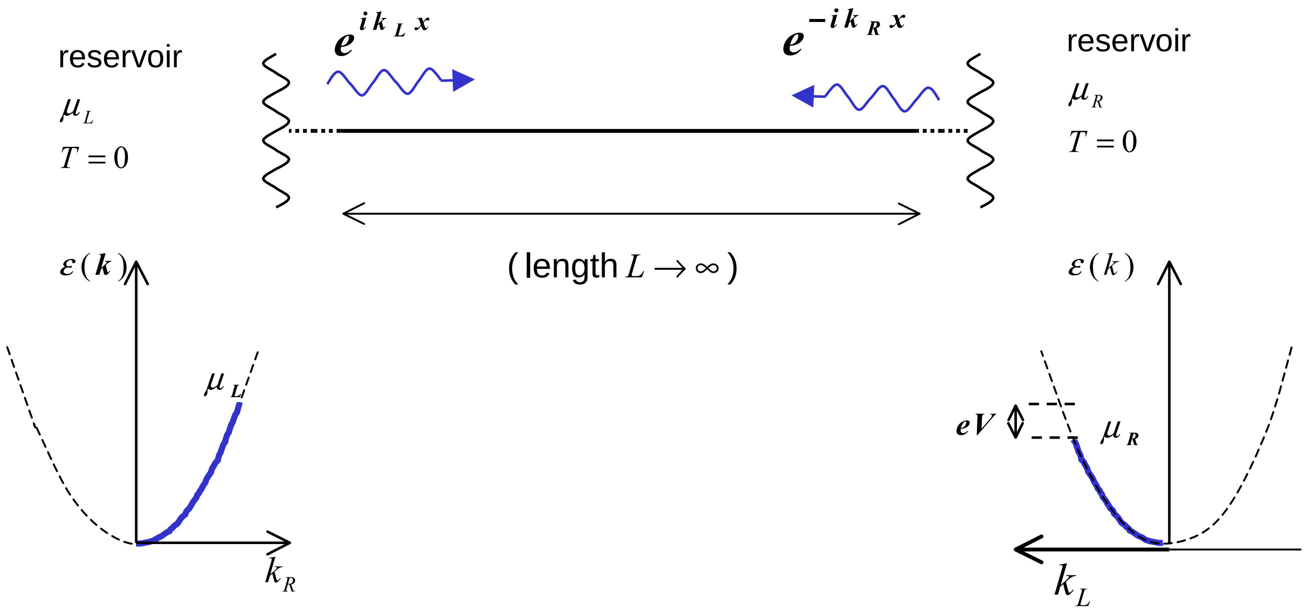


electron reservoirs = - "black-body sources" of electrons
- emit electrons according to Fermi distribution
- absorb perfectly all electrons

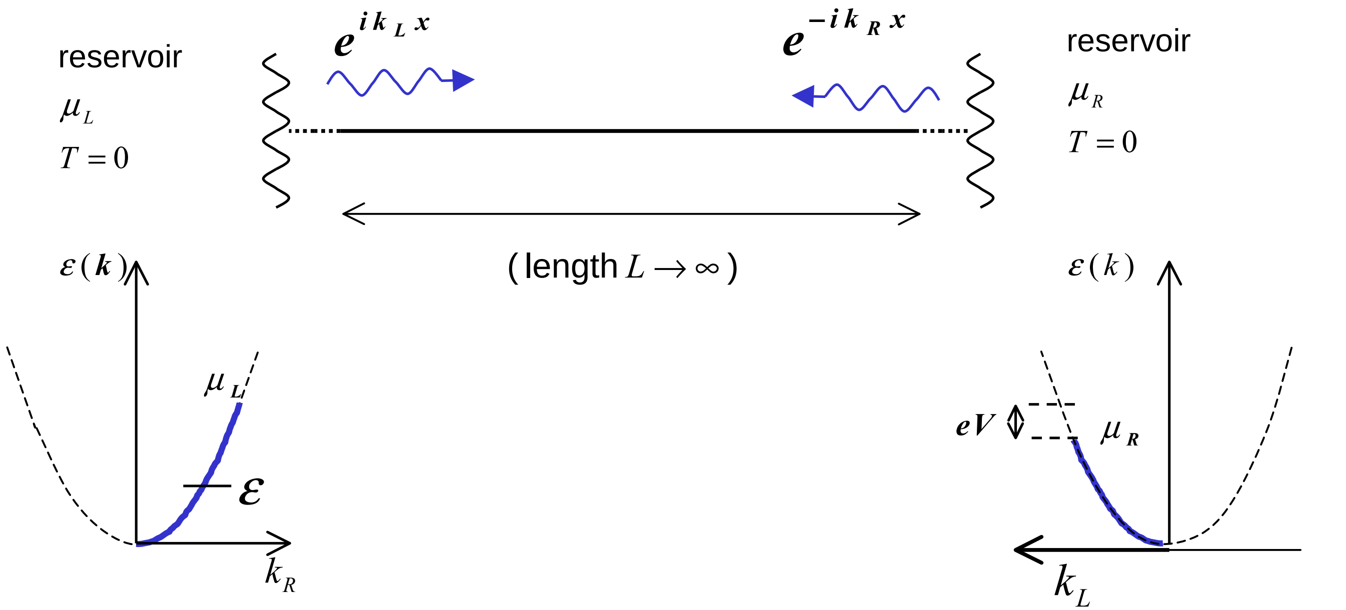
incoming electron (i) $\rightarrow$ superposition of transmitted (t) and reflected (r) state



I.1 the ideal 1D wire (… or one mode conductor)



$$k_{n,L(R)} = n\frac{2\pi}{L} \quad \text{and} \quad \varepsilon_n = \frac{\hbar^2 k_n^{\,2}}{2m} \quad , \quad \varphi_{\varepsilon_n,L(R)}(x) = \frac{1}{\sqrt{L}} e^{\pm i k_{L(R)} x}$$



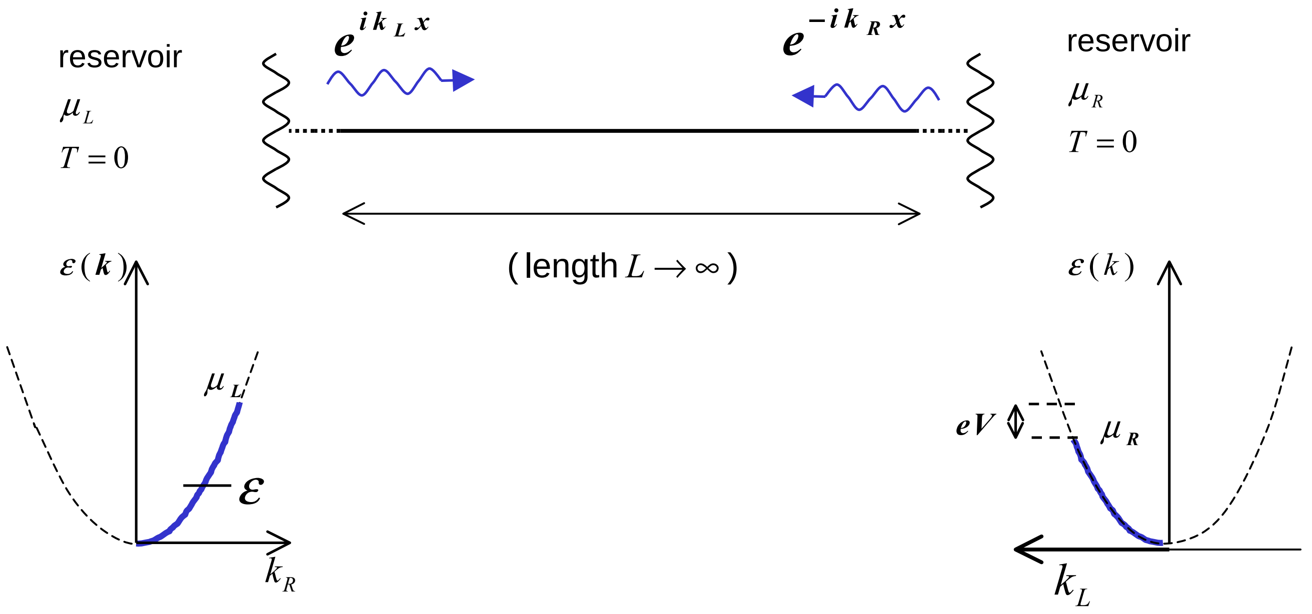
$$k_{L(R)}(\varepsilon) \quad \text{and} \quad \varepsilon = \frac{\hbar^2 k_{L(R)}{}^2}{2m} \quad , \quad \varphi_{\varepsilon,L(R)}(x) = \frac{1}{\sqrt{2\pi\hbar v_{L(R)}(\varepsilon)}} e^{\pm i k_{L(R)}(\varepsilon) x}$$

$$\boxed{\left\langle \varphi_{\varepsilon',\beta} \middle| \varphi_{\varepsilon,\alpha} \right\rangle = \delta_{\alpha,\beta}\,\delta(\varepsilon' - \varepsilon)}$$

density of state
per unit of energy
per unit length

$$\boxed{D(\varepsilon) = \frac{1}{2\pi\hbar v(\varepsilon)}}$$

energy is a natural parameter for elastic scattering.
(… and all results are simpler)



$$k_{L(R)}(\varepsilon) \quad \text{and} \quad \varepsilon = \frac{\hbar^2 k_{L(R)}{}^2}{2m} \quad , \quad \varphi_{\varepsilon,L(R)}(x) = \frac{1}{\sqrt{2\pi\hbar v_{L(R)}(\varepsilon)}} e^{\pm i k_{L(R)}(\varepsilon)x}$$

$$dI_{L,R} = \pm e \left\langle \varphi_{\varepsilon,L(R)} \middle| \frac{\hbar k_{L,R}(\varepsilon)}{m} \middle| \varphi_{\varepsilon,L(R)} \right\rangle d\varepsilon$$

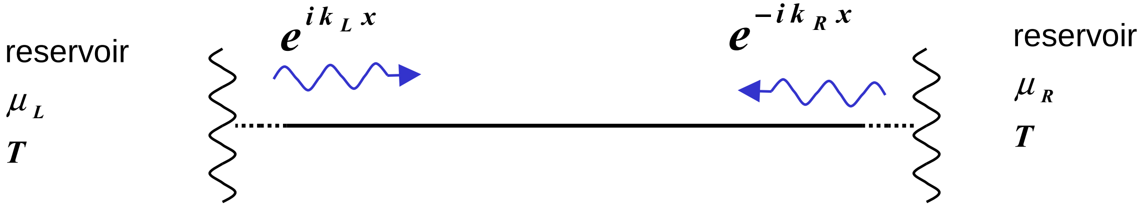
$$\boxed{dI_{L(R)} = \pm \frac{e}{h} d\varepsilon}$$

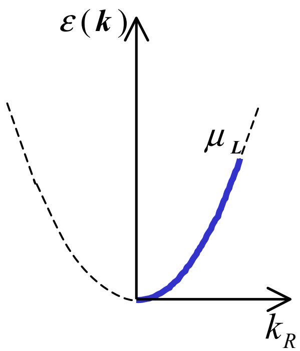
… is the current carried by one mode within energy range $d\varepsilon$

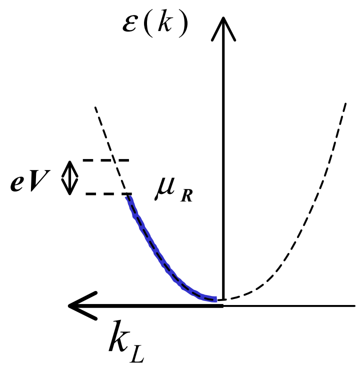
(independent on specific energy band dispersion relation)

second quantification representation

(to be ready to go further than simple scattering: … shot noise, ac transport, entanglement …)







$$\hat{\psi}_L(x,t) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\, \hat{a}_L(\varepsilon)\, e^{i(k_L x - \varepsilon t)}$$

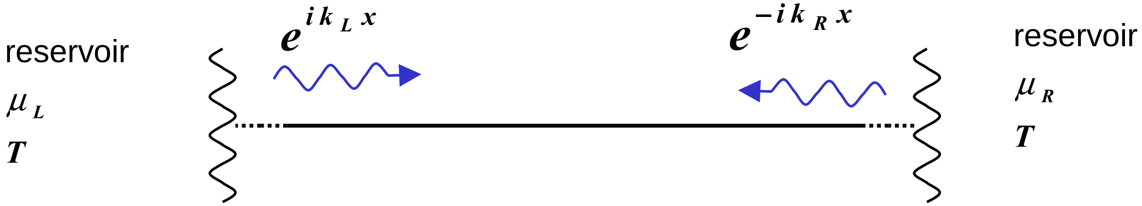
$$\hat{\psi}_R(x,t) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_R(\varepsilon)}}\, \hat{a}_R(\varepsilon)\, e^{i(-k_R x - \varepsilon t)}$$

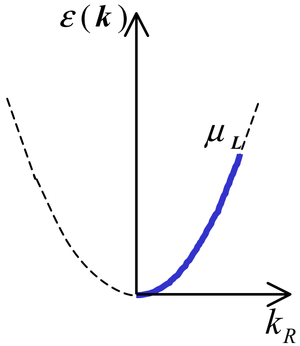
$$e\frac{\partial\left(\hat{\psi}^+\hat{\psi}\right)}{\partial t} + \frac{\partial}{\partial x}\hat{I} = 0$$

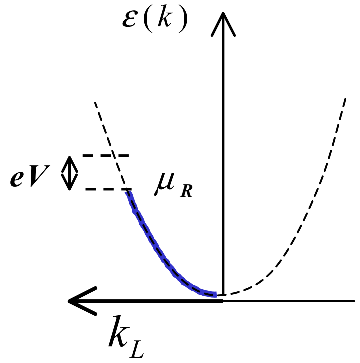
$$\boxed{\begin{gathered}\left\{\hat{a}_\beta(\varepsilon'), \hat{a}_\alpha{}^+(\varepsilon)\right\} = \delta_{\alpha,\beta}\,\delta(\varepsilon'-\varepsilon)\\ \left\{\hat{a}_\beta(\varepsilon'), \hat{a}_\alpha(\varepsilon)\right\} = 0\\ \left\langle \hat{a}_\alpha{}^+(\varepsilon).\hat{a}_\beta(\varepsilon')\right\rangle = f_\alpha(\varepsilon)\,\delta_{\alpha,\beta}\,\delta(\varepsilon'-\varepsilon)\end{gathered}}$$

$\hat{a}_{\alpha(\beta)}$, act on the Fock space of the reservoirs

$\left|L\right\rangle = \prod_{\varepsilon=0,\mu_L} \hat{a}^+{}_L(\varepsilon)\left|0\right\rangle_L$ and $\left|R\right\rangle = \prod_{\varepsilon=0,\mu_R} \hat{a}^+{}_R(\varepsilon)\left|0\right\rangle_R$







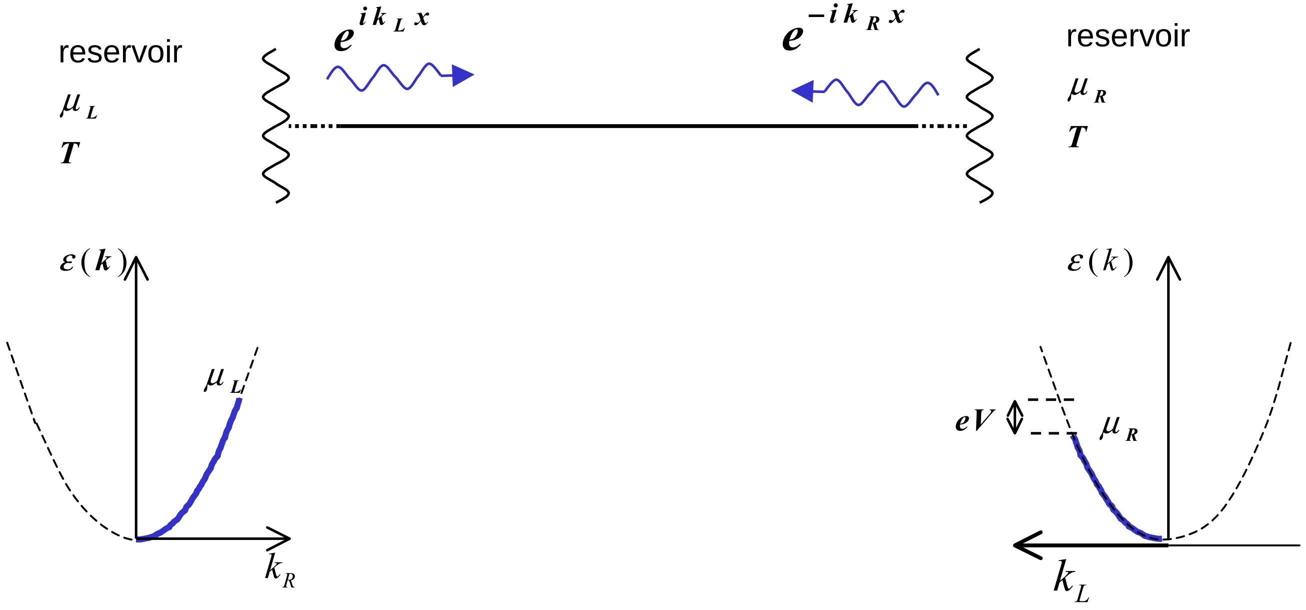
$$\hat{\psi}_L(x,t) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\ \hat{a}_L(\varepsilon)\, e^{i(k_L x - \varepsilon t)}$$

$$\hat{\psi}_R(x,t) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_R(\varepsilon)}}\ \hat{a}_R(\varepsilon)\, e^{i(-k_R x - \varepsilon t)}$$

$$\widehat{I}_L(x,t) = e\frac{\hbar}{i2m}\left(\widehat{\Psi}_L^{\dagger}(x,t)\frac{\partial \widehat{\Psi}_L(x,t)}{\partial x} - \frac{\partial \widehat{\Psi}_L^{\dagger}(x,t)}{\partial x}\widehat{\Psi}_L(x,t)\right)$$

$$= \frac{e}{h}\int d\varepsilon d\varepsilon' \widehat{a}_L^{\dagger}(\varepsilon')\widehat{a}_L(\varepsilon)\frac{v(\varepsilon)+v(\varepsilon')}{2\sqrt{v(\varepsilon)}\sqrt{v(\varepsilon')}} e^{i(k(\varepsilon)-k(\varepsilon'))x} e^{i(\varepsilon'-\varepsilon)t} \longrightarrow \left\langle \hat{I}_L(x,t) \right\rangle = \boxed{I_L = \frac{e}{h}\int d\varepsilon\, f_L(\varepsilon)}$$

the quantum of conductance



$$I = \int_0^{\infty} \left(f_L(\varepsilon) - f_R(\varepsilon)\right) \frac{e}{h} d\varepsilon$$

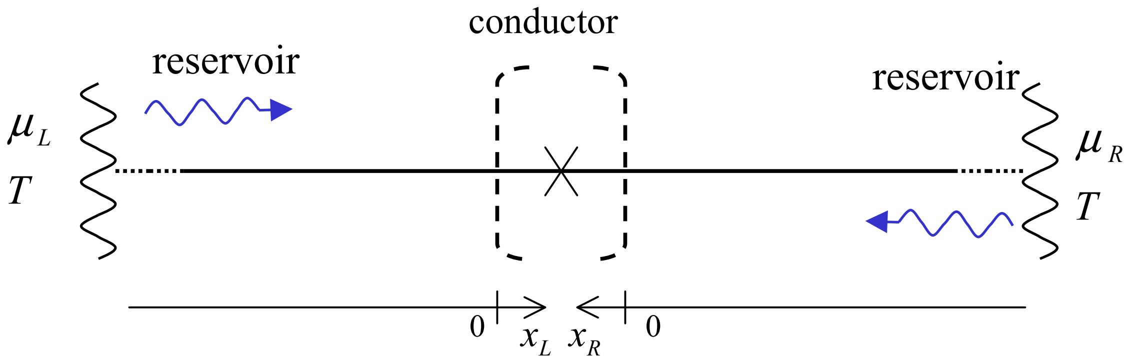
$$I = \frac{e^2}{h} V \qquad \mu_L = \mu_R + eV \qquad \forall\ V,\ T$$

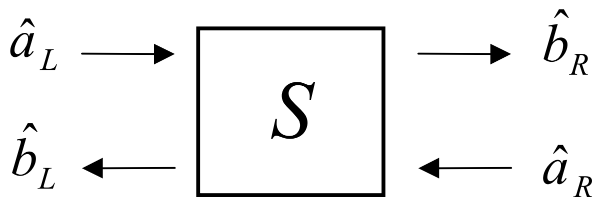
the conductance : $\dfrac{e^2}{h}$

is independent on temperature and voltage

$$I = e\frac{eV}{h} \quad \text{Pauli } (1 \times e) \ + \ \text{Heisenberg}\,(eV/h)$$

I . 2 ONE MODE SCATTERING





$$S = \begin{pmatrix} s_{LL} & s_{LR} \\ s_{RL} & s_{RR} \end{pmatrix} \equiv \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

$$S\,S^{+} = S^{+}S = 1$$

scattering states in the reservoirs:

Idea: write the total fermion operator as:

$$\hat{\psi}(x_L) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\left(\hat{a}_L(\varepsilon)e^{ik_L x_L} + \hat{b}_L(\varepsilon)e^{-ik_L x_L}\right)$$

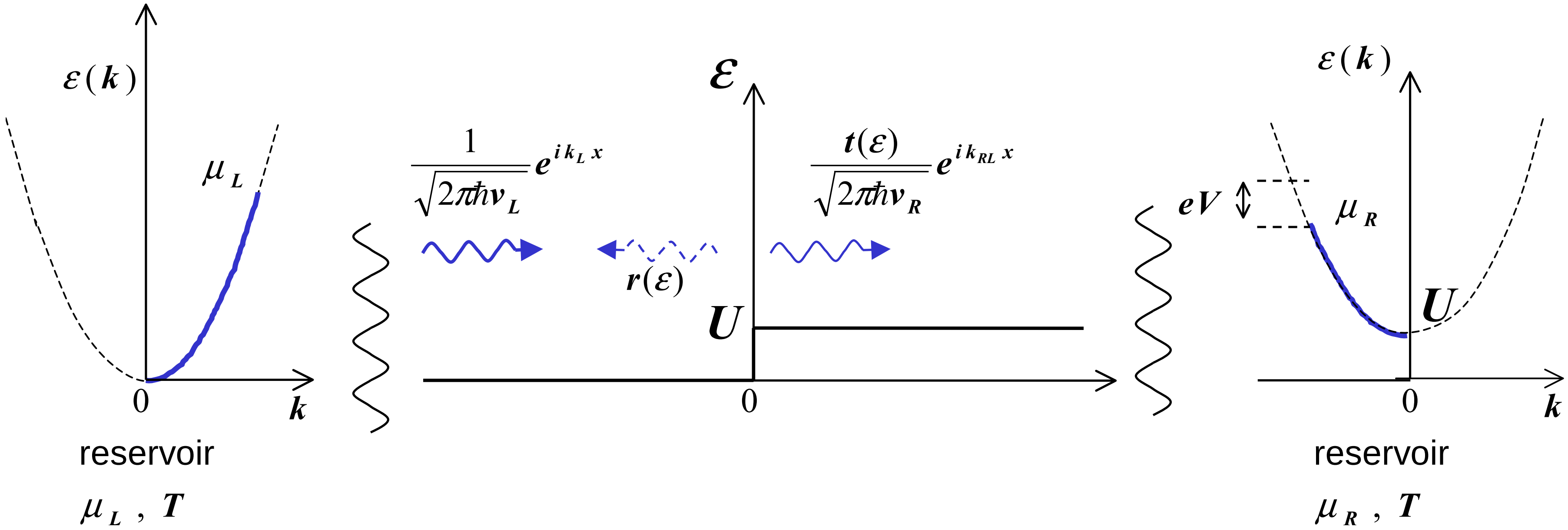
$$\hat{\psi}(x_R) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_R(\varepsilon)}}\left(\hat{a}_R(\varepsilon)e^{ik_R x_R} + \hat{b}_R(\varepsilon)e^{-ik_R x_R}\right)$$

S contains all the transport information on the conductor.

links the outgoing amplitudes to the incoming wave amplitudes

amplitude are replaced by the annihilation operator acting on the Fock space of each reservoir L,R

a simple example:



$$\varepsilon(k_L)=\frac{\hbar^2 k_L{}^2}{2m} \qquad \varepsilon(k_R)=U+\frac{\hbar^2 k_R{}^2}{2m}$$

1. scattering states: electrons emitted by the left reservoir

$$\hat{\psi}_{\varepsilon,L}(x,t)=\int d\varepsilon\ \frac{\hat{a}_L(\varepsilon)}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\left(e^{ik_L x_L}+r(\varepsilon)e^{-ik_L x_L}\right)e^{-i\varepsilon t} \qquad x_L<0$$

$$=\int d\varepsilon\ \frac{\hat{a}_L(\varepsilon)}{\sqrt{2\pi\hbar v_R(\varepsilon)}}t(\varepsilon)e^{-ik_R x_R}e^{-i\varepsilon t} \qquad x_R<0$$

(describes electrons emitted by the left reservoir)

(current probability amplitude)

$$\hat{\psi}_{\varepsilon,L}(x,t)=\int d\varepsilon\,\frac{\hat{a}_L(\varepsilon)}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\left(e^{ik_Lx_L}+r(\varepsilon)e^{-ik_Lx_L}\right)e^{-i\varepsilon t}\qquad x_L<0$$

$$=\int d\varepsilon\,\frac{\hat{a}_L(\varepsilon)}{\sqrt{2\pi\hbar v_R(\varepsilon)}}t(\varepsilon)\,e^{-ik_Rx_R}e^{-i\varepsilon t}\qquad x_R<0$$

similarly, for electrons emitted from the right reservoir

$$\hat{\psi}_{\varepsilon,R}(x\ ,t)=\int d\varepsilon\,\frac{\hat{a}_R(\varepsilon)}{\sqrt{2\pi\hbar v_R(\varepsilon)}}\left(e^{ik_Rx_R}+r'(\varepsilon)e^{-ik_Rx_R}\right)e^{-i\varepsilon t}\qquad x_R<0$$

$$=\int d\varepsilon\,\frac{\hat{a}_R(\varepsilon)}{\sqrt{2\pi\hbar v_L(\varepsilon)}}t'(\varepsilon)\,e^{-ik_Lx_L}e^{-i\varepsilon t}\qquad x_L<0$$

total state in the left lead

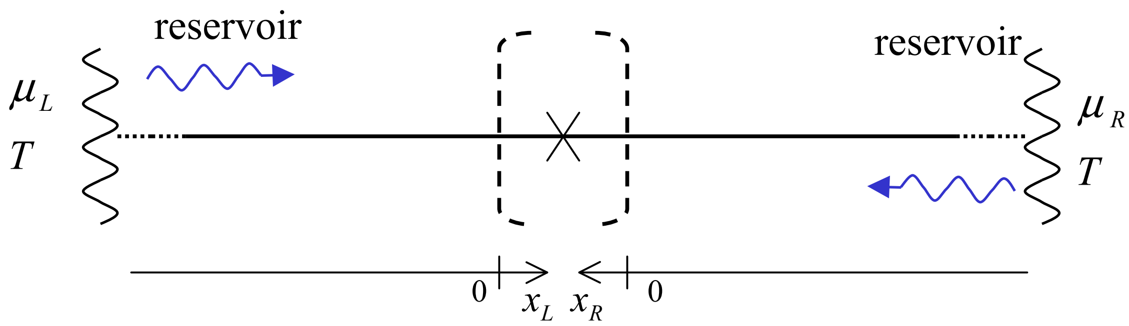
$$\hat{\psi}_{\varepsilon}(x_L,t)=\int d\varepsilon\,\frac{\hat{a}_L(\varepsilon)}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\left(\hat{a}_L(\varepsilon)e^{ik_Lx_L}+\hat{a}_L(\varepsilon)\,r(\varepsilon)e^{-ik_Lx_L}+\hat{a}_R(\varepsilon)\,t'(\varepsilon)e^{-ik_Lx_L}\right)e^{-i\varepsilon t}$$

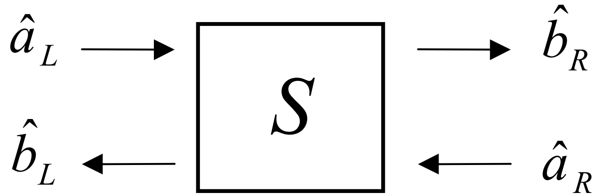
$$=\int d\varepsilon\,\frac{1}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\left(\hat{a}_L(\varepsilon)\,e^{ik_Lx_L}+\hat{b}_L(\varepsilon)e^{-ik_Lx_L}\right)e^{-i\varepsilon t}$$

… and similarly in the right lead

$$\longrightarrow\qquad\begin{pmatrix}\hat{b}_L\\ \hat{b}_R\end{pmatrix}=\begin{pmatrix}r & t'\\ t & r'\end{pmatrix}\begin{pmatrix}\hat{a}_L\\ \hat{a}_R\end{pmatrix}$$

to summarize:





$$\boxed{\begin{array}{l} S = \begin{pmatrix} s_{LL} & s_{LR} \\ s_{RL} & s_{RR} \end{pmatrix} \\ \\ S\,S^{+} = S^{+}S = 1 \end{array}}$$

$$I = \frac{e}{h}\int d\varepsilon \left\{ f_L(\varepsilon) - \left(|s_{LL}|^2 f_L(\varepsilon) + |s_{LR}|^2 f_R(\varepsilon) \right) \right\}$$

Landauer formula

$$\boxed{I = \frac{e}{h}\int d\varepsilon |s_{LR}|^2 \left(f_L(\varepsilon) - f_R(\varepsilon) \right)}$$

$$G = \frac{e^2}{h}\int d\varepsilon \left(-\frac{\partial f}{\partial \varepsilon} \right) D(\varepsilon)$$

$$D = |S_{LR}|^2 = |S_{RL}|^2 = 1 - |S_{LL}|^2 = 1 - |S_{RR}|^2 = 1 - R$$

$$G = \frac{e^2}{h}\; D(\varepsilon_F) \quad @\ \mathrm{T} = 0$$

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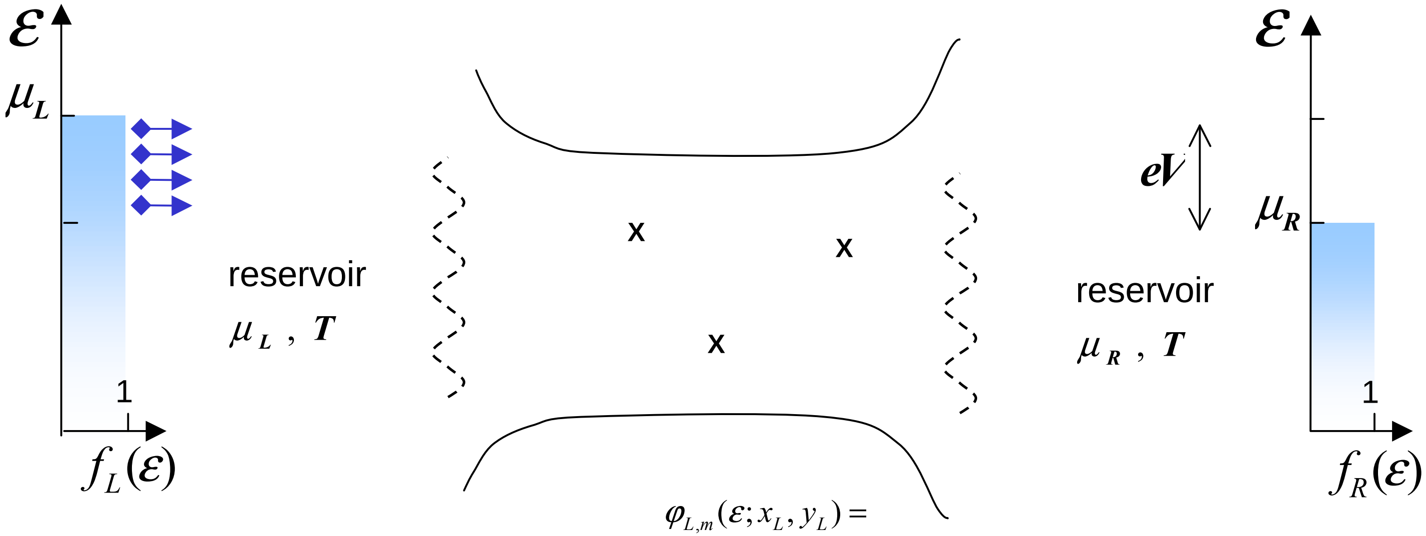
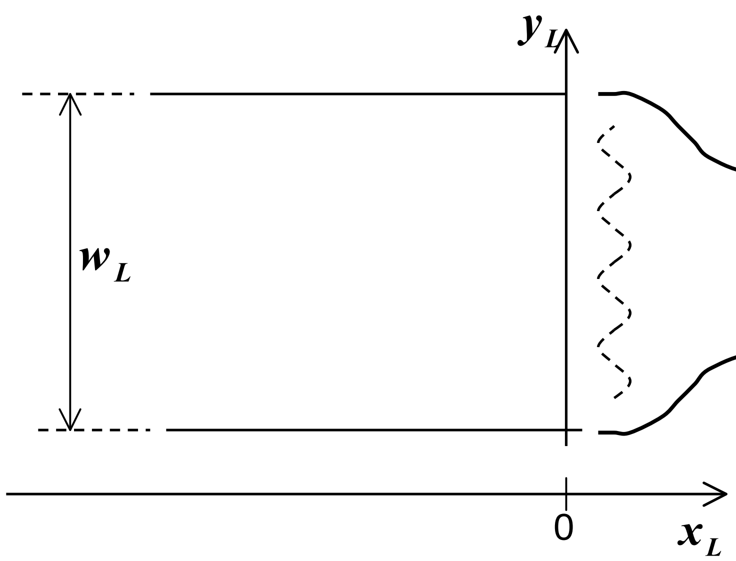
I . 3 Multiple mode scattering : Landauer fromula

modeling of the reservoirs for a two-terminal conductor: (2D here for simpler notations)





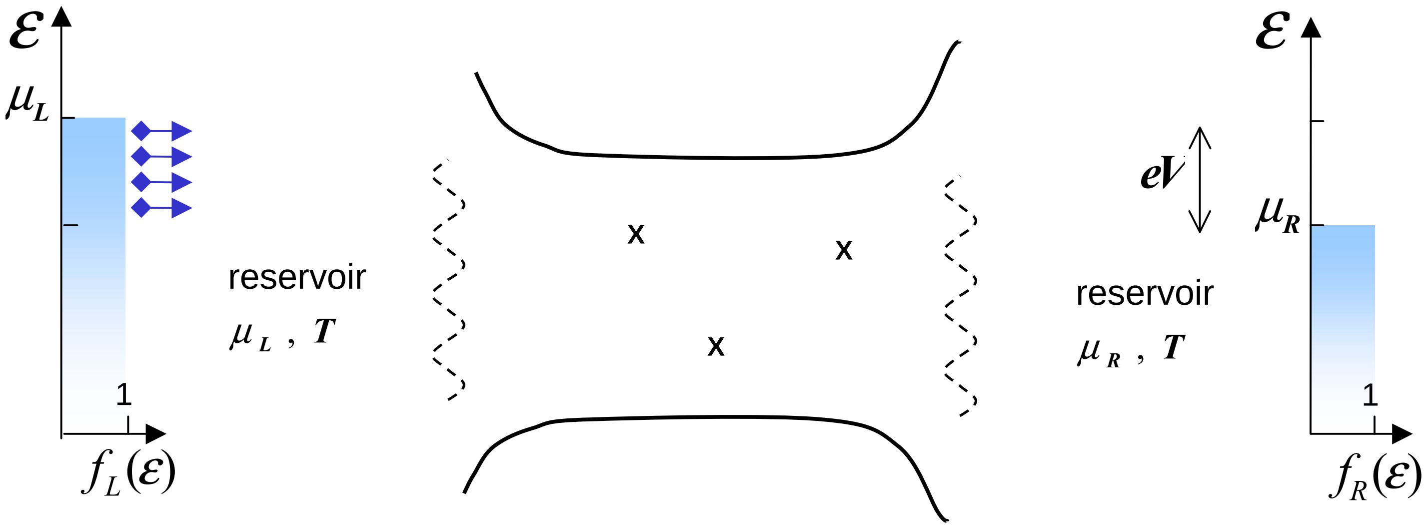
ideal multi-mode lead

$$\varepsilon = \frac{\hbar^2}{2m^*}\left[\left(\frac{m\pi}{w_L}\right)^2 + k_{L,m}(\varepsilon)^2\right]$$

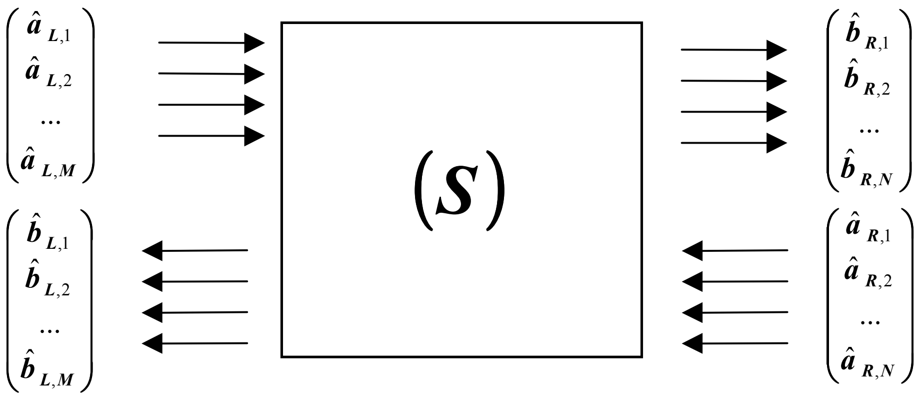
Incoming wave : emitted by (L) mode (m)

$$\chi_{L,m}(y_L)\frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{\hbar v_{L,m}(\varepsilon)}}e^{ik_{L,m}(\varepsilon)x_L}$$

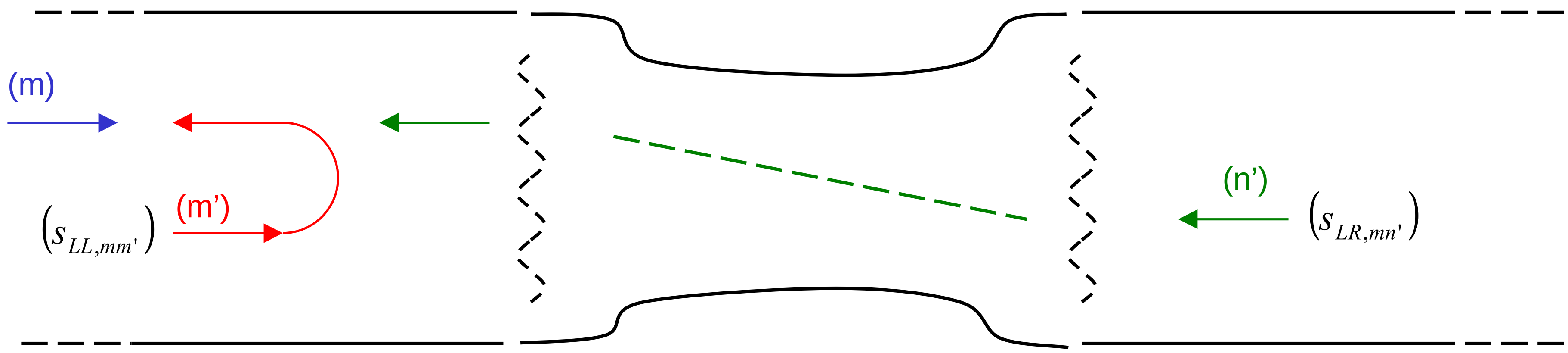
$$\chi_{L,m}(y) = \left(\frac{2}{W_L}\right)^{1/2}\sin\left(\frac{\pi m}{W_L}y\right)$$



scattering matrix for many modes:

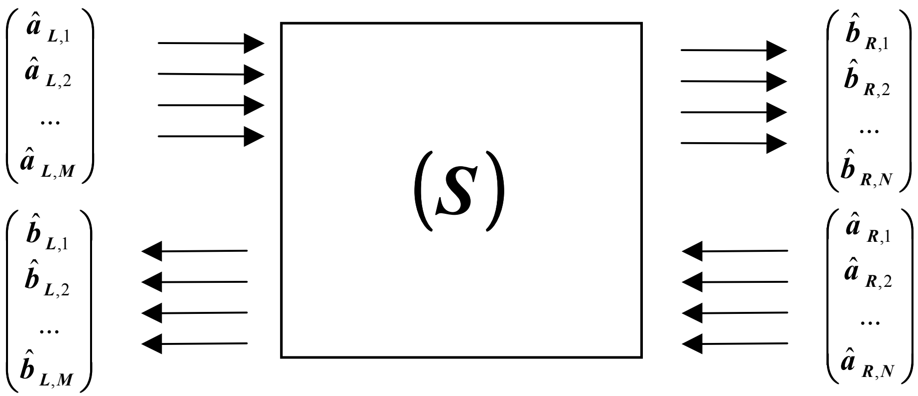


$$S = \begin{pmatrix} (S_{LL}) & (S_{LR}) \\ (S_{RL}) & (S_{RR}) \end{pmatrix}$$



(reservoir 'L') (conductor : S) (reservoir 'R')

$$I = \frac{e}{h}\int d\varepsilon \sum_{m}\left\{ f_L(\varepsilon) - \left(f_L(\varepsilon)\sum_{m'}\left|s_{LL,mm'}\right|^2 + f_R(\varepsilon)\sum_{n}\left|s_{LR,mn'}\right|^2 \right)\right\}$$

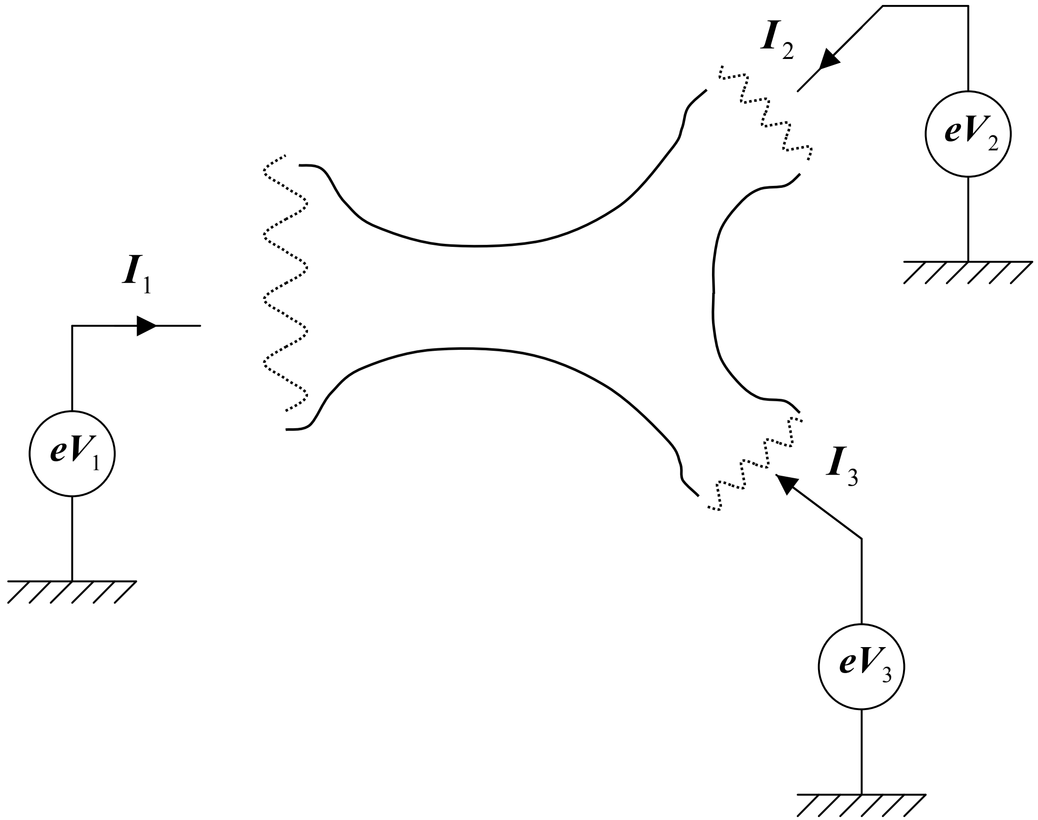


$$\boxed{\begin{array}{c} S = \begin{pmatrix} (s_{LL}) & (s_{LR}) \\ (s_{RL}) & (s_{RR}) \end{pmatrix} \\ \\ S\,S^{+} = S^{+}S = 1 \end{array}}$$

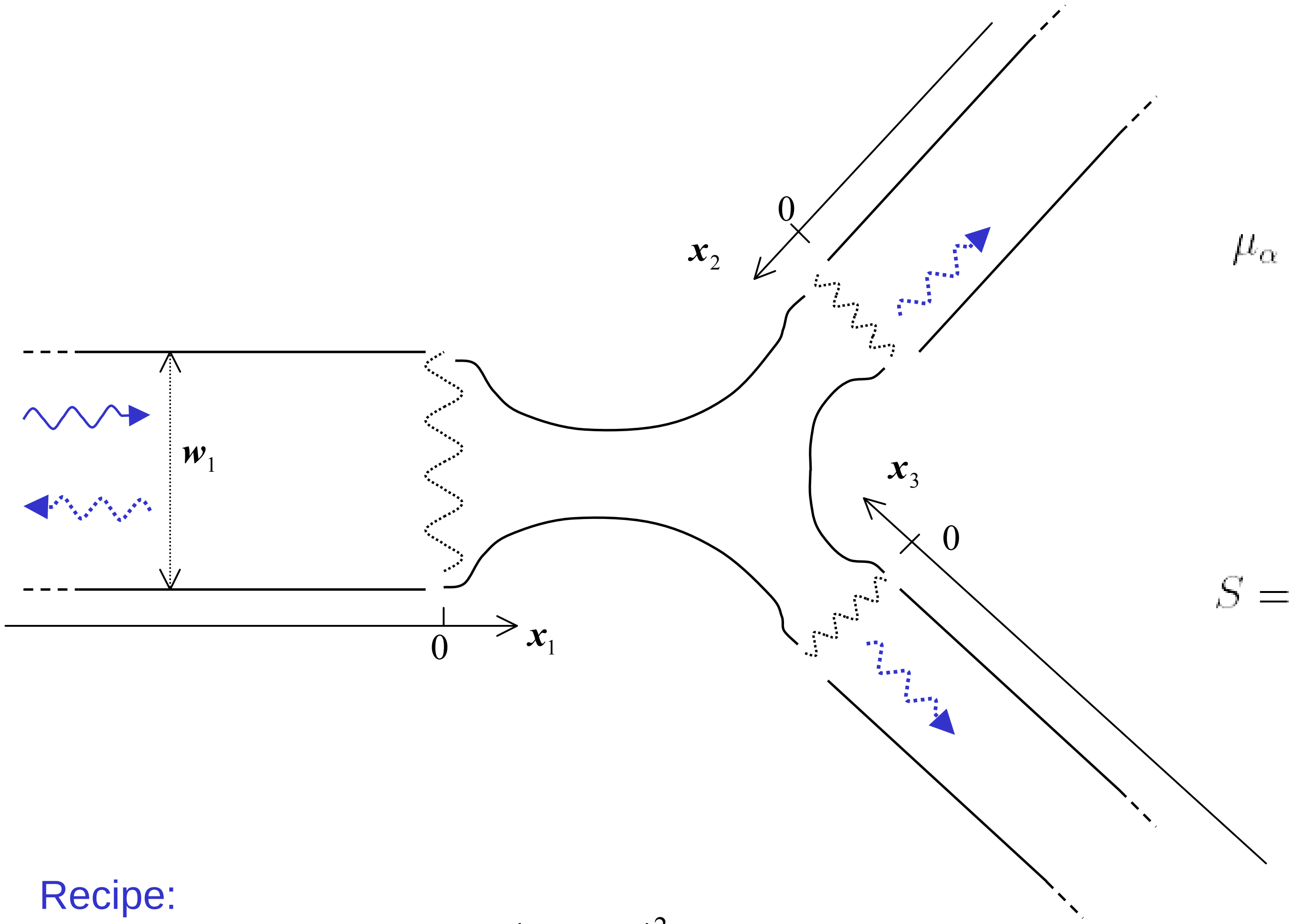
I . 4 multi-terminal conductors: Landauer-Büttiker formula.

the quantum Kirchhof law:

Electrochemical potential of the contacts: $\mu_\alpha = E_F + eV_\alpha$



find the linear relation between the I_β and the V_α $\qquad I_\beta = G_{\alpha\beta} V_\alpha$



$$\mu_\alpha = E_F + eV_\alpha .$$

$$S = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix}$$

Recipe:

add the quantum probablities $\left|s_{\alpha\beta,mn}\right|^2$ with the right sign and the thermodynamical weight $f_\beta(\varepsilon)$

$$I_\alpha = \frac{e}{h}\int d\varepsilon \sum_{m=1}^{M_\alpha}\left\{ f_\alpha(\varepsilon) - \left(f_\beta(\varepsilon)\sum_{n=1}^{M_\beta}\left|s_{\alpha\beta,mn}\right|^2 + f_\alpha(\varepsilon)\sum_{m'=1}^{M_\alpha}\left|s_{\alpha\alpha,m'm}\right|^2 \right)\right\}$$

incoming from α (occupation f_α)

emitted by β and transmitted into α (occupation f_β)

emitted by and reflected into α (occupation f_α)

(number of occupied channels
at energy ε in lead α)

$Tr(s_{\alpha\alpha} s_{\alpha\alpha}{}^{+})$ $\qquad$ $Tr(s_{\alpha\beta} s_{\alpha\beta}{}^{+})$

$$I_\alpha = \frac{e}{h}\int d\varepsilon \left\{ f_\alpha(\varepsilon)\left(M_\alpha - R_{\alpha\alpha}\right) - \sum_\beta T_{\alpha\beta}\, f_\beta(\varepsilon) \right\}$$

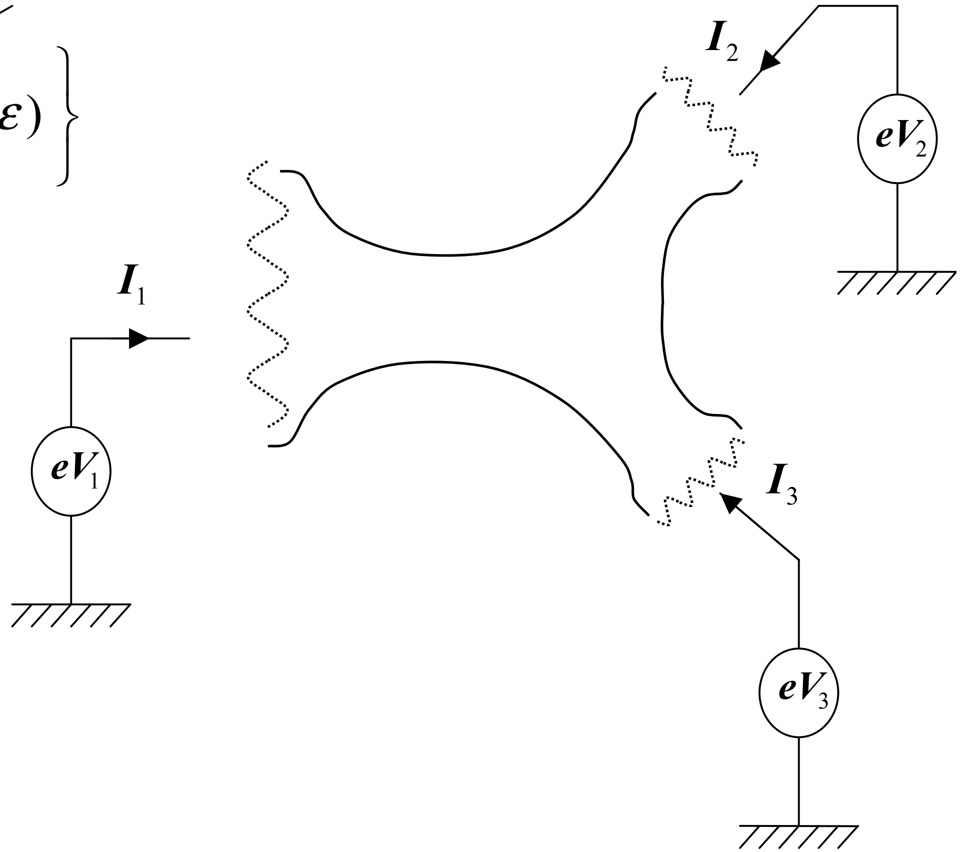
using $S^+ S = S S^+ = 1$

quantum Kirchhof law:

$$\boxed{I_\alpha = \sum_\beta G_{\alpha\beta} V_\beta}$$

$$G_{\alpha\alpha} = \frac{e^2}{h}\left(M_\alpha - R_{\alpha\alpha}\right)$$

$$G_{\alpha\beta} = \frac{e^2}{h}\, T_{\alpha\beta}$$



Introduction

I. electronic scattering (a brief introduction)

1- one ideal mode (+ electron flux/photon flux analogy)
2- one mode scattering
3- multi-mode scattering: Landauer formula
4- multi-terminal Büttiker formula (quantum Kirchhoff law)
5- chiral transport with QHE edge states.

II. Quantum Shot noise

IV. Shot noise: ***the*** tool to detect entanglement

V. Shot noise and high frequencies

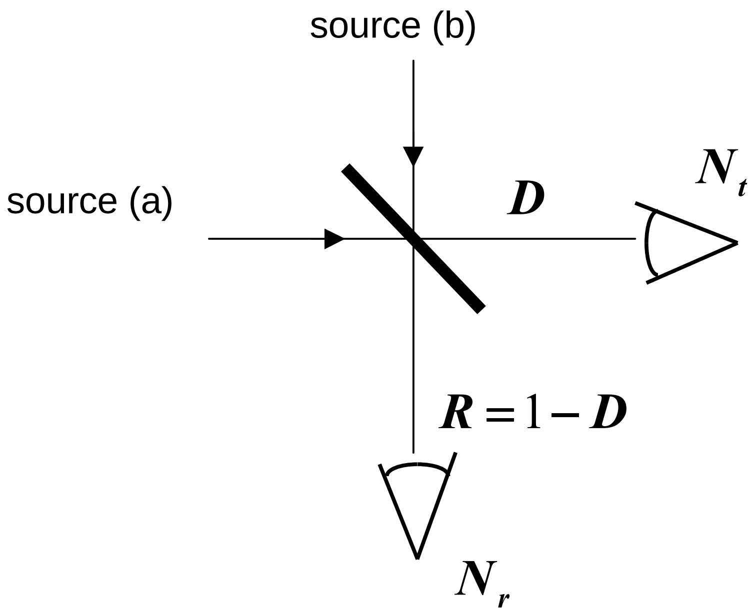
I . 5 chiral transport : edge states

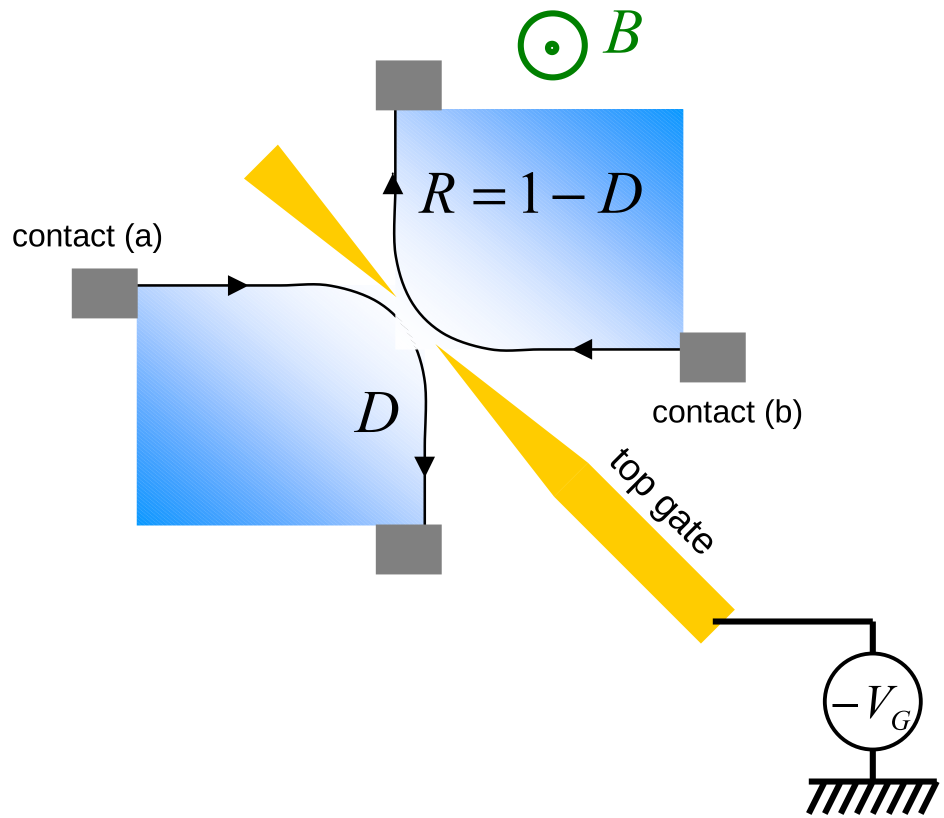
WHY ?

- Fermi statistics properties are best seen with ballistic electrons
- ballistic electrons allow a direct comparison with most quantum optics experiments
- edge states in high field provide ‘super’-ballistic transport.

ballistic 2DEG in high field (QHE regime)

example of the beam splitter:





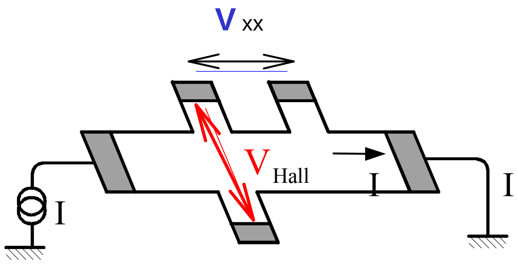
advantage:

- 1D (chiral) modes : QHE edge states

- no backscattering (robust to disorder)

gate control of the transparency D

(macroscopic) Quantum Hall Effect



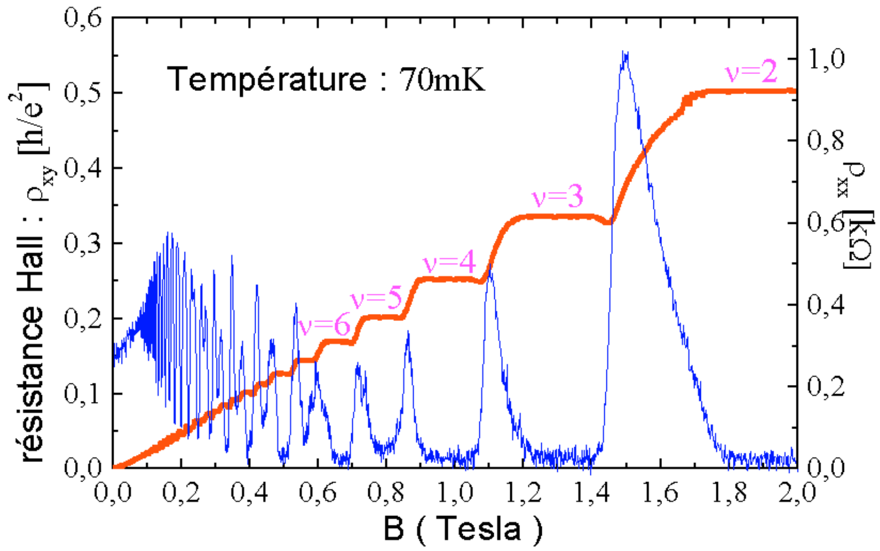
Edwin Hall
1879

$$\vec{E}_{Hall} = -\vec{v} \times B\,\hat{z}$$

$$\boxed{R_{Hall} = \frac{B}{e\,n_s}}$$

$$\boxed{R_{Hall} = \frac{h}{e^2} \cdot \frac{1}{Integer}}$$

K. von Klitzing
G. Dorda
M. Pepper 1980



$$B = \frac{h}{e} n_\Phi$$

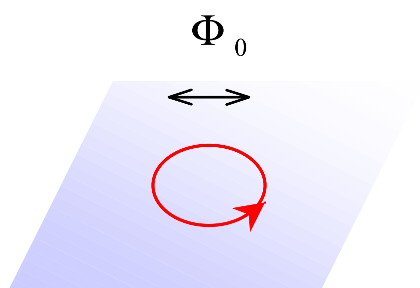
$$\boxed{R_{Hall} = \frac{h}{e^2} \cdot \frac{n_\Phi}{n_s}}$$

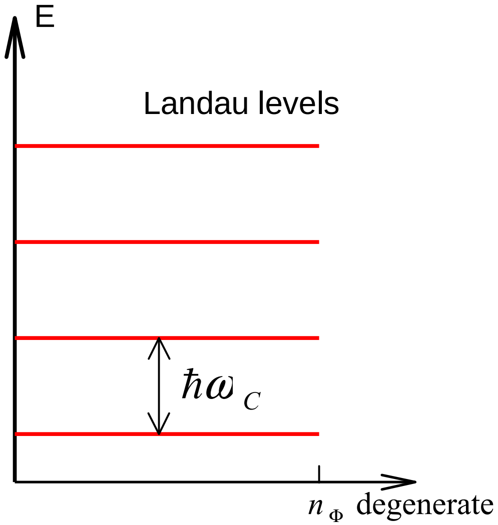
n_Φ = density of flux quantum $\Phi_0 = \dfrac{h}{e}$

$$\boxed{n_s = (Integer) \times n_\Phi}$$

$$\frac{h}{e^2} = 25812.805(6) \text{ Ohms}$$

Exactness : gauge invariance (B. Laughlin 1981)





$$H = \frac{1}{2m}\left(\vec{p} - e\vec{A}\right)^2 \qquad \vec{A} = \left(-yB, 0, 0\right) \qquad p_x = \hbar k$$

$$H = \frac{p_y^{\,2}}{2m} + \frac{1}{2} m \omega_C^{\,2}\left(y - k.l_C^{\,2}\right)^2 \qquad ; \; l_C = \left(\frac{\hbar}{eB}\right)^{1/2}$$

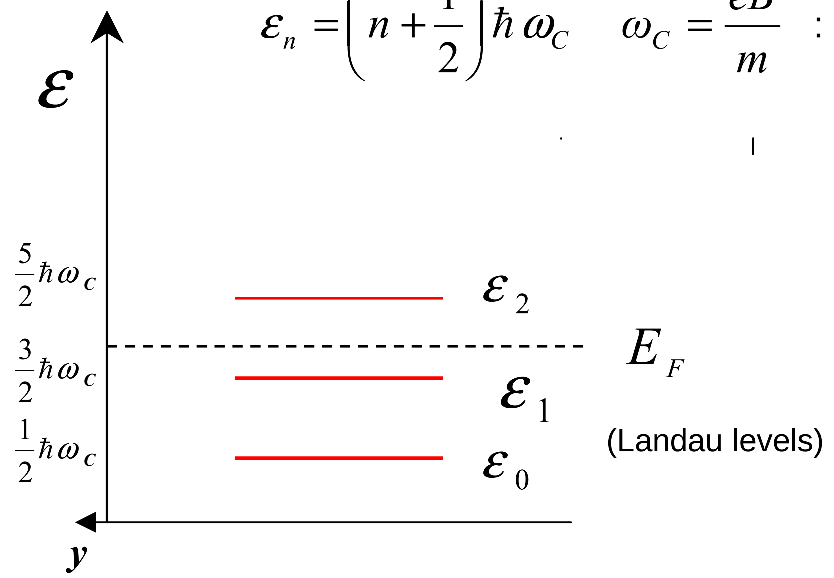
$$\Phi_{n,k} = \frac{1}{\sqrt{L_X}} e^{ikx} H_n(y - y_k) e^{-\frac{1}{2}\frac{(y-y_k)^2}{l_c^2}}$$

electrons form highly degenerated
Landau levels with energies e_n , n = 0, 1, 2, ...

$$\Delta y_k = \frac{2\pi}{L} l_C^{\,2} = \frac{eB}{hL}$$

$$\varepsilon_n = \left(n + \frac{1}{2}\right)\hbar\,\omega_C \qquad \omega_C = \frac{eB}{m}$$: the cyclotron frequency

(spin disregarded)



the mesoscopic quantum Hall effect:

$$H = \frac{1}{2m}\left(\vec{p} - e\vec{A}\right)^2 + U(y) \qquad \vec{A} = (-yB, 0, 0)$$

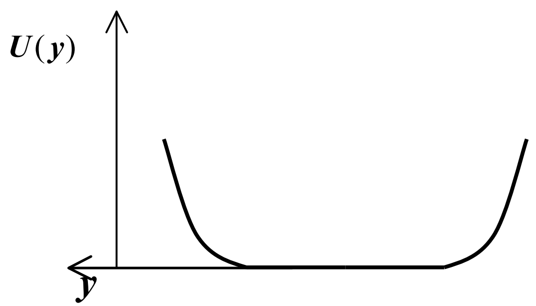
$$H = \frac{p_y^{\ 2}}{2m} + \frac{1}{2} m\omega_C^{\ 2}\left(y - k.l_C^{\ 2}\right)^2 + U(y)$$

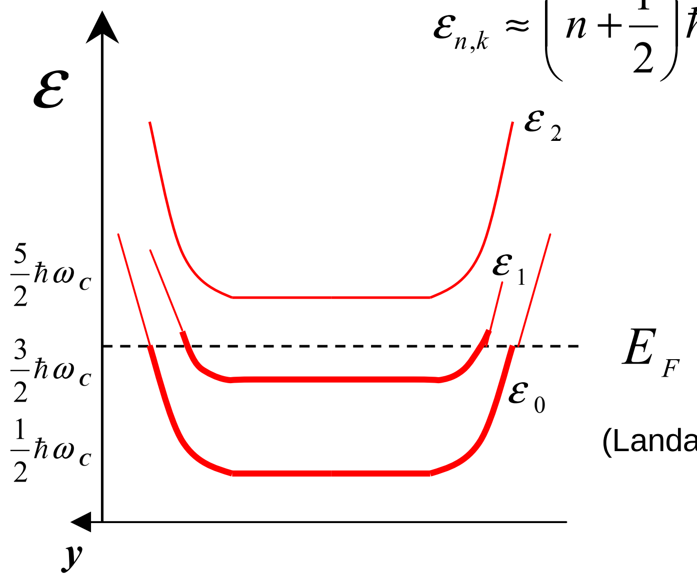
$$l_C = \left(\frac{\hbar}{eB}\right)^{1/2}$$

Landau level degeneracy is lifted by the confining potential for each $\boldsymbol{\varepsilon_n}$, n = 0, 1, 2, ...

$$\varepsilon_{n,k} \approx \left(n + \frac{1}{2}\right)\hbar\,\omega_C + U(y_k = k.l_C^{\ 2})$$

confining potential





(Landau levels)

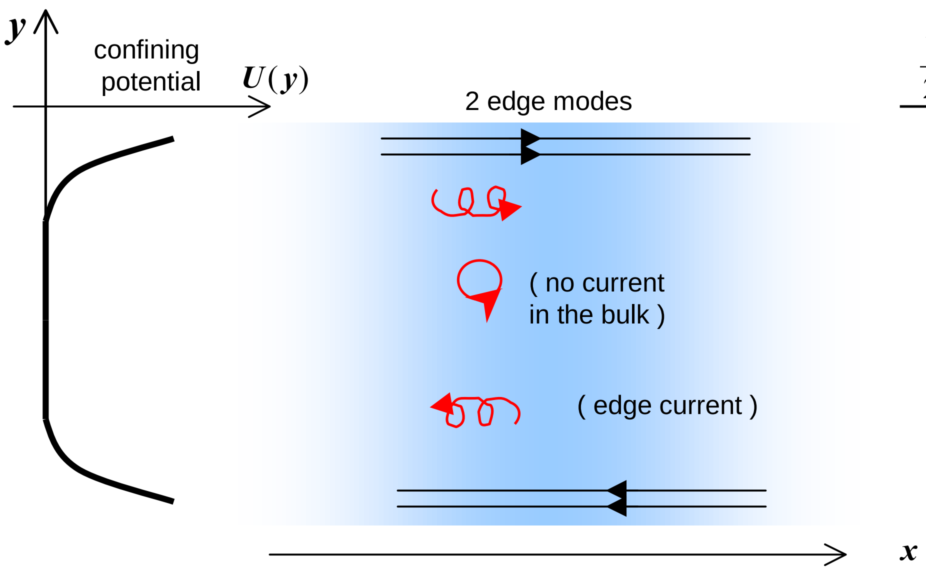
$$\varepsilon_{n,k} \approx \left(n+\frac{1}{2}\right)\hbar\omega_C + U(y_k = k.l_C^{\,2})$$

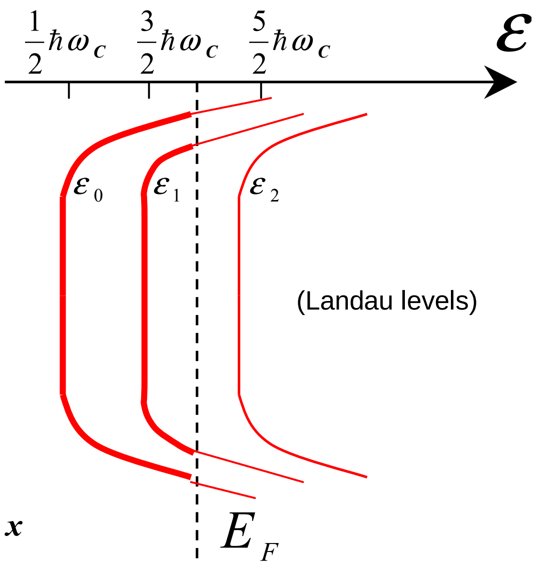
bend Landau levels give p chiral 1D modes (edge channels) if p Landau level are filled.

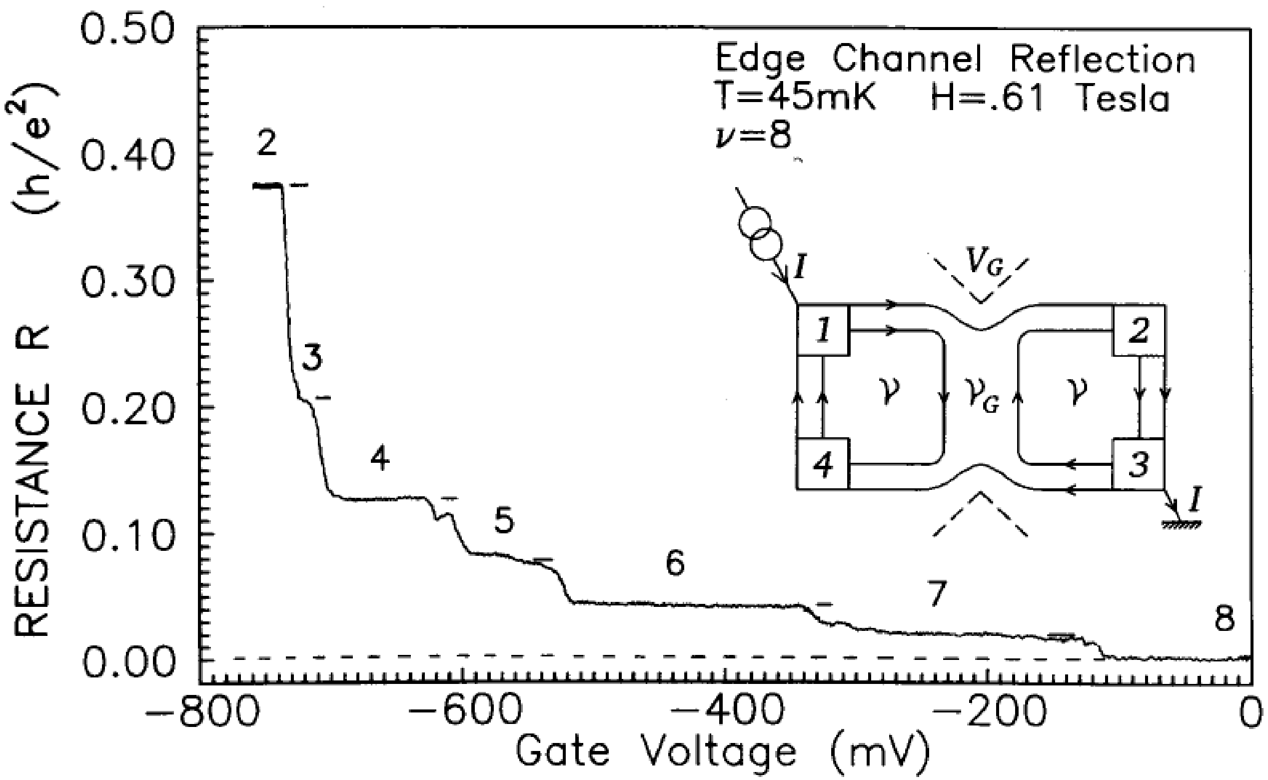
$$G = p\frac{e^2}{h}$$ if p Landau level filled in the bulk (Landauer)

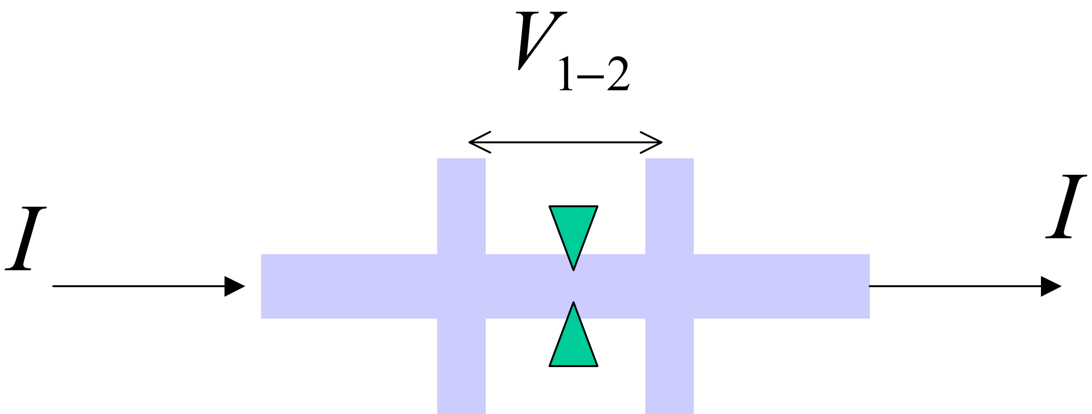
electrons drift along along the edge with velocity:

$$_{G}v_x^{(n)}(k) = \frac{1}{\hbar}\frac{\partial \varepsilon_n}{\partial k} \approx \frac{1}{\hbar}\frac{\partial U}{\partial k} = \frac{1}{eB}\frac{\partial U}{\partial y}$$







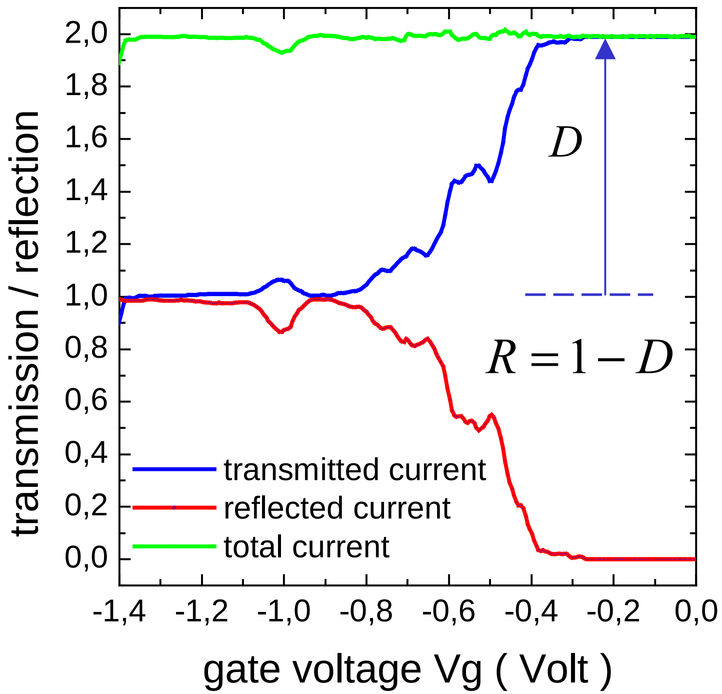


$$V_{1-2} = \left(\frac{1}{\nu_G} - \frac{1}{\nu} \right) \frac{h}{e^2} I$$

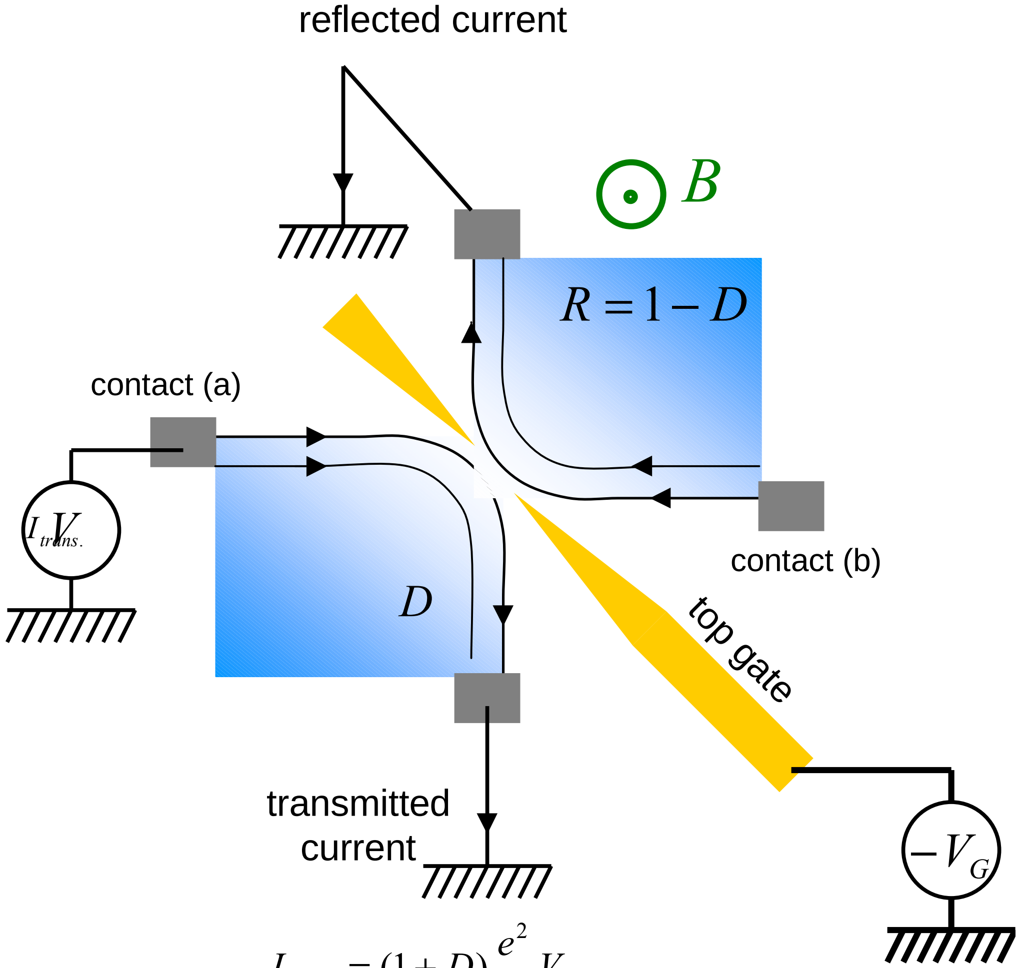
obtained using :

$$I_\alpha = \sum_\beta G_{\alpha\beta} V_\beta$$

the QPC beam splitter



(© from P. Roche/ F. Portier, nanoelectronics group Saclay)

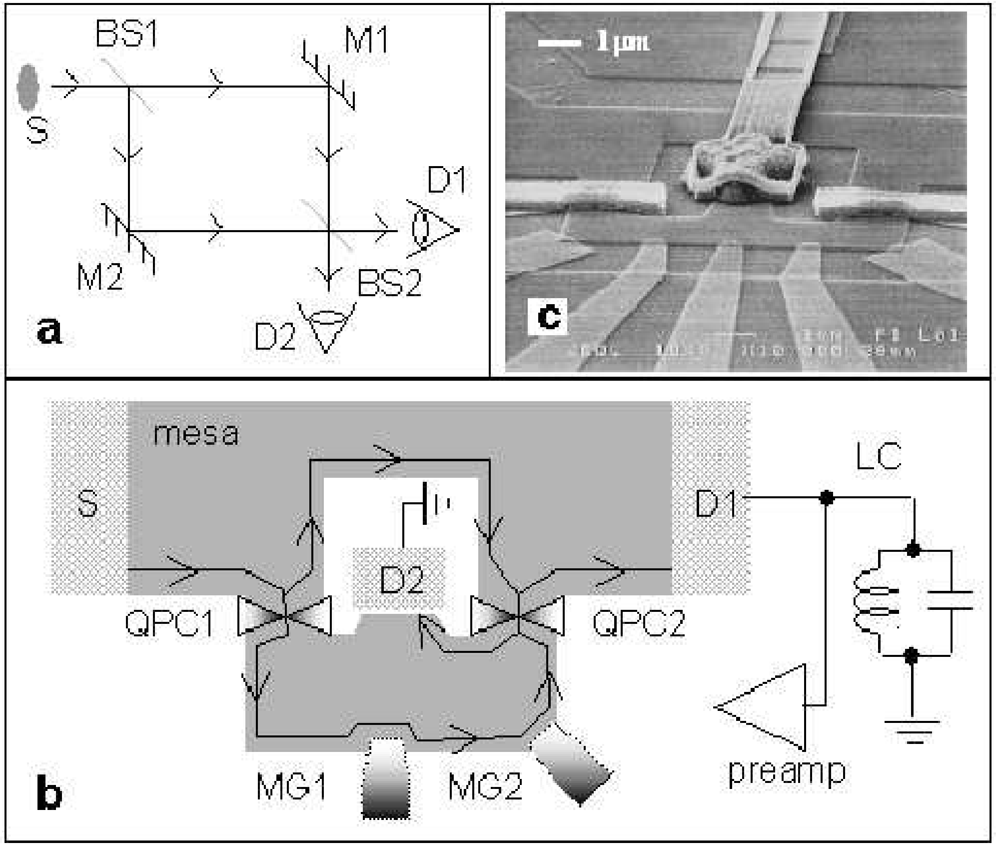


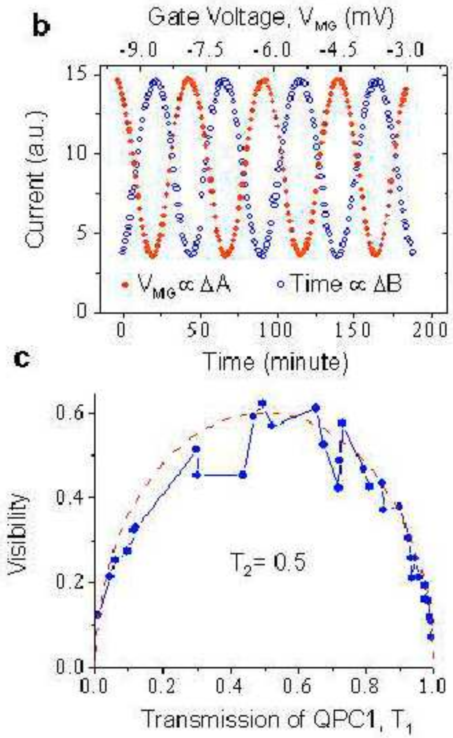
$$I_{trans.} = (1 + D)\frac{e^2}{h}V$$

$$I_{refl.} = R\frac{e^2}{h}V$$

$$I_{trans.} + I_{refl.} = 2.\frac{e^2}{h}V$$

electronic Mach - Zehnder interferometer *(from Yang Ji et al, M. Heiblum's group, Weizmann)*





edge states of Quantum Hall effect
= chiral 1D ideal conductors.

Area of the loop = 45 μm^2
electron temperature : 20mK

$$I_{D1} \sim \left| t_1 t_2 + r_1 r_2 e^{i\phi} \right|^2$$

$$\phi = 2\pi \frac{\Phi}{\Phi_0}$$

$$T_1 = |t_1|^2 = 1 - |r_1|^2$$

$$T_2 = |t_2|^2 = 1 - |r_2|^2$$

maximum visibility for $t_1 = t_2 = 1/2$