Introduction

I. Electronic scattering (a brief introduction)

II. Quantum Shot noise

1 - Quantum partition noise
- one and two particle partitioning :electrons/ photons
- electronic shot noise

2- scattering derivation of quantum shot noise
a- $S(\omega)$ for an ideal one mode conductor
b- quantum shot noise for a single mode
c-zero frequency shot noise and multimode case

3- experimental examples

4- current noise cross-correlations
- scattering derivations
- electronic analog of the optical Hanbury-Brown Twiss experiment
- electronic quantum exchange

III Shot Noise and Interactions:

IV. Shot noise: ***the*** tool to detect entanglement

V. Shot noise and high frequencies

II. 1 Quantum Partition Noise

single -particle partitioning :

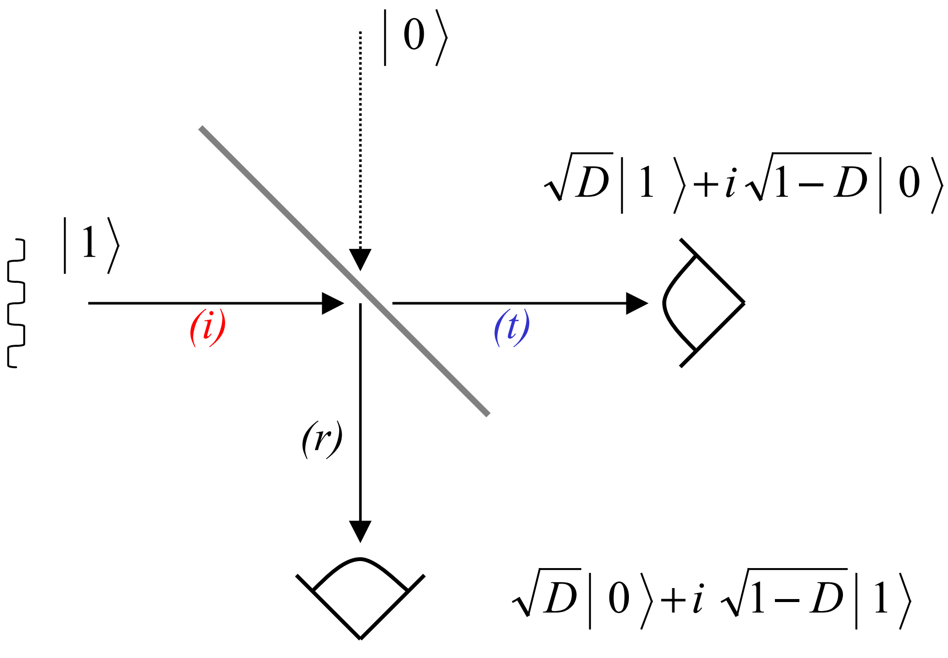
assume only one particle in a wave incident on a scatter

- scattering $\Rightarrow$ the wave is diffracted

- diffraction+reservoirs $\Rightarrow$ the particle is randomly partitioned

$\Rightarrow$ quantum partition noise is *diffraction* (there is no classical analog) + particle-wave duality

(quantum noise exemplifies particle-wave duality)



$$n_i = 1, 1, 1, 1, 1, 1, 1, 1, \ldots$$
$$n_t = 1, 1, 0, 1, 0, 1, 0, 0, \ldots$$
$$n_r = 0, 0, 1, 0, 1, 0, 1, 1, \ldots$$

$$\overline{n_t} = D \qquad \overline{n_r} = 1 - D$$

$$\overline{(\Delta n_t)^2} = \overline{n_t}(1 - \overline{n_t}) = D(1-D) = \overline{(\Delta n_r)^2}$$

binomial = Poisson X reduction factor (1 - *D*)

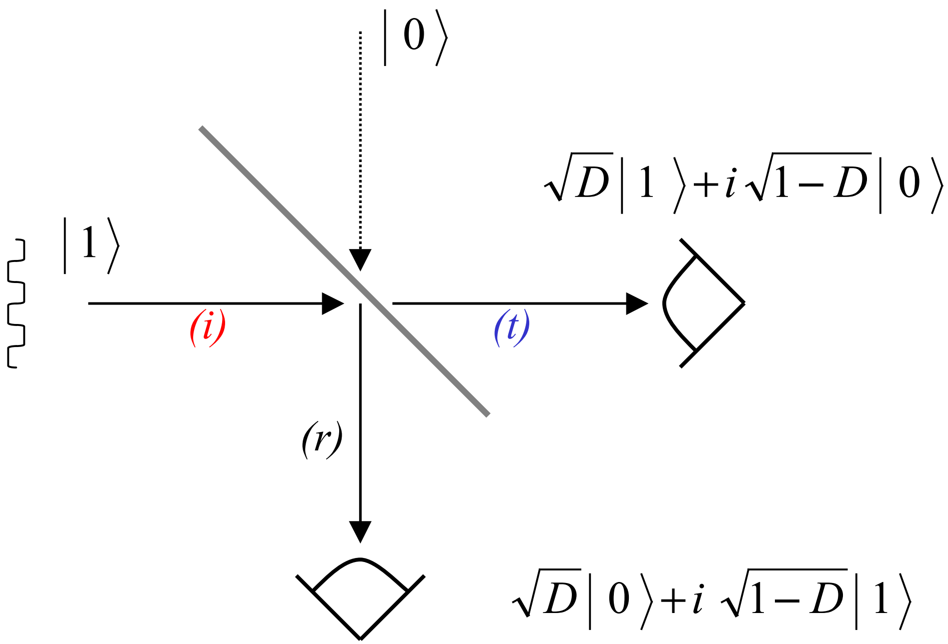
$$\overline{\Delta n_t . \Delta n_r} = -D(1-D)$$

assume only one particle in a wave incident on a scatter

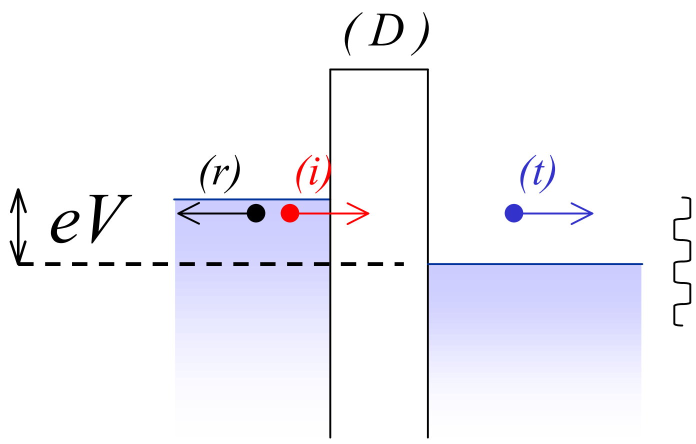
- scattering $\Rightarrow$ the wave is diffracted
- diffraction+reservoirs $\Rightarrow$ the particle is randomly partitioned

$\Rightarrow$ quantum partition noise is *diffraction* (there is no classical analog)

(quantum noise exemplifies particle-wave duality)

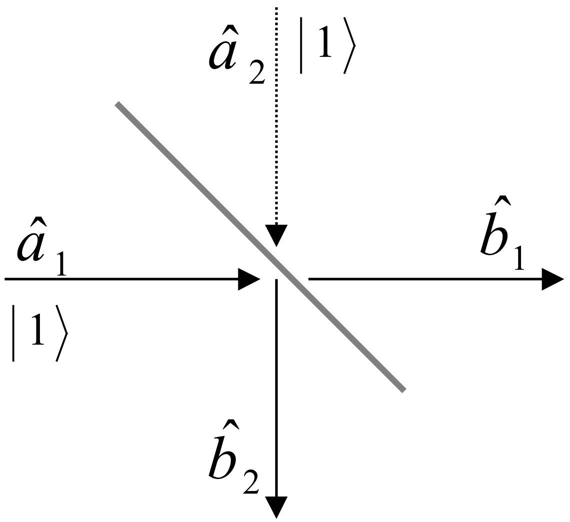


this applies also to electronic waves:



$\Rightarrow$ responsible for current fluctuations or quantum shot noise

two-particle partitioning : difference between Bosons and Fermions



$$\begin{pmatrix}\hat{b}_1\\ \hat{b}_2\end{pmatrix} = S\begin{pmatrix}\hat{a}_1\\ \hat{a}_2\end{pmatrix} \qquad S = \begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}$$

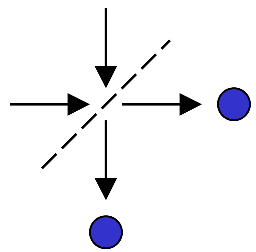
$$\Rightarrow \qquad \begin{pmatrix}\hat{a}_1{}^{+}\\ \hat{a}_2{}^{+}\end{pmatrix} = S\begin{pmatrix}\hat{b}_1{}^{+}\\ \hat{b}_2{}^{+}\end{pmatrix}, \quad \text{as } S^{+}S = 1$$

initial state : $|i\rangle = \hat{a}_1{}^{+}\hat{a}_2{}^{+}|0\rangle_{in}$

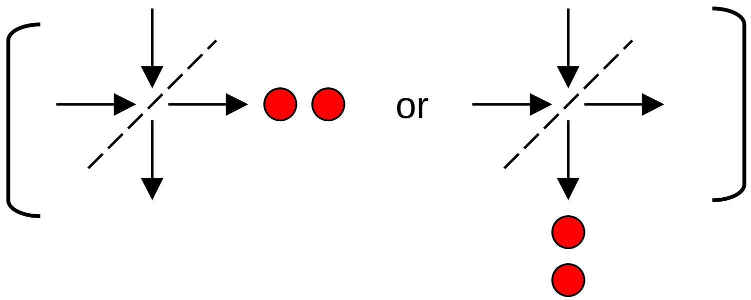
final state: $|f\rangle = \frac{1}{2}\left(\hat{b}_1{}^{+}\hat{b}_1{}^{+} - \hat{b}_2{}^{+}\hat{b}_2{}^{+}\right) + \frac{1}{2}\left(\hat{b}_1{}^{+}\hat{b}_2{}^{+} - \hat{b}_2{}^{+}\hat{b}_1{}^{+}\right)|0\rangle_{out}$

= 0 (Fermion) = 0 (Boson)

Fermions:



Bosons:



no bunching, no noise

bunching, binomial two-particle partition noise

electron sources versus photon source : reservoir

electrons

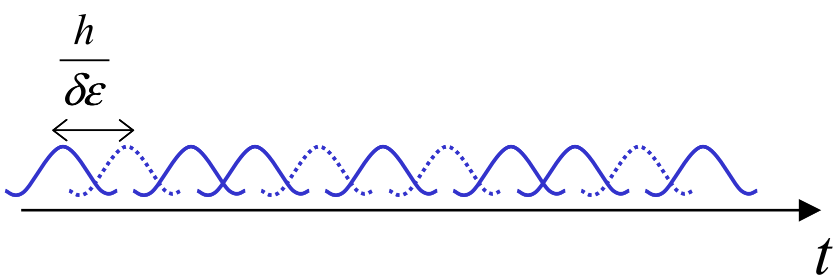
$$dI_{el.} = \frac{e}{h} f_{F.D.}(\varepsilon) d\varepsilon$$

$$\overline{N}_{\tau} = f_{F.D.}(\varepsilon)\frac{\delta\varepsilon}{h}\tau$$

$$\overline{(\Delta N)^2}_{\tau} = \overline{\left(N - \overline{N}_{\tau}\right)^2}_{\tau}$$

$$\overline{(\Delta N)^2}_{\tau} = f_{F.D.}(1 - f_{F.D.})\frac{\delta\varepsilon}{h}\tau = \overline{N}_{\tau}(1 - f_{F.D.})$$

sub - poissonian
(anti-bunching)



in particular : noiseless Fermi sea at T=0

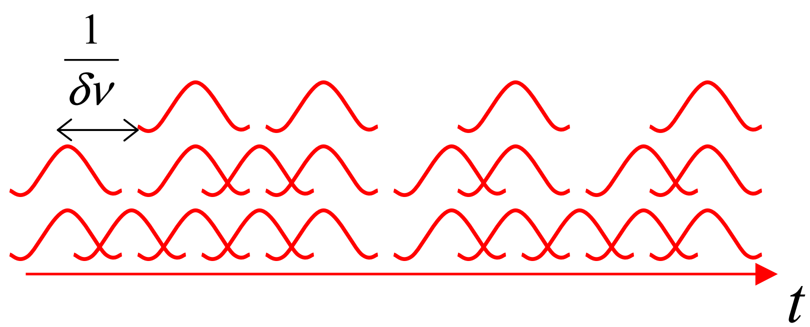
photons

$$dI_{ph.} = h\nu\, f_{B.E.}(h\nu) d\nu$$

$$\overline{N}_{\tau} = f_{B.E.}(\varepsilon)\delta\nu\tau$$

$$\overline{(\Delta N)^2}_{\tau} = f_{B.E.}(1 + f_{B.E.})\delta\nu\tau = \overline{N}_{\tau}(1 + \frac{\overline{N}_{\tau}}{\delta\nu\tau})$$

super - poissonian
(thermal photon bunching)



simple derivation of the electronic quantum shot noise for a single mode conductor

incoming current :

$$I_0 = e\left(eV/h\right)$$

(noiseless thanks
to Fermi statistics)

transmitted current :

$$I = D\, I_0 = D\frac{e^2}{h}V$$

current noise in B.W. Δf :

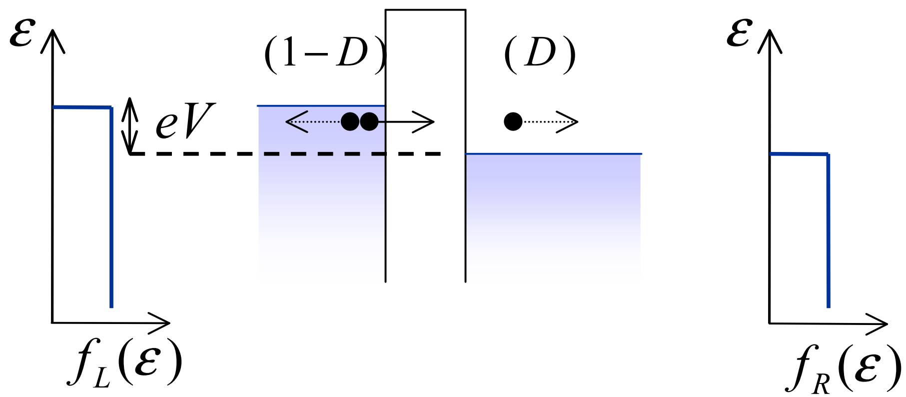
$$\left\langle (\Delta I)^2 \right\rangle = 2eI_0\, \Delta f \cdot D(1-D)$$

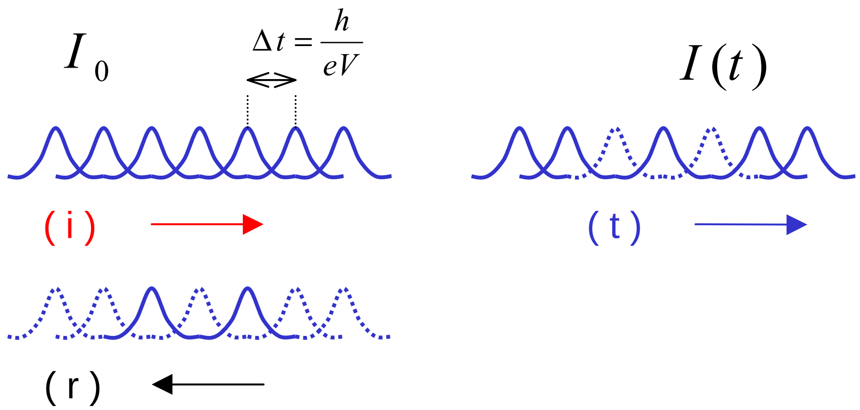
Variance of partioning
binomial statistics

$$S_I = 2eI\,(1-D) = 2eI\,.F$$

Poisson (Schottky)

reduction factor
(Fano)

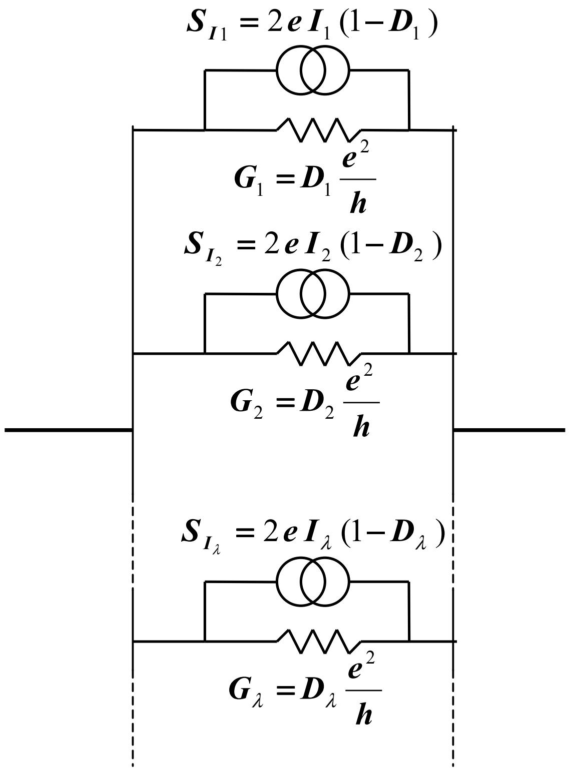




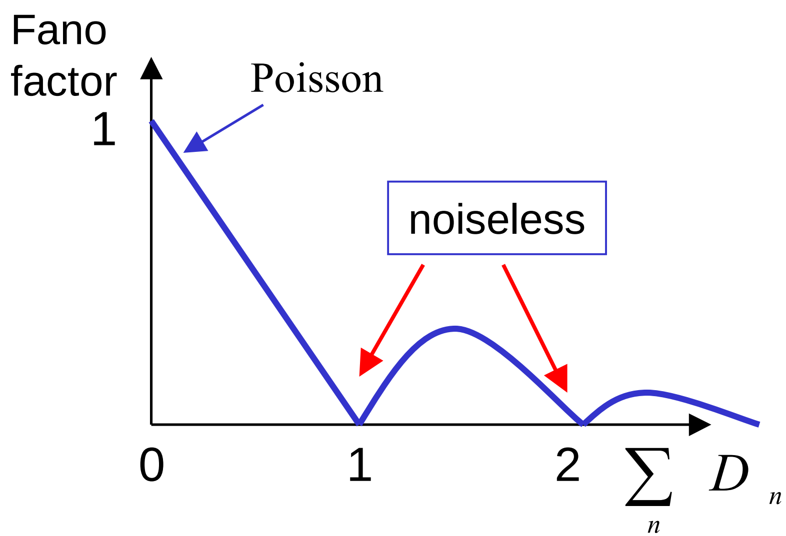
anti-correlation of transmitted
and reflected current fluctuations

V. A. Khlus, Zh. Eksp. Teor. Fiz. 93 (1987) 2179 [Sov. Phys. JETP 66 (1987) 1243].
G. B. Lesovik, Pis'ma Zh. Eksp. Teor. Fiz. 49 (1989) 513 [JETP Lett. 49 (1989) 592].

electronic shot noise for a multi-mode conductor



$$S_I = 2eI \frac{\sum_n D_n(1-D_n)}{\sum_n D_n} = 2eI.F$$



V. A. Khlus, Zh. Eksp. Teor. Fiz. 93 (1987) 2179 [Sov. Phys. JETP 66 (1987) 1243].
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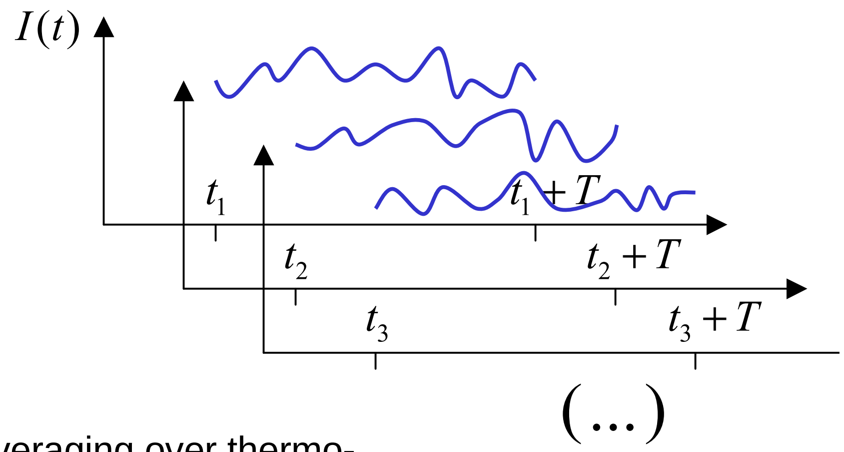
II. 2 Scattering derivation of quantum shot noise



some classical definitions to start:

$$I(t)=\sum_{n=-\infty}^{+\infty} x_n e^{-i2\pi n\frac{t}{T}} \quad ; \quad T\to\infty \quad ; \quad x_n^{\ *}=x_{-n}$$

$$\overline{I(t)}=\sum_{n=-\infty}^{+\infty} \overline{x_n} e^{-i2\pi n\frac{t}{T}} \quad \text{and} \quad \overline{I}=\overline{x_0}$$

(ensemble averaging over thermo-dynamically equivalent realizations)

$$\overline{I^2(t)}=\sum_n \sum_m \overline{x_n x_m}\, e^{-i2\pi\frac{(n+m)t}{T}} \qquad \overline{x_n x_m}=0 \text{ if } m\neq -n$$

(stationary condition)

$$=\overline{x_0^{\ 2}}+2\sum_{n=1}^{\infty}\overline{x_n x_n^{\ *}}$$

$$\overline{\Delta I^2(t)}=\overline{(I-\overline{I})^2}=2\sum_{n=1}^{\infty}\overline{x_n x_n^{\ *}}$$

$$\overline{\Delta I^2}=\int_0^{\infty} S_I(\nu)\,d\nu \quad \text{with } \nu\equiv\frac{n}{T} \quad \text{and } d\nu\equiv\frac{1}{T}$$

$$\boxed{S_I(\nu)=\lim_{T\to\infty} 2.T.\overline{x_n x_n}^{\,2}}$$

spectral density of the current

$$\overline{I(t)I(t+\tau)} = \sum_n \sum_m \overline{x_n x_m}\, e^{-i2\pi\frac{(n+m)t}{T}}\, e^{-i2\pi\frac{n\tau}{T}} \qquad \overline{x_n x_m} = 0 \text{ if } m \neq -n$$

$$= 2\sum_{n=1}^{\infty} \overline{x_n x_n^{\,*}} \cos 2\pi \frac{n\tau}{T} = \int_0^{\infty} d\nu\, S_I(\nu) \cos 2\pi\nu\tau$$

$$S_I(\nu) = 2\int_{-\infty}^{\infty} d\tau\, \overline{I(t)I(t+\tau)}\, e^{i2\pi\nu\tau}$$

this classical expression will be used to define the quantum noise spectral density

II. 2.a. quantum noise of an ideal one mode wire

- *no scattering* (no shot noise, here *only reservoir noise* is considered)
- *useful* to point out some specific and general properties of quantum noise

- first consider the noise contribution coming from the left reservoir : $\hat{\psi}_L(x,t) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\hat{a}_L(\varepsilon)e^{i(k_L x - \varepsilon t)}$

$$\hat{I}_L(x,t) = \frac{e}{h}\int d\varepsilon d\varepsilon' \frac{v_L(\varepsilon') + v_L(\varepsilon)}{2\sqrt{{}_L(\varepsilon')v_L(\varepsilon)}}\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon)e^{i(k_L(\varepsilon) - k_L(\varepsilon'))x}e^{-i(\varepsilon - \varepsilon')t}$$

$$\langle I_L \rangle = \frac{e}{h}\int d\varepsilon\, f_L(\varepsilon) \qquad \langle \hat{a}_L^{+}(\varepsilon')\hat{a}_L(\varepsilon)\rangle = f_L(\varepsilon)\delta(\varepsilon' - \varepsilon)$$

we want to compute : $S_I(\nu) = 2\int_{-\infty}^{\infty} d\tau \langle \hat{I}(0)\hat{I}(\tau)\rangle e^{i2\pi\nu\tau}$

$$\left\langle \hat{I}(x_L,0)\hat{I}(x_L,\tau)\right\rangle = \left(\frac{e}{h}\right)^2 \int d\varepsilon''' d\varepsilon'' d\varepsilon' d\varepsilon \, \frac{v_L(\varepsilon''')+v_L(\varepsilon'')}{2\sqrt{v_L(\varepsilon''')v_L(\varepsilon'')}} \frac{v_L(\varepsilon')+v_L(\varepsilon)}{2\sqrt{v_L(\varepsilon')v_L(\varepsilon)}} \; e^{i\left(k_L(\varepsilon''')-k_L(\varepsilon'')\right)x} \, e^{i\left(k_L(\varepsilon)-k_L(\varepsilon')\right)x} \ldots$$

$$\ldots \left\langle \hat{a}_L^+(\varepsilon''')\hat{a}_L(\varepsilon'')\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon) \right\rangle e^{-i(\varepsilon-\varepsilon')\tau}$$

normal pairing : $\varepsilon''' = \varepsilon''$ and $\varepsilon' = \varepsilon$

gives the contribution: $\left\langle \hat{I}(x_L,0)\right\rangle \left\langle \hat{I}(x_L,\tau)\right\rangle = \left\langle I_L \right\rangle^2$

$\rightarrow$ the fluctuations : $\left\langle \Delta\hat{I}(x_L,0).\Delta\hat{I}(x_L,\tau)\right\rangle = \left\langle \hat{I}(x_L,0)\,\hat{I}(x_L,\tau)\right\rangle - \left\langle \hat{I}(x_L,0)\right\rangle\left\langle \hat{I}(x_L,\tau)\right\rangle$

only come from the

exchange pairing term : $\varepsilon''' = \varepsilon$ and $\varepsilon' = \varepsilon''$

$$\left\langle \hat{a}_L^+(\varepsilon''')\hat{a}_L(\varepsilon'')\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon) \right\rangle_{exchange} \equiv f_L(\varepsilon)\,\delta(\varepsilon'''-\varepsilon)\left(1-f_L(\varepsilon')\right)\delta(\varepsilon''-\varepsilon')$$

$$\left\langle \hat{I}(x_L,0)\hat{I}(x_L,\tau)\right\rangle = \left(\frac{e}{h}\right)^2 \int d\varepsilon' d\varepsilon \, \frac{\left(v_L(\varepsilon') + v_L(\varepsilon)\right)^2}{4 v_L(\varepsilon') v_L(\varepsilon)} \, f_L(\varepsilon)\left(1 - f_L(\varepsilon')\right) e^{i2\left(k_L(\varepsilon) - k_L(\varepsilon')\right)x} \, e^{-i\left(\varepsilon - \varepsilon'\right)\tau/\hbar}$$

spectral density of the current fluctuations:

$$S_I(\nu) = 2\int_{-\infty}^{\infty} d\tau \left\langle \hat{I}(0)\hat{I}(\tau)\right\rangle e^{i2\pi\nu\tau}$$

$$S_{I_L}(\nu) \cong 2\frac{e^2}{h}\int d\varepsilon \, f_L(\varepsilon)\left(1 - f_L(\varepsilon - \hbar\omega)\right)$$

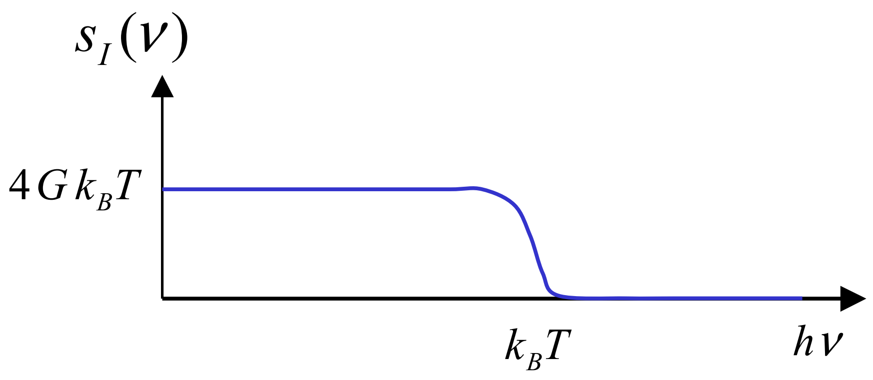
$(\hbar\omega << E_F$ and $v_F/\omega >> L)$

adding contribution of right the reservoir:

$$S_I(\nu) = 4\frac{e^2}{h}\int d\varepsilon \, f_L(\varepsilon)\left(1 - f_L(\varepsilon - \hbar\omega)\right)$$

$$= 4\frac{e^2}{h}\frac{\hbar\omega}{e^{\frac{\hbar\omega}{k_B T}} - 1}$$

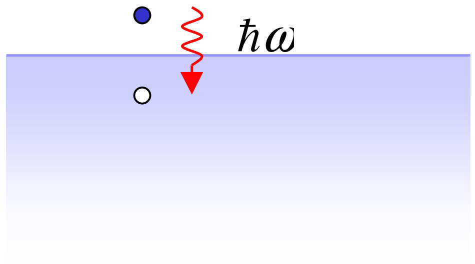
$$= 4\frac{e^2}{h}\hbar\omega \, N(\omega)$$



no (detectable) reservoir noise at zero temperature

$$S_I(\nu) = 4\frac{e^2}{h}\int d\varepsilon\, f_L(\varepsilon)\big(1 - f_L(\varepsilon - \hbar\omega)\big)$$
$$= 4\frac{e^2}{h}\frac{\hbar\omega}{e^{\frac{\hbar\omega}{k_B T}} - 1}$$
$$= 4\frac{e^2}{h} h\nu\, N(\nu)$$

('spontaneous' fluctuations)



only fluctuations corresponding to electronic transitions *down in energy* are considered in this definition of S_I

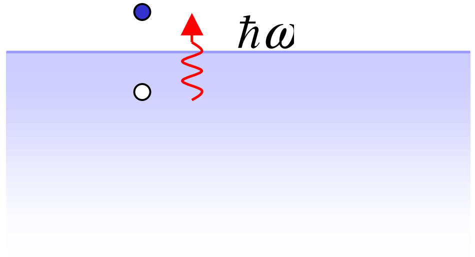
$$S_I(\nu) = 2\int_{-\infty}^{\infty} d\tau \left\langle \hat{I}(0)\hat{I}(\tau)\right\rangle e^{i2\pi\nu\tau}$$

does not commute

$$S_I'(\nu) = S_I(-\nu) = 2\int_{-\infty}^{\infty} d\tau \left\langle \hat{I}(0)\hat{I}(\tau)\right\rangle e^{-i2\pi\nu\tau} = 2\int_{-\infty}^{\infty} d\tau \left\langle \hat{I}(\tau)\hat{I}(0)\right\rangle e^{+i2\pi\nu\tau}$$

$$S'_I(\nu) = 4\frac{e^2}{h}\int d\varepsilon\, f_L(\varepsilon)\big(1 - f_L(\varepsilon + \hbar\omega)\big)$$
$$= 4\frac{e^2}{h} h\nu\big(N(\nu) + 1\big)$$

('stimulated' fluctuations)

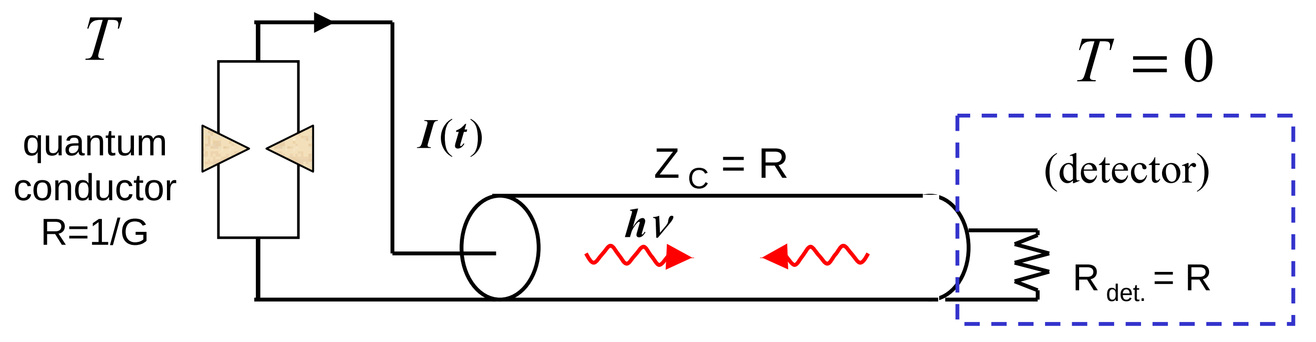


transitions up in energy: these fluctuations are revealed when connecting to an 'active' detector able to excite the Fermi sea

-fluctuation -dissipation (here calculated in the frame of the scattering approach)

$$S_I(-\nu) - S_I(\nu) = 2\,G\,h\nu$$ (Kubo)

- meaning of $S_I(\nu)$:



$$P = R S_I(\nu) d\nu$$

(noise detectable with detector in ground state) (also Nyquist 1928)

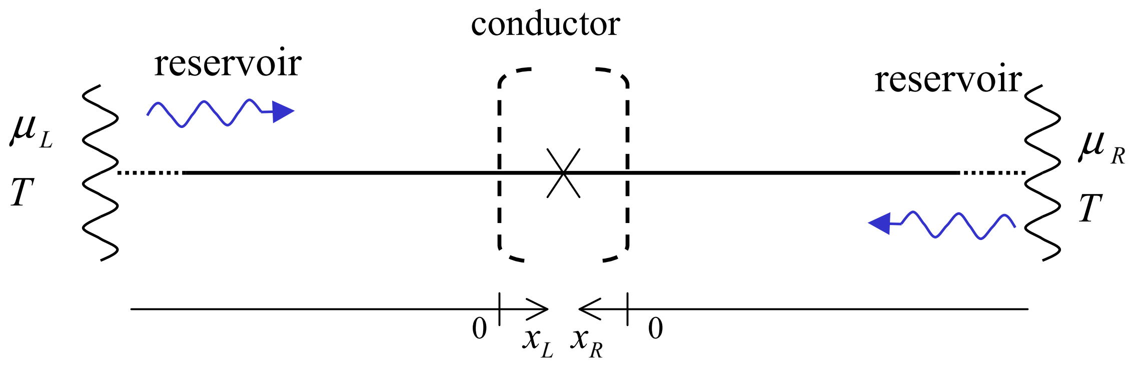
(see more in last part of the talk, if time permits)

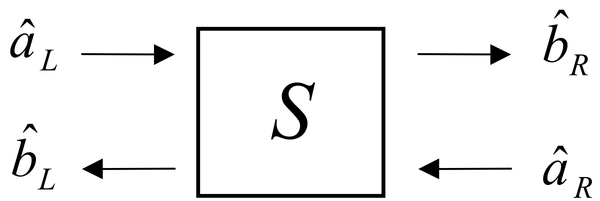
For a detector at finite temperature

$$P \propto S_I(\nu).(N_{Det.}+1) - S_I(-\nu).N_{Det.}$$

Lesovik and Loosen, JETP. Lett. 65, 295 (1997)
Y. Gavish, Y. Imry and Y. Levinson, Phys.Rev.B. 62, 10637 (2000)

II . 2.b. quantum shot noise for a single mode





$$S = \begin{pmatrix} s_{LL} & s_{LR} \\ s_{RL} & s_{RR} \end{pmatrix} \equiv \begin{pmatrix} r & it \\ it & r \end{pmatrix}$$

$$S\,S^{+} = S^{+}S = 1$$

scattering states in the reservoirs:

$$\hat{\psi}(x_L) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\left(\hat{a}_L(\varepsilon)e^{ik_L x_L} + \hat{b}_L(\varepsilon)e^{-ik_L x_L}\right)$$

$$\hat{\psi}(x_R) = \frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_R(\varepsilon)}}\left(\hat{a}_R(\varepsilon)e^{ik_R x_R} + \hat{b}_R(\varepsilon)e^{-ik_R x_R}\right)$$

$$t^2 = D = 1 - R$$

$$r^2 = R$$

Current fluctuations will be calculated in the left reservoir lead

$$\hat{\psi}(x_L,t)=\frac{e}{h}\int d\varepsilon \frac{1}{\sqrt{2\pi\hbar v_L(\varepsilon)}}\left(\hat{a}_L(\varepsilon)e^{ik_L x_L}+\hat{b}_L(\varepsilon)e^{-ik_L x_L}\right)e^{-i\varepsilon t/\hbar}$$

$$\hat{b}_L = r\hat{a}_L + it\hat{a}_R$$

$$\hat{I}(x_L,t)=\frac{e}{h}\int d\varepsilon d\varepsilon'\left\{\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon)-\hat{b}_L^+(\varepsilon')\hat{b}_L(\varepsilon)\right\}\frac{v_L(\varepsilon')+v_L(\varepsilon)}{2\sqrt{_L(\varepsilon')v_L(\varepsilon)}}e^{i(k_L-k_L')x_L}\ e^{i(\varepsilon'-\varepsilon)t}$$

... plus a factor

$$\propto \frac{(v_L(\varepsilon')-v_L(\varepsilon))}{\left(v_L(\varepsilon')v_L(\varepsilon)\right)^{1/2}}\left(a_L{}^+b_L \ldots b_L{}^+a_L\right)$$

also : ~ 1

negligeable if ε'-ε~ $h\nu$ << E_F

contribute to fluctuations within reservoirs

$$\hat{I}(x_L,t)=\frac{e}{h}\int d\varepsilon d\varepsilon'\left\{\begin{array}{l} t^2\left(\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon)-\hat{a}_R^+(\varepsilon')\hat{a}_R(\varepsilon)\right)-\ldots \\ \ldots irt\left(\hat{a}_L^+(\varepsilon')\hat{a}_R(\varepsilon)-\hat{a}_R^+(\varepsilon')\hat{a}_L(\varepsilon)\right)\end{array}\right\}\ e^{i(k_L-k_L')x_L}\ e^{i(\varepsilon'-\varepsilon)t}$$

contribute to fluctuations between reservoirs (partitionning)

$$\hat{I}(0)\hat{I}(\tau)=\left(\frac{e}{h}\right)^2\int d\varepsilon'''d\varepsilon''d\varepsilon'd\varepsilon\ \left[t^2\left(\hat{a}_L^+(\varepsilon''')\hat{a}_L(\varepsilon'')-\hat{a}_R^+(\varepsilon''')\hat{a}_R(\varepsilon'')\right)-irt\left(\hat{a}_L^+(\varepsilon''')\hat{a}_R(\varepsilon'')-\hat{a}_R^+(\varepsilon''')\hat{a}_L(\varepsilon'')\right)\right]....$$

$$...\times\left[t^2\left(\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon)-\hat{a}_R^+(\varepsilon')\hat{a}_R(\varepsilon)\right)-irt\left(\hat{a}_L^+(\varepsilon')\hat{a}_R(\varepsilon)-\hat{a}_R^+(\varepsilon')\hat{a}_L(\varepsilon)\right)\right]e^{-i(\varepsilon-\varepsilon')\tau}$$

$$\left\langle \hat{I}(x_L,0)\,\hat{I}(x_L,\tau)\right\rangle=??$$

normal pairing : $\varepsilon'''=\varepsilon''$ and $\varepsilon'=\varepsilon$

gives the contribution: $\left\langle\hat{I}(0)\right\rangle\left\langle\hat{I}(\tau)\right\rangle=\left\langle I_L\right\rangle^2$

→ the *fluctuations* : $\left\langle\Delta\hat{I}(x_L,0).\Delta\hat{I}(x_L,\tau)\right\rangle=\left\langle\hat{I}(x_L,0)\,\hat{I}(x_L,\tau)\right\rangle-\left\langle\hat{I}(x_L,0)\right\rangle\left\langle\hat{I}(x_L,\tau)\right\rangle$

again come from the exchange term : $\varepsilon'''=\varepsilon$ and $\varepsilon'=\varepsilon''$

first part:

$$\begin{cases} D^2\left\langle\hat{a}_L^+(\varepsilon''')\hat{a}_L(\varepsilon'')\,\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon)\right\rangle\equiv f_L(\varepsilon)\,\left(1-f_L(\varepsilon')\right)\delta(\varepsilon'''-\varepsilon)\,\delta(\varepsilon''-\varepsilon') \\ D^2\left\langle\hat{a}_R^+(\varepsilon''')\hat{a}_R(\varepsilon'')\,\hat{a}_R^+(\varepsilon')\hat{a}_R(\varepsilon)\right\rangle\equiv f_R(\varepsilon)\,\left(1-f_R(\varepsilon')\right)\delta(\varepsilon'''-\varepsilon)\,\delta(\varepsilon''-\varepsilon') \end{cases}$$

= emission noise of each reservoir. the D^2 term indicates two-particle emitted by a reservoir and transmitted

$$\hat{I}(0)\hat{I}(\tau)=\left(\frac{e}{h}\right)^2\int d\varepsilon'''d\varepsilon''d\varepsilon'd\varepsilon\ \left[t^2\left(\hat{a}_L^+(\varepsilon''')\hat{a}_L(\varepsilon'')-\hat{a}_R^+(\varepsilon''')\hat{a}_R(\varepsilon'')\right)-irt\left(\hat{a}_L^+(\varepsilon''')\hat{a}_R(\varepsilon'')-\hat{a}_R^+(\varepsilon''')\hat{a}_L(\varepsilon'')\right)\right]....$$

$$...\times\left[t^2\left(\hat{a}_L^+(\varepsilon')\hat{a}_L(\varepsilon)-\hat{a}_R^+(\varepsilon')\hat{a}_R(\varepsilon)\right)-irt\left(\hat{a}_L^+(\varepsilon')\hat{a}_R(\varepsilon)-\hat{a}_R^+(\varepsilon')\hat{a}_L(\varepsilon)\right)\right]e^{-i(\varepsilon-\varepsilon')\tau}$$

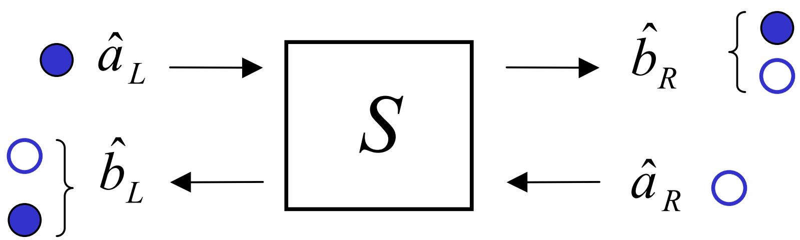
$\rightarrow$ the *fluctuations* : $\left\langle\Delta\hat{I}(x_L,0).\Delta\hat{I}(x_L,\tau)\right\rangle=\left\langle\hat{I}(x_L,0)\,\hat{I}(x_L,\tau)\right\rangle-\left\langle\hat{I}(x_L,0)\right\rangle\left\langle\hat{I}(x_L,\tau)\right\rangle$

again come from the exchange term : $\varepsilon'''=\varepsilon$ and $\varepsilon'=\varepsilon''$

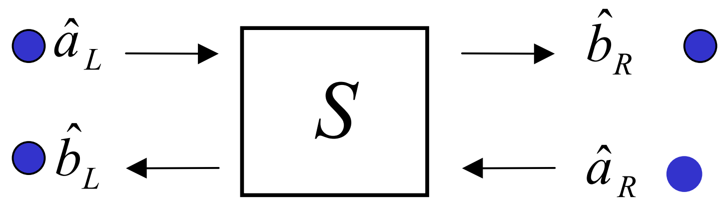
second part:

$$\begin{cases} (-irt)^2\left\langle\ \hat{a}_L^+(\varepsilon''')\hat{a}_R(\varepsilon'')\left(-\hat{a}_R^+(\varepsilon')\hat{a}_L(\varepsilon)\right)+\left(-\hat{a}_R^+(\varepsilon''')\hat{a}_L(\varepsilon'')\right)\hat{a}_L^+(\varepsilon')\hat{a}_R(\varepsilon)\ \right\rangle \\ \Rightarrow\qquad RD\left[f_L(\varepsilon)\ \left(1-f_R(\varepsilon')\right)+f_R(\varepsilon)\ \left(1-f_l(\varepsilon')\right)\right]\delta(\varepsilon'''-\varepsilon)\,\delta(\varepsilon''-\varepsilon') \end{cases}$$

= partition shot noise..



partition noise



'Pauli blocking' of partition noise

$$S_I(\nu) = 2\int_{-\infty}^{\infty} d\tau\, \overline{I(t)I(t+\tau)}\, e^{i2\pi\nu\tau}$$

complete finite frequency, finite temperature and voltage formula:

$$S_I(\nu)=2\frac{e^2}{h}\int d\varepsilon\left\{D^2\left[f_L(\varepsilon)\left(1-f_L(\varepsilon-h\nu)\right)+f_R(\varepsilon)\left(1-f_R(\varepsilon-h\nu)\right)\right]+RD\left[f_L(\varepsilon)\left(1-f_R(\varepsilon-h\nu)\right)+f_R(\varepsilon)\left(1-f_l(\varepsilon-h\nu)\right)\right]\right\}$$

reservoir emission noise

shot noise

EQUILIBRIUM : $(f_R = f_L = f)$

$$S_I(\nu)=4D\frac{e^2}{h}\frac{\hbar\omega}{e^{\frac{\hbar\omega}{k_BT}}-1} \qquad G = D\frac{e^2}{h}$$

$$S_I(\nu)=4Gk_BT \qquad h\nu << k_BT$$

$$D = D^2 + D(1-D)$$

thermal noise of reservoirs

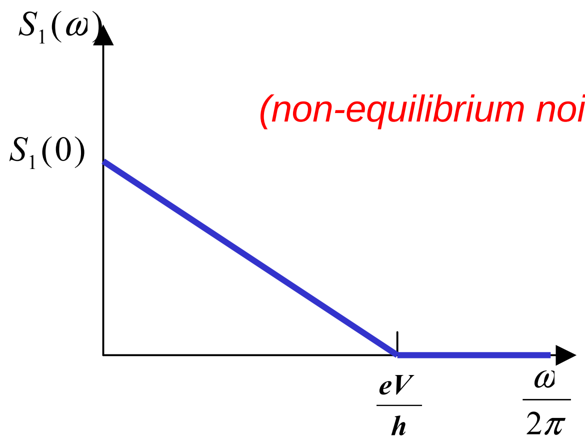
partition noise of thermal electrons

equilibrium noise
=
thermal noise
+
partition noise
(equal amount)

NON EQUILIBRIUM, and T=0 : $(\mu_L = \mu_R + eV)$

$$S_I(\nu)=2.D(1-D)\frac{e^2}{h}\left(eV-h\nu\right) \qquad eV \geq h\nu$$

$$=0 \qquad eV < h\nu$$



II. 2. C. zero frequency shot noise and multimode case

shot noise at low frequency and zero temperature

(one mode)

$$S_I = 2eI\,(1-D) = 2eI\,.F \qquad \text{for} \quad \nu \to 0$$

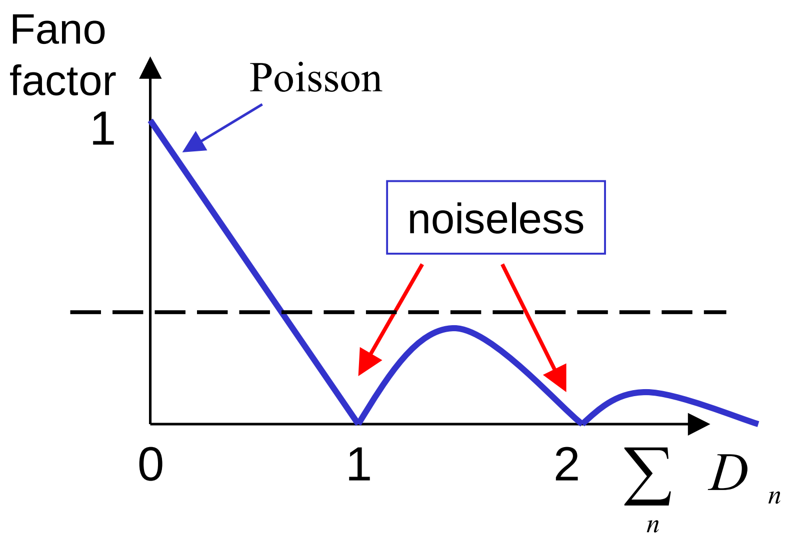
Poisson (Schottky)

reduction factor (Fano)

generalization to multiple modes:

$$Tr\left[S_{21}S_{21}^{+}(1-S_{21}S_{21}^{+})\right]$$

$$S_I = 2eI\,\frac{\sum_n D_n(1-D_n)}{\sum_n D_n} = 2eI\,.F$$



G. B. Lesovik, Pis'ma Zh. Eksp. Teor. Fiz. 49 (1989) 513 [JETP Lett. 49 (1989) 592].

Introduction

I. Electronic scattering (a brief introduction)

II. Quantum Shot noise

1 - Quantum partition noise
- one and two particle partitioning :electrons/ photons
- electronic shot noise

2- scattering derivation of quantum shot noise
a- $S(\omega)$ for an ideal one mode conductor
b- quantum shot noise for a single mode
c-zero frequency shot noise and multimode case

→ 3- experimental examples

4- current noise cross-correlations
- scattering derivations
- electronic analog of the optical Hanbury-Brown Twiss experiment
- electronic quantum exchange

III Shot Noise and Interactions:

IV. Shot noise: ***the*** tool to detect entanglement

V. Shot noise and high frequencies

EXPERIMENTAL EXAMPLES

numbers:

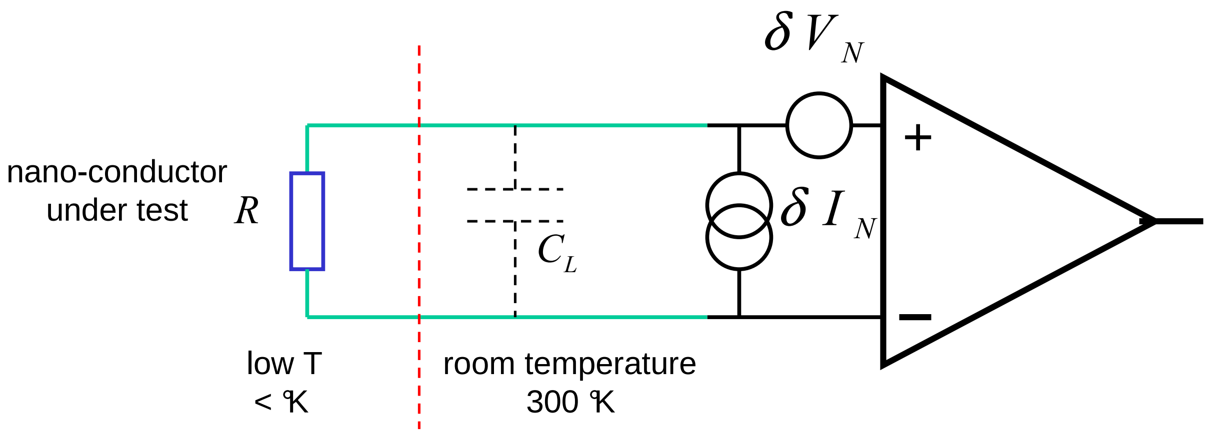
$$100\text{mK} \approx 10\mu\text{V}$$

$$\frac{2e^2}{h}\times 10\mu\text{V} \approx 0.8\,\text{nA}$$

$$S_I = 2eI \approx 2.610^{-38}\,\text{A}^2/\text{Hz} = \left(16\,\text{fA}/\sqrt{\text{Hz}}\right)^2$$

$$S_V = \left(200\text{pV}/\sqrt{\text{Hz}}\right)$$

detection noise:



noise power added by the amplifier reffered to resistor R:

$$k_B T_N \Delta f = \frac{1}{4R}\left(\delta V_N\right)^2 + \frac{R}{4}\left(\delta I_N\right)^2$$

optimal resistor:

$$R_{opt.} = \frac{\sqrt{\left(\delta V_N\right)^2}}{\sqrt{\left(\delta I_N\right)^2}}$$

lowest T_N:

$$k_B T_N \Delta f = \frac{1}{2}\sqrt{\left(\delta V_N\right)^2}\sqrt{\left(\delta I_N\right)^2}$$

excellent room temperature commercial LNA (100kHz range) :

$$\sqrt{(\delta V_N)^2} \approx 1.3\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$$
$$\sqrt{(\delta I_N)^2} \approx 13\,\mathrm{fA}/\sqrt{\mathrm{Hz}}$$
$$R_{opt.} \approx 100\mathrm{kOhms}$$
$$\mathrm{T_N}^{opt} \approx 700\mathrm{mK}$$

(LI75A from NF)

well adapted to quantum point contacts,quantum dots, STM,etc,... provided microphonic noise sources in the audio range are carefully eliminated

(one meter coax limits to f < 16 kHz for 100kOhm sample)

good room temperature commercial 80MHz range LNA:

$$\sqrt{(\delta V_N)^2} \approx 0.45\,\mathrm{nV}/\sqrt{\mathrm{Hz}}$$
$$\sqrt{(\delta I_N)^2} \approx 130\,\mathrm{fA}/\sqrt{\mathrm{Hz}}$$
$$R_{opt.} \approx \text{few kOhms}$$
$$\mathrm{T_N}^{opt} \approx 2.5\mathrm{K}$$

(220 FS from NF)

well adapted to diffusive wire in semi-conductors, superconducting/2DEG hybride junctions, ...
Higher frequency (up to few MHz without appreciable capacitive shunting)

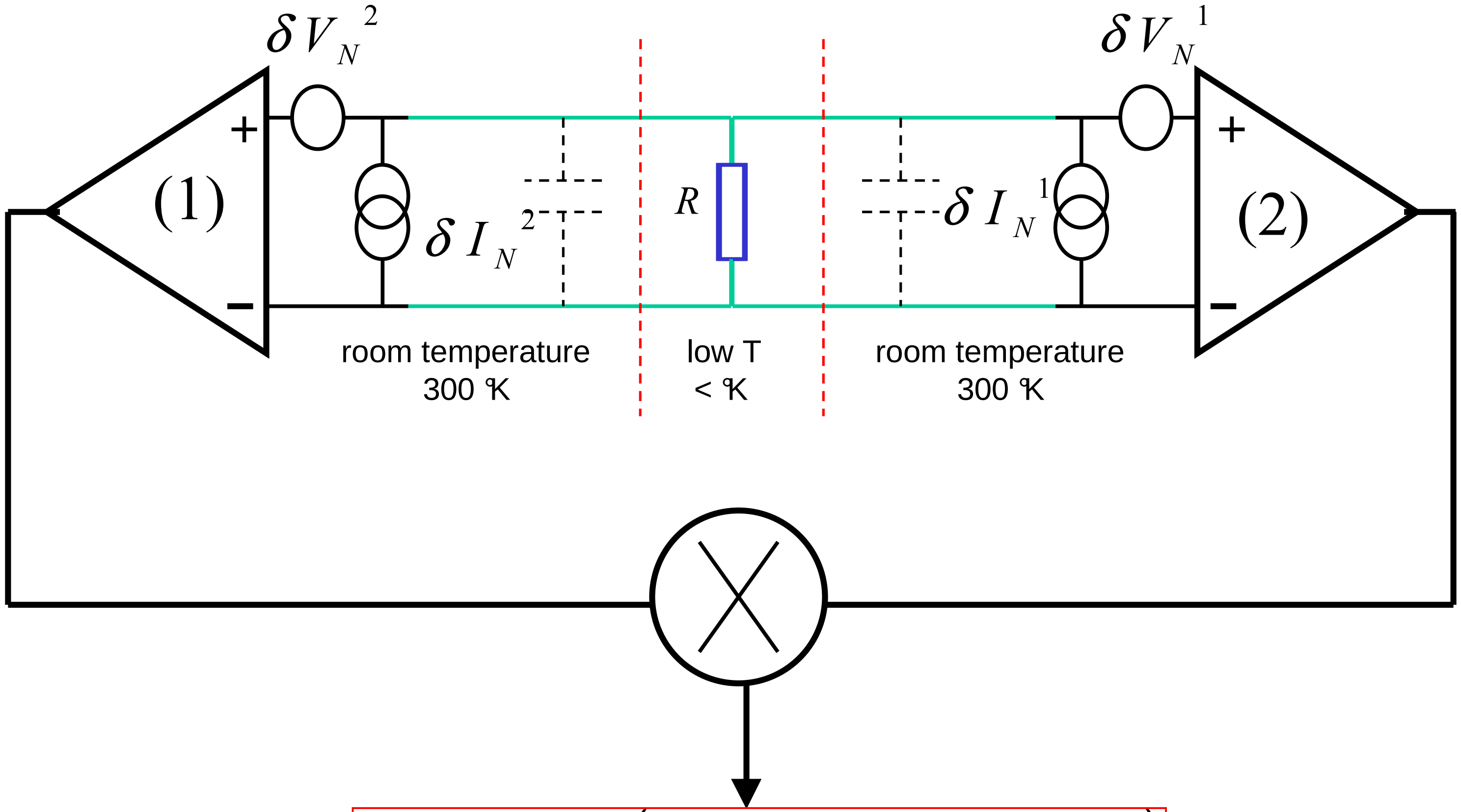
microwave LNAs (GHz range):

bad coupling : 50 Ohms versus 13kOhms
but fast:

$$\sqrt{(\delta V_N)^2} \approx 0.30\,\mathrm{nV}/\sqrt{\mathrm{Hz}} \quad \text{(room temperature)} \quad T_N = 30°K \text{ on } 50\,\Omega$$
$$\sqrt{(\delta V_N)^2} \approx 100-80\,\mathrm{pV}/\sqrt{\mathrm{Hz}} \quad \text{(cooled} < \text{20Kelvin)} \quad T_N = 3-2°K \text{ on } 50\,\Omega$$

$$\delta T_N \equiv \frac{T_N}{\sqrt{\Delta f.\tau}}$$

cross-correlations



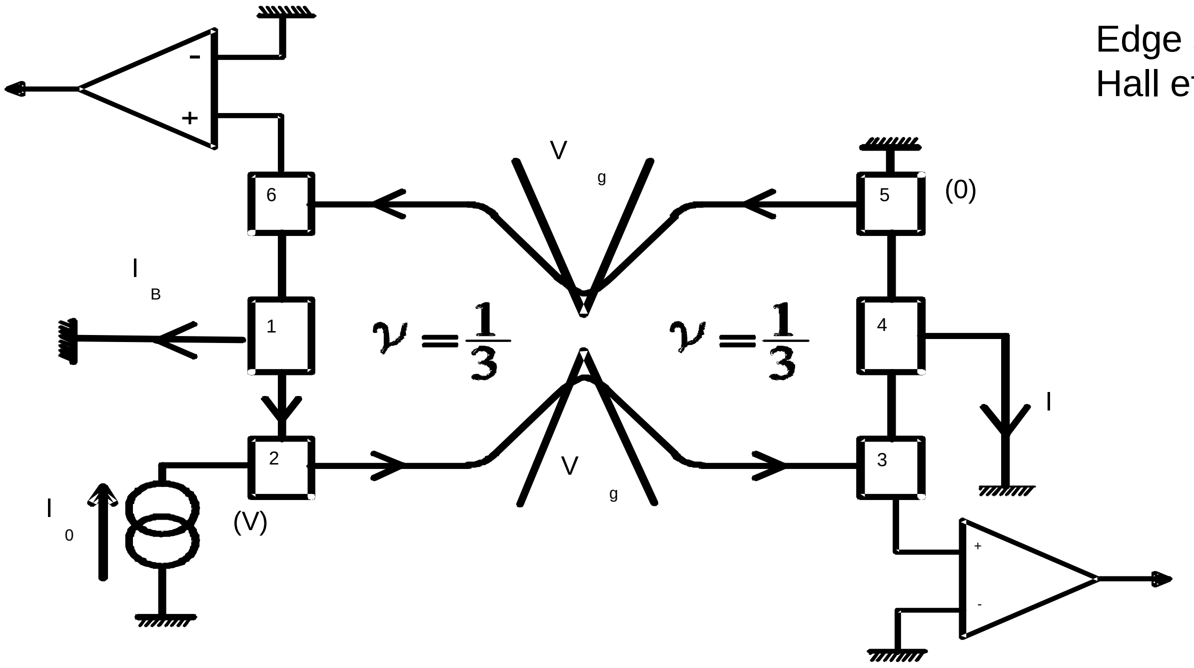
$$S_{V_1V_2}\Delta f = R^2.\left(S_I \Delta f + \left(\delta I_N^1\right)^2 + \left(\delta I_N^2\right)^2 \right)$$

physical sample noise to be measured

(white) current noise of each amplifier added

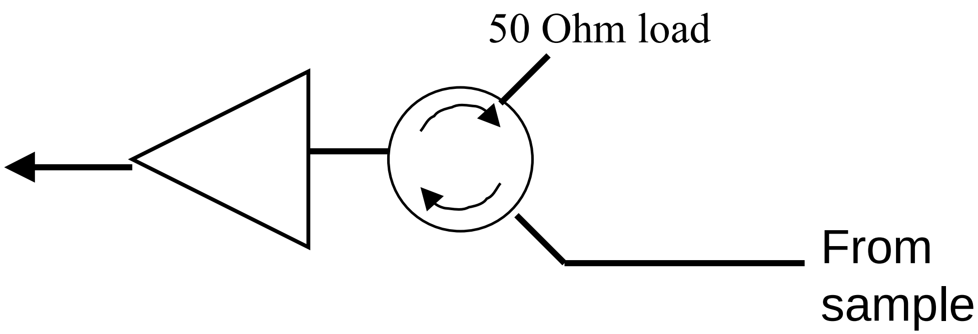
eliminates uncorrelated amplifier voltage noise, thermal noise of leads, reduces microphonic noise.
(like four-point resistance measurements). Improvement in reliability (not noise sensitivity):

trick: make use of chirality, when possible to eliminate current noise of the amplifiers



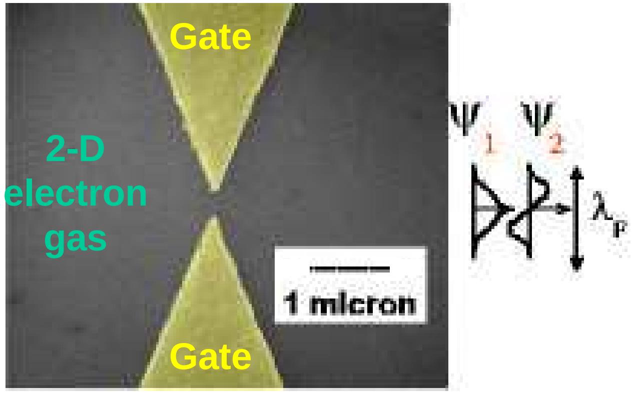
Edge states of the Quantum Hall effect

High frequency (microwaves): isolators (also called circulators)



in both cases chirality helps!!

quantum point contact



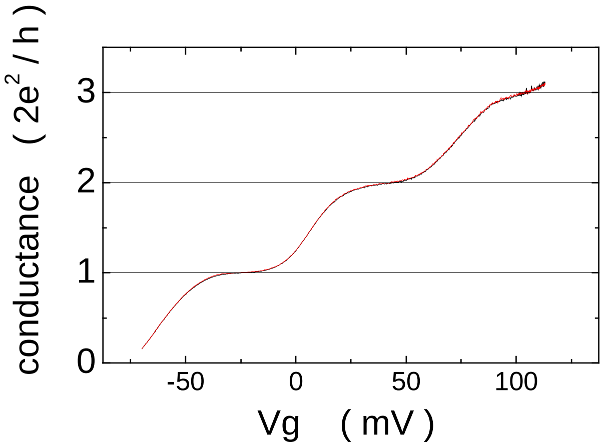
$\lambda_F \approx 70$ nm

$l_{elast.} \approx 10-20\ \mu$m

(ballistic conductor)

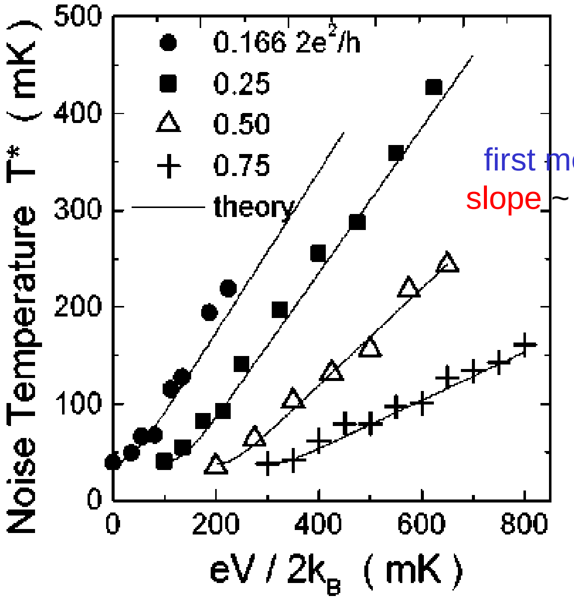
$$F = \frac{\sum_n D_n(1-D_n)}{\sum_n D_n}$$

$$S_I = F.2eI$$

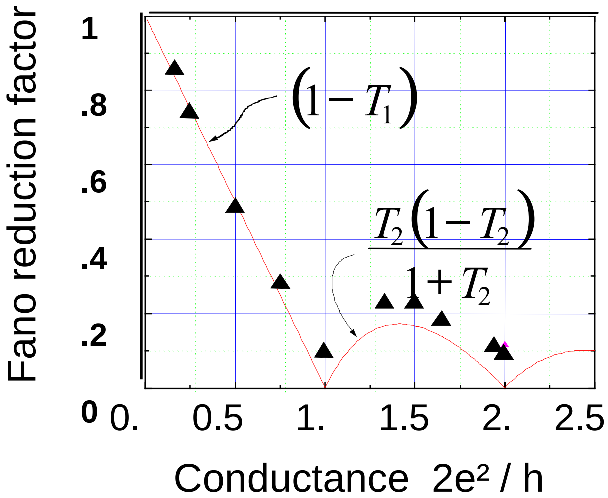


$$G = \frac{2e^2}{h}.\sum_n D_n$$

$$T^* = \frac{S_I}{4Gk_B}$$



first mode :
slope ~ (1 - D1)



D.C. Glattli, NTT-BRL School, 03 november 05

A. Kumar et al. Phys. Rev. Lett. 76 (1996) 2778..
M. I. Reznikov et al., Phys. Rev. Lett. 75 (1995) 3340.

Thermal to shot noise cross-over regime

$$S_I = 2\frac{e^2}{h} k_B T . \sum_n D_n^2 + 2\frac{e^2}{h} eV . \sum_n D_n (1 - D_n) \coth\left(\frac{eV}{2 k_B T}\right)$$

thermal emission noise of reservoirs

shot noise

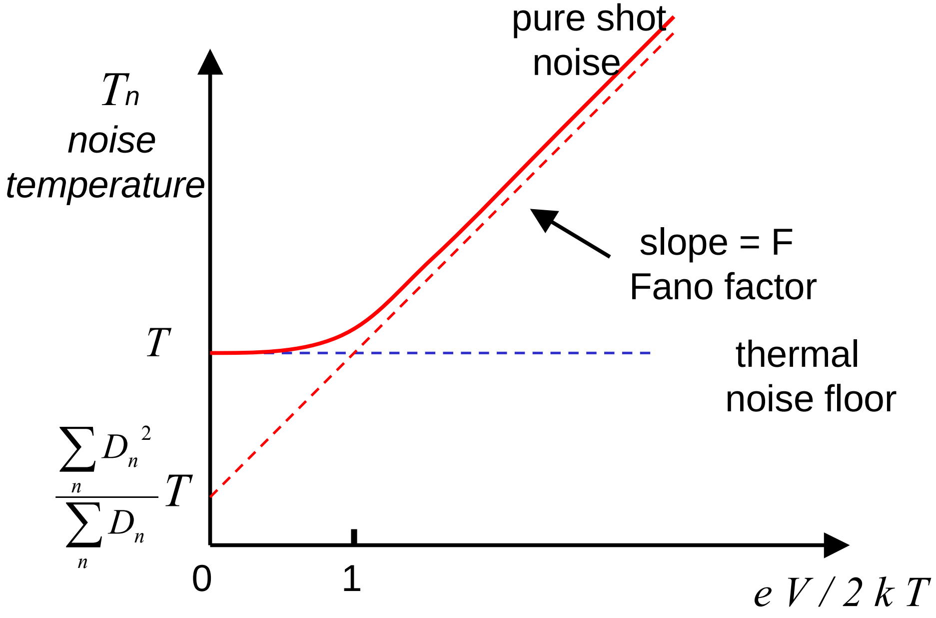
M. Büttiker, Phys. Rev. Lett. 65 (1990) 2901.
R. Landauer and Th. Martin, Physica B 175 (1991)

Johnson Nyquist noise at $V = 0$

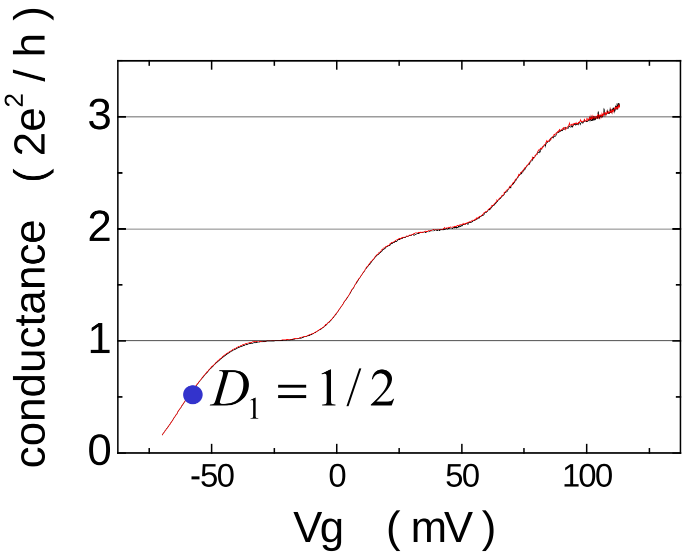
$$S_I = 4\frac{e^2}{h} k_B T . \sum_n D_n = 4 G k_B T$$

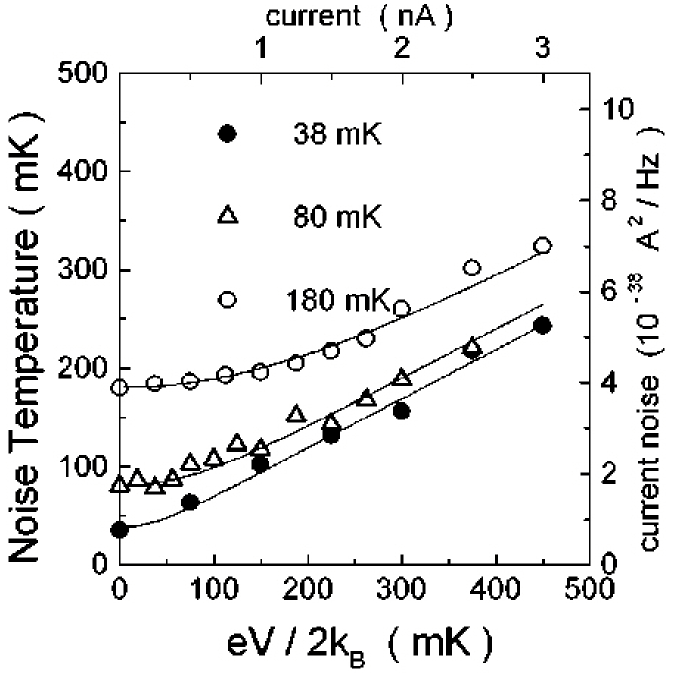
Def. : noise temperature T_n

$$T_n = \frac{S_I}{4 G k_B}$$



thermal to shot-noise cross-over checked using a QPC





A. Kumar et al. Phys. Rev. Lett. 76 (1996) 2778..

no adjustable parameter

$$S_I = 2\,\frac{e^2}{h}\,k_B T\,.\,D_1^2 + 2\,\frac{e^2}{h}\,eV\,.\,D_1(1-D_1)\,\coth\left(\frac{eV}{2k_B T}\right)$$

- electron shot noise reaches quantum partition noise limit

- in general quantum conductor show sub-poisonian noise

various Fano factor have been observed in agreement with theory

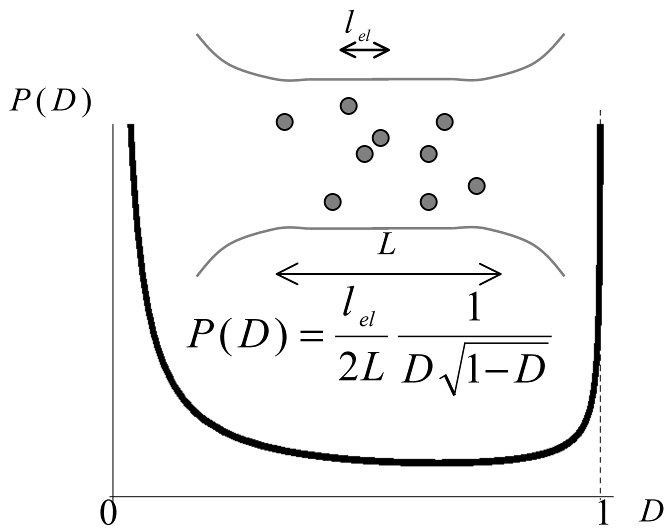
- F =1/3 : diffusive conductors
- F =1/4 : electron billards (quantum chaos)
- F =1/2 : quantum dots
- ….

$$F = \frac{\sum_\lambda D_\lambda (1-D_\lambda)}{\sum_\lambda D_\lambda} \equiv \frac{\langle D(1-D)\rangle}{\langle D \rangle}$$

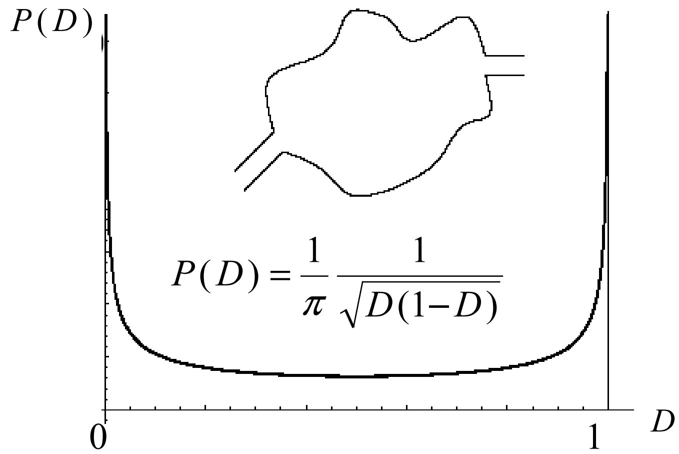
where : $\langle \; \rangle$ is the average over the probability distribution $P(\{D\})$ of transmissions D_λ

diffusive : $\langle D \rangle = \frac{l_{el}}{L} << 1$

chaotic :



$$P(D) = \frac{l_{el}}{2L} \frac{1}{D\sqrt{1-D}}$$



$$P(D) = \frac{1}{\pi} \frac{1}{\sqrt{D(1-D)}}$$

(question : what is quantum in the 1/3 Fano factor of diffusive electronic systems ?)

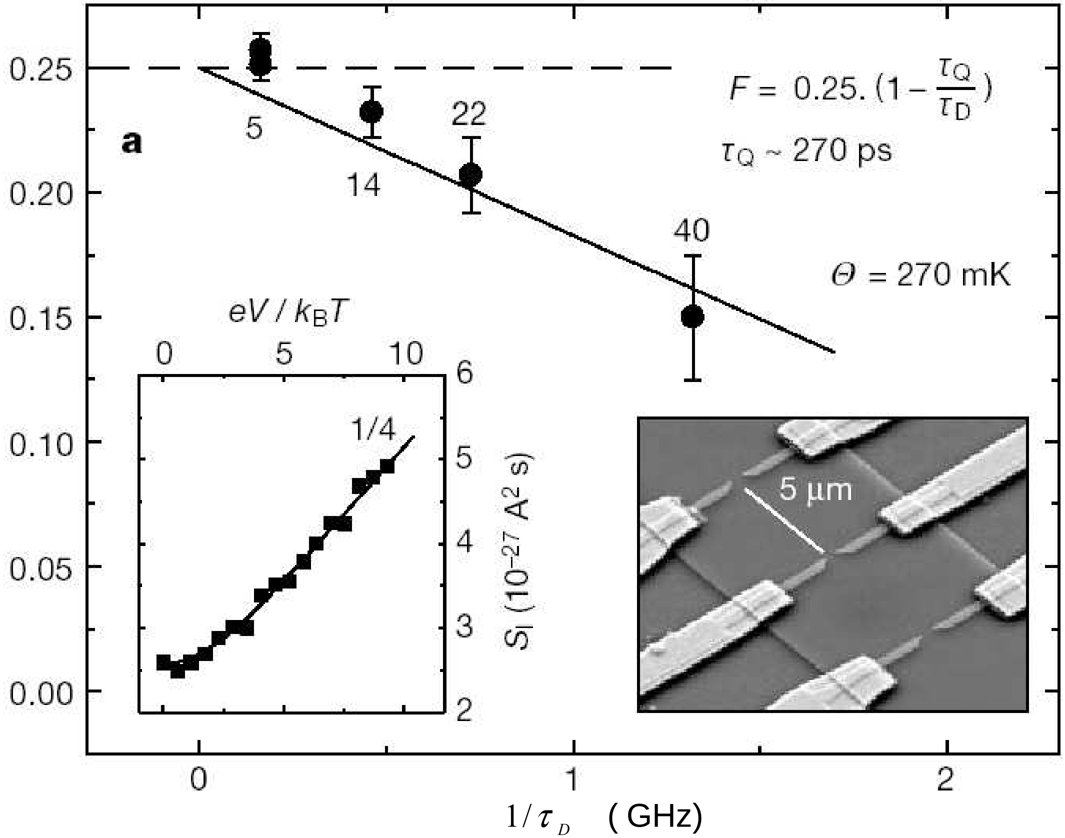
Chaotic cavity (electron billard)

cross-over from quantum to classical (no noise) regime.

noise is quantum !

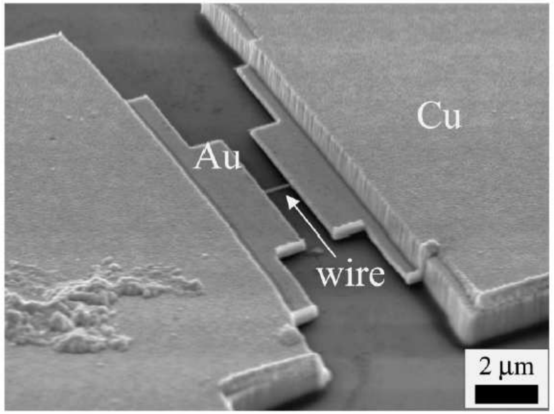
FANO Factor

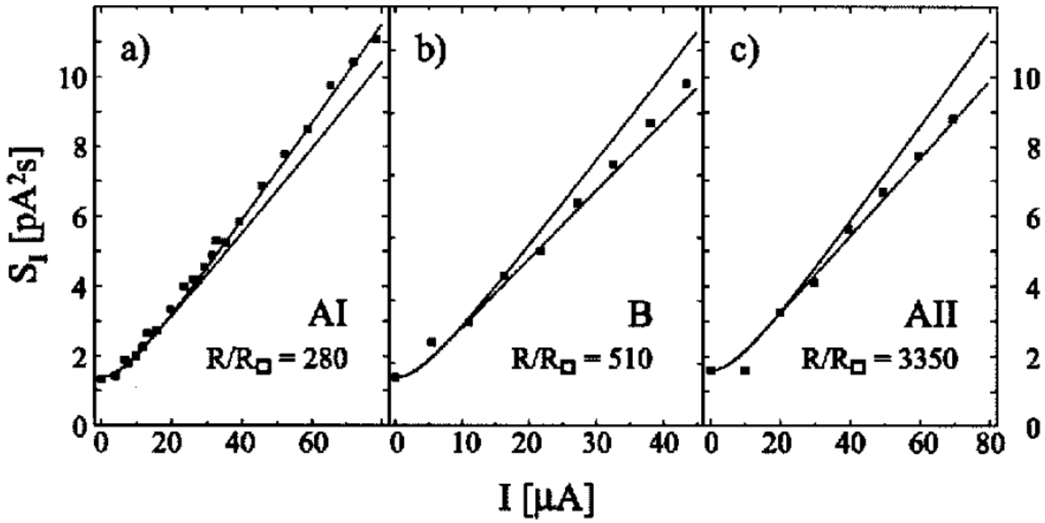
$S_I / 2e|I|$



Oberholzer et al. Nature (2002)

$$S_I = F \, .2eI$$





lower slope : diffusive regime

$$F = 1/3$$

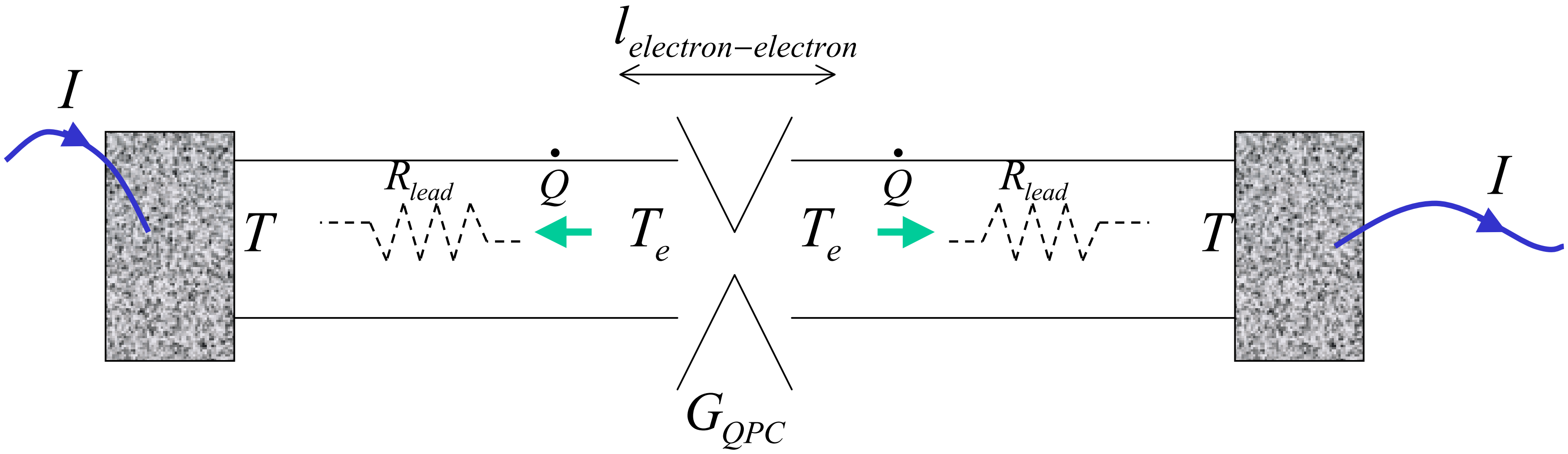
upper slope : hot electron regime
(see later)

$$"F" = \sqrt{3}/4$$

M. Henny, S. Oberholzer, C. Strunk, and C. Schonenberger,
Phys. Rev. B 59 (1999) 2871.

(question : what is quantum in the 1/3 Fano factor of diffusive electronic systems ?)

heating effect : apparent shot noise



$$\dot{Q} = \frac{1}{2} G_{QPC} V^2$$ heat produced

$$\dot{Q} = \frac{\pi^2}{3}\left(\frac{k_B}{e^2}\right)^2 \frac{T_e^{\,2} - T^2}{R_{lead}}$$ heat flow Wiedeman-Franz

$$T_e^{\,2} = T^2 + \frac{3}{2\pi^2}\left(\frac{e^2}{k_B}\right)^2 G_{QPC} R_{lead} V^2$$

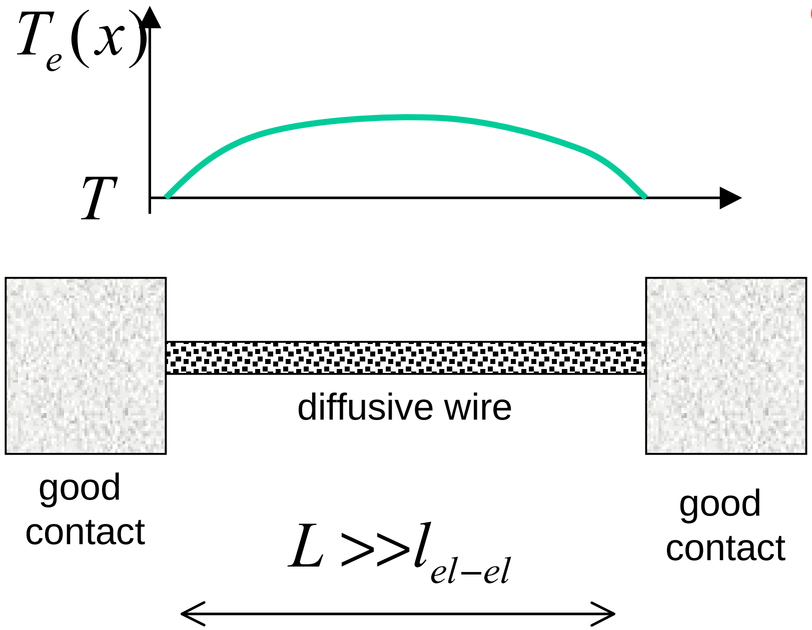
$$S_I = 4\, G_{QPC}\, k_B T_e$$

$$S_I = 2eI\sqrt{\frac{6}{\pi^2} G_{QPC}\, R_{lead}} \quad \text{for } eV > k_B T$$

not shot noise, just **heating**,
apparent fano factor F

Important : good, low resistive contacts

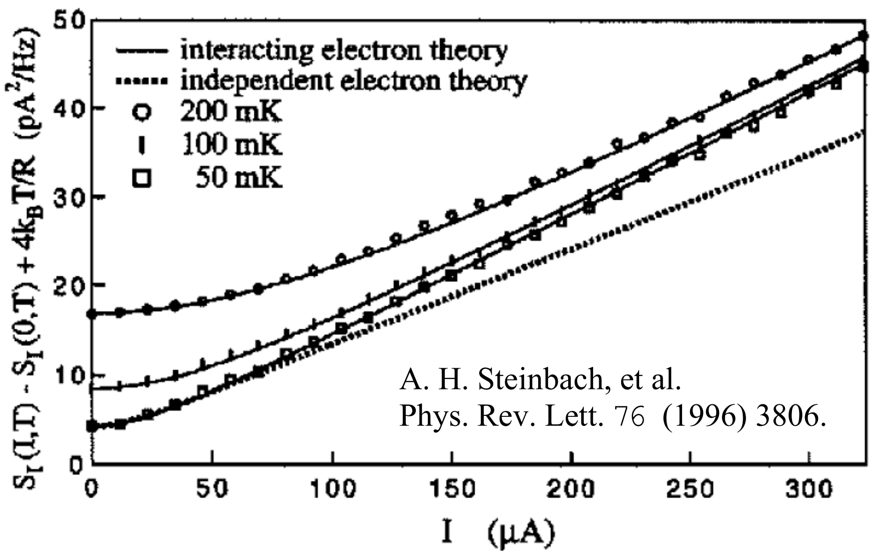
electron heating effect in a diffusive wire



$$S_I = 2eI \times "F"$$

$$"F" = \sqrt{3}/4$$

(just electron heating, not transport shot noise)



A. H. Steinbach, et al.
Phys. Rev. Lett. 76 (1996) 3806.

Also :
M. Henny, S. Oberholzer, C. Strunk, and C. Schonenberger,
Phys. Rev. B 59 (1999) 2871.

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- electronic shot noise

2- scattering derivation of quantum shot noise

a- $S(\omega)$ for an ideal one mode conductor
b- quantum shot noise for a single mode
c-zero frequency shot noise and multimode case

3- experimental examples

→ 4- current noise cross-correlations

- scattering derivations
- electronic analog of the optical Hanbury-Brown Twiss experiment
- electronic quantum exchange

III Shot Noise and Interactions:

IV. Shot noise: ***the*** tool to detect entanglement

V. Shot noise and high frequencies

II. 4 . current noise correlations.

Zero temperature expression:

$$S_{\alpha\beta} = 2\frac{e^2}{h}\sum_{\gamma\neq\delta}\int d\varepsilon\ \left[s^*_{\alpha\gamma}s_{\alpha\delta}s^*_{\beta\delta}s_{\beta\gamma}\right] \times \{f_\gamma(\varepsilon)(1-f_\delta(\varepsilon)) + f_\delta(\varepsilon)(1-f_\gamma(\varepsilon))\}$$

use the property $\sum_\delta s_{\alpha\delta}s^*_{\beta\delta} = 0$

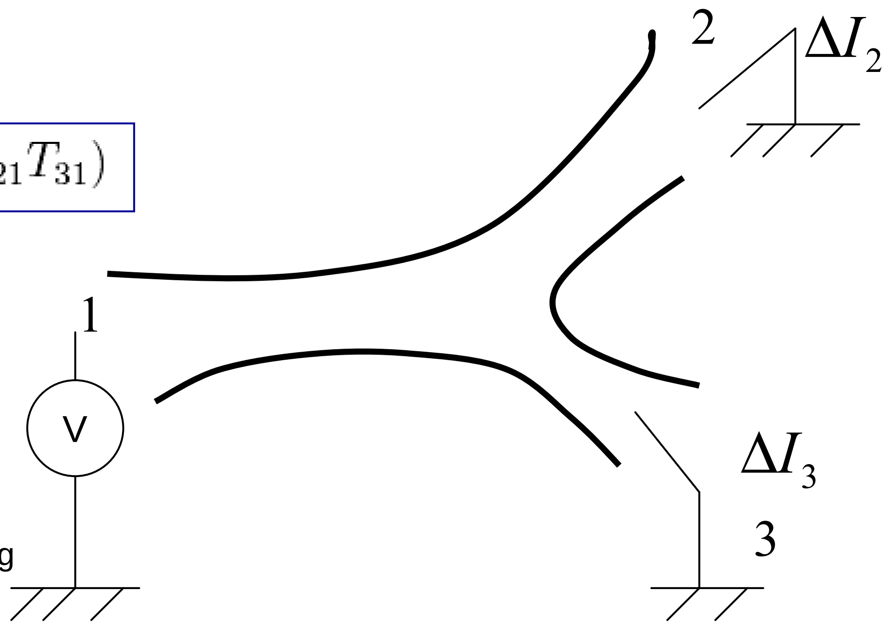
$$S_{\alpha\beta} = -2\frac{e^2}{h}\int d\varepsilon\ \left(\sum_\gamma s_{\alpha\gamma}s^*_{\beta\gamma}f_\gamma(\varepsilon)\right)\left(\sum_\delta s_{\alpha\delta}s^*_{\beta\delta}f_\delta(\varepsilon)\right)$$

cross-correlation between two different leads are always *negative*

Special case of a three terminal branch :
one finds :

$$\boxed{S_{23} = -2\frac{e^2}{h}eV(s_{21}s^*_{21}s_{31}s^*_{31}) = -2\frac{e^2}{h}eV.(T_{21}T_{31})}$$

$$\Delta I_2\,\Delta I_3 = S_{23}\Delta f < 0$$



binomial partitioning is here replaced by multinomial partitioning
(just ‘gambling’ law!)

Hanbury Brown & Twiss experiment with electrons

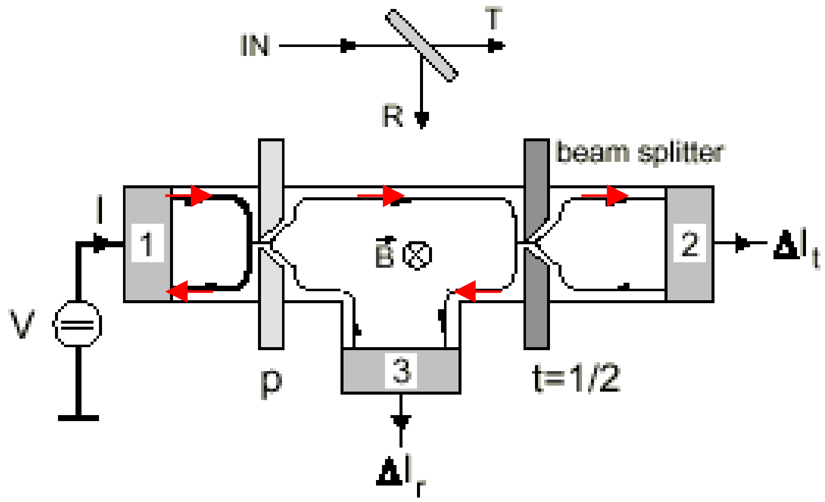
Oberholzer et al. ‘ 00

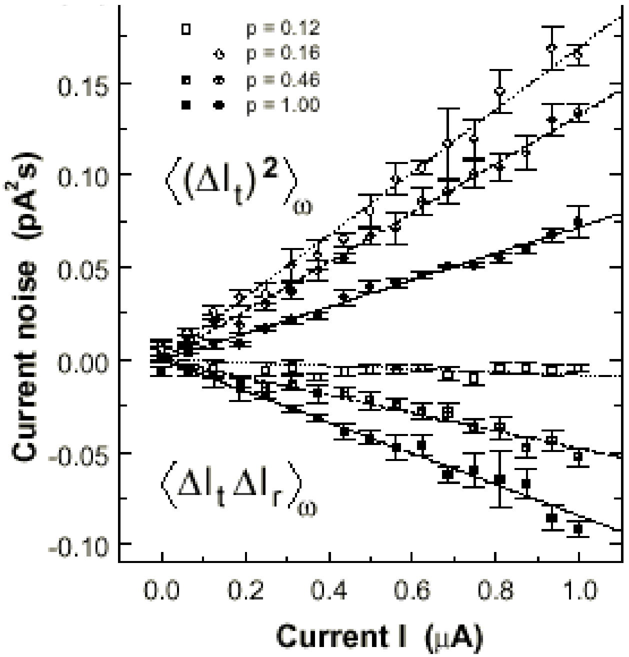
from sub-poissonian statistics

$$\Delta I_t \Delta I_r < 0 \quad = -\Delta I_t^{\,2} = -\Delta I_r^{\,2}$$

to poissonian statistics

$$\Delta I_t \Delta I_r = 0$$





Four terminal lead :

(A) $V_1 = V$; $V_3 = 0$

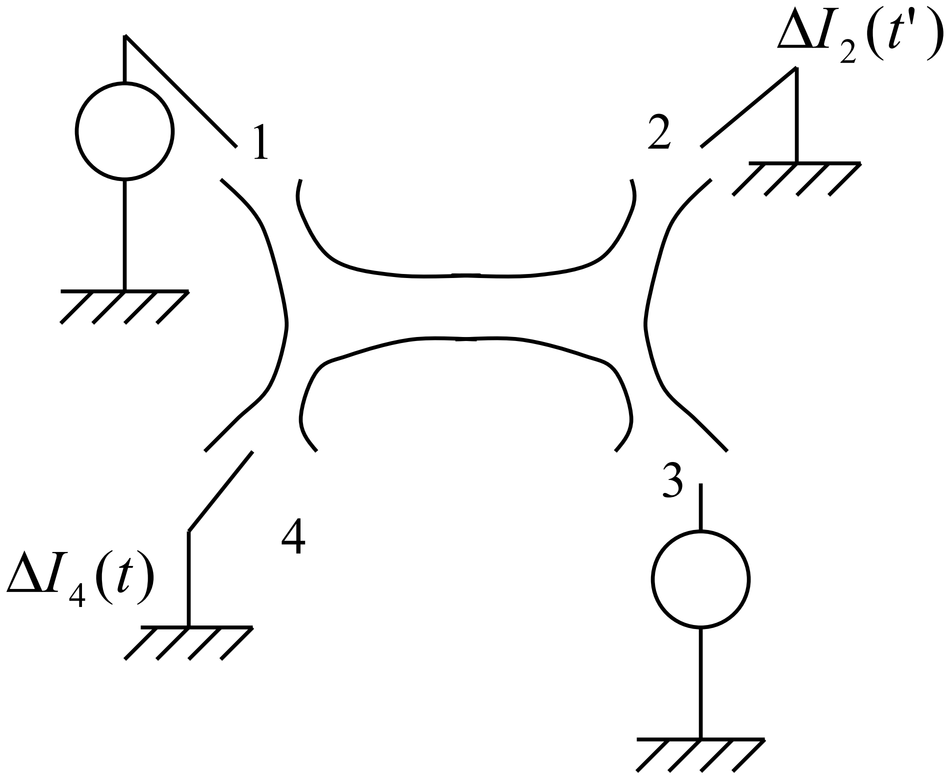
(B) $V_1 = 0$; $V_3 = V$

(A+B) $V_1 = V$; $V_3 = V$

$$S_{(A)} = -2\frac{e^2}{h}eV(s_{21}s_{21}^{*}s_{41}s_{41}^{*})$$

$$S_{(B)} = -2\frac{e^2}{h}eV(s_{23}s_{23}^{*}s_{43}s_{43}^{*})$$

$$S_{(A+B)} \neq S_{(A)} + S_{(B)}$$



(M. Büttiker, Phys. Rev. B 46 (1992) 12485.

$$S_{(A+B)} - (S_{(A)} + S_{(B)}) = -2\frac{e^2}{h}eV\left[\left(s_{21}s_{23}{}^{*}s_{43}s_{41}{}^{*}\right) + \left(s_{23}s_{21}{}^{*}s_{41}s_{43}{}^{*}\right)\right]$$

exchange terms : non classical

Introduction

I. Electronic scattering (a brief introduction)

II. Quantum Shot noise

III Shot Noise and Interactions:

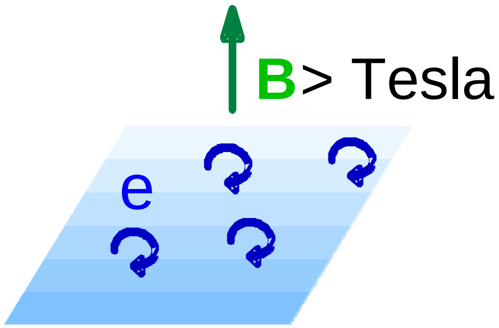
1. Fractional Quantum Hall effect
2.. Superconducting-Normal mesoscopic interfaces
3. Interactions in a QPC : 0.7 structure

IV. Shot noise: ***the*** tool to detect entanglement

V. Combining electrons and photons

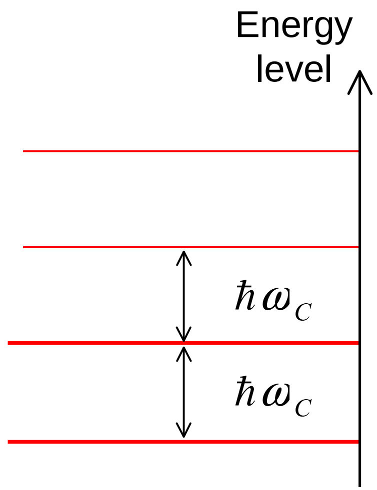
III 1. Quantum Hall effect

Integer Q.H.E. (von Klitzing 80)

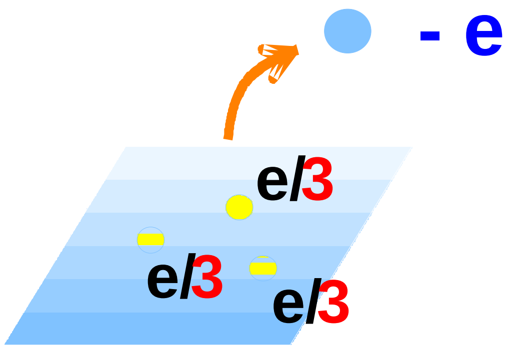


$$R_{\text{Hall}} = \frac{h}{e^2} \cdot \frac{1}{\text{integer}}$$

$$\omega_C = \frac{eB}{m}$$ cyclotron frequency



Fractional Q.H.E. (Tsui, Störmer,Gossard 1982)
(Laughlin 1983)



$$R_{\text{Hall}} = \frac{h}{e^2} \cdot \frac{1}{\text{fraction}}$$

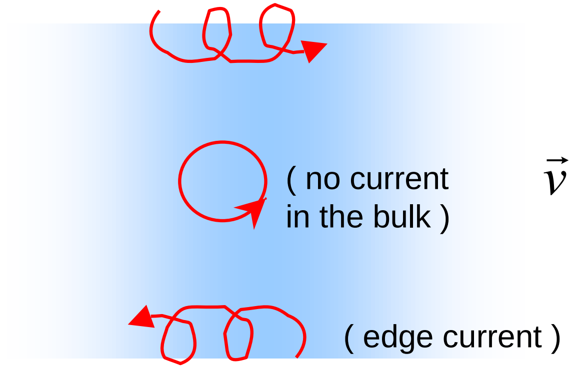
$$\nu = \frac{\text{number of electrons}}{\text{number of quantum states}}$$

$\nu = 1/3$:

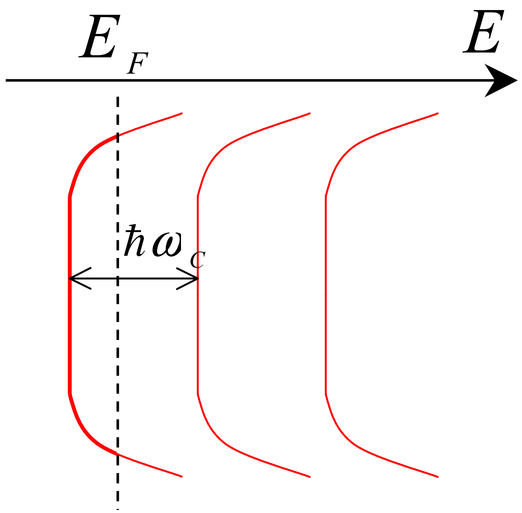
1 electron for 3 quantum states

elementary excitation
≡ empty a quantum state
≡ carry fractional charge e/3

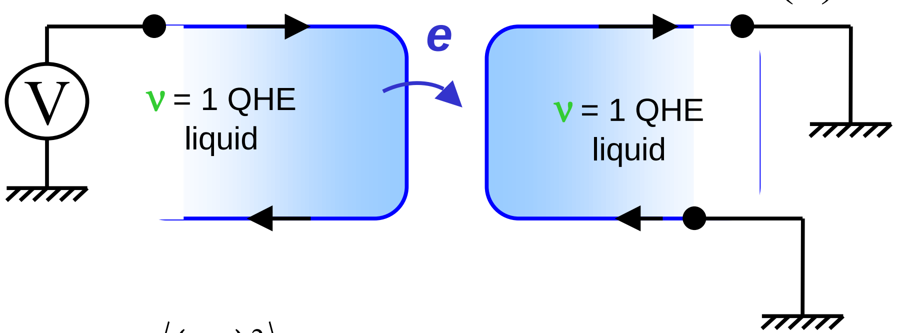
available current is at the edges of the sample



$$\vec{v}_{drift} = \vec{E}_{Confinement} \times B\,\hat{z}$$

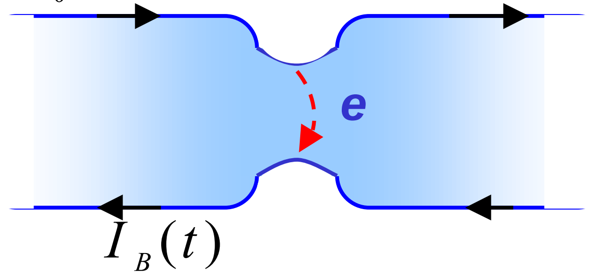


Tunneling trough barrier :

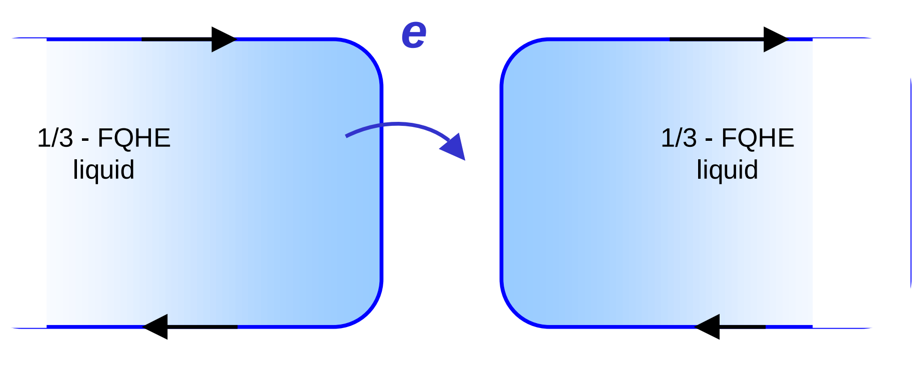


$$\langle (\Delta I)^2 \rangle = 2eI\Delta f \qquad D << 1$$

Transfer of hole trough Q.H.E. fluid:



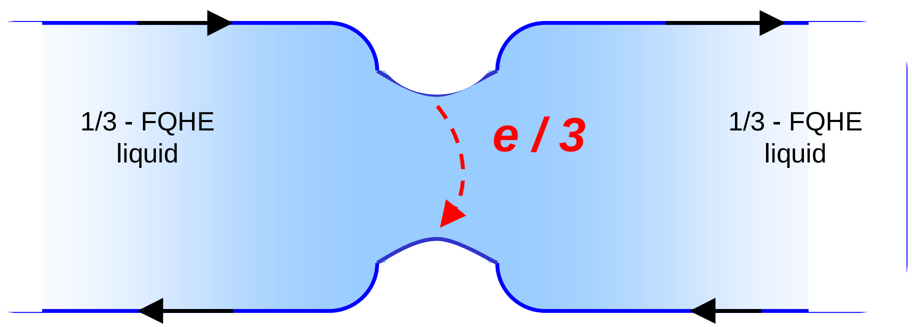
$$\langle (\Delta I)^2 \rangle = 2eI_B\Delta f \qquad D \approx 1$$



Tunneling trough
barrier :

q = e

$$\left\langle (\Delta I)^2 \right\rangle = 2eI\,\Delta f \qquad D \ll 1$$



Transfer trough
1/3 FQHE fluid:

q = e/3

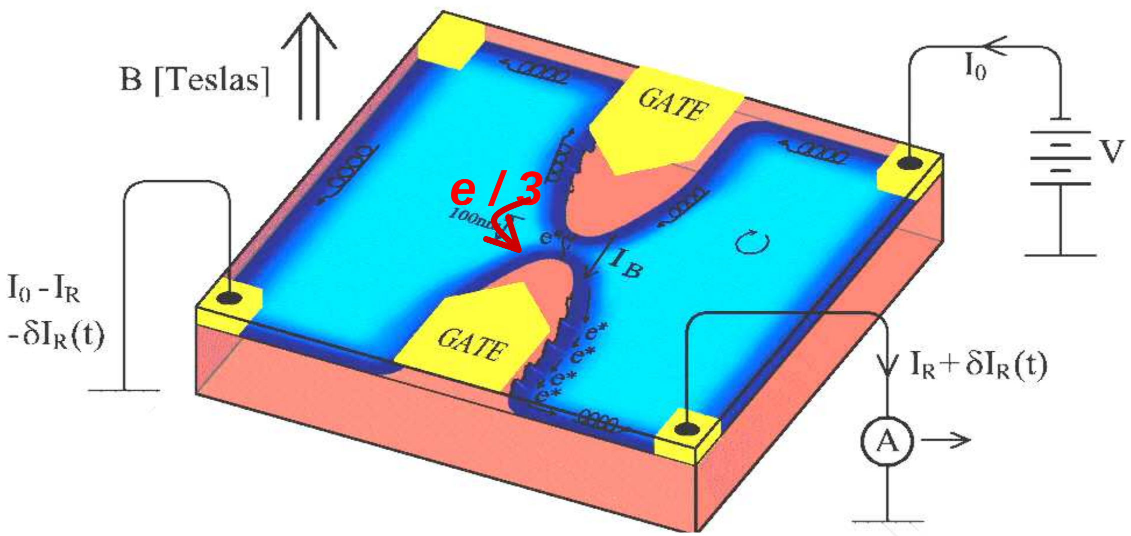
$$\left\langle (\Delta I)^2 \right\rangle = 2(e/3) I_B\,\Delta f \qquad D \approx 1$$

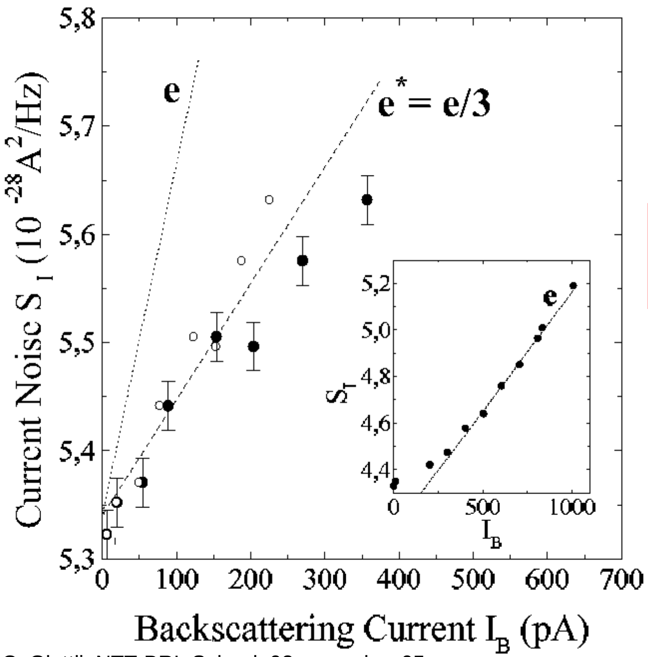
$$\frac{\left\langle \Delta I^2 \right\rangle}{\Delta f} = S_I = 2qI$$

measuring both quasiparticle shot noise (Poissonian regime!)
+ mean current gives the charge with no adjustable parameters.

current is carried by fractional charges
(direct evidence, no unknown parameters)

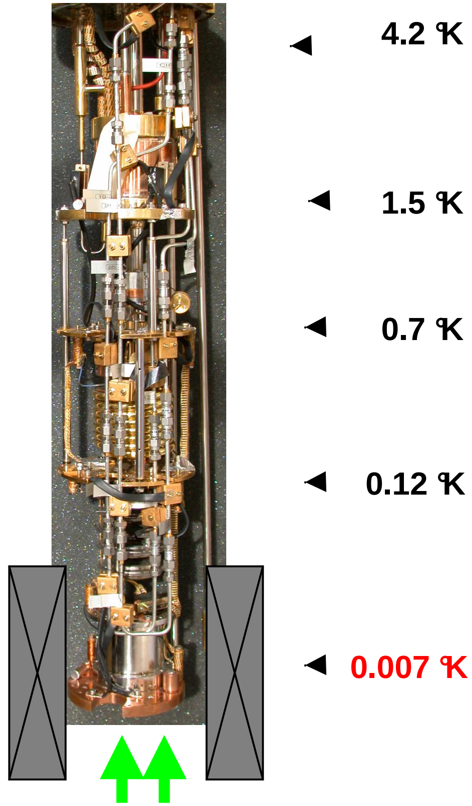
L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. 79, 2526 (1997).



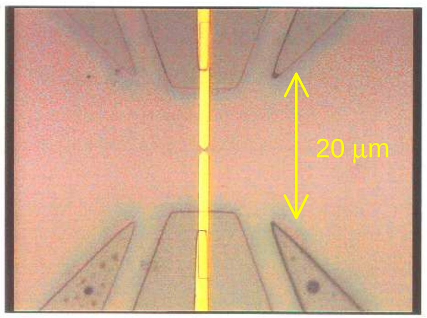


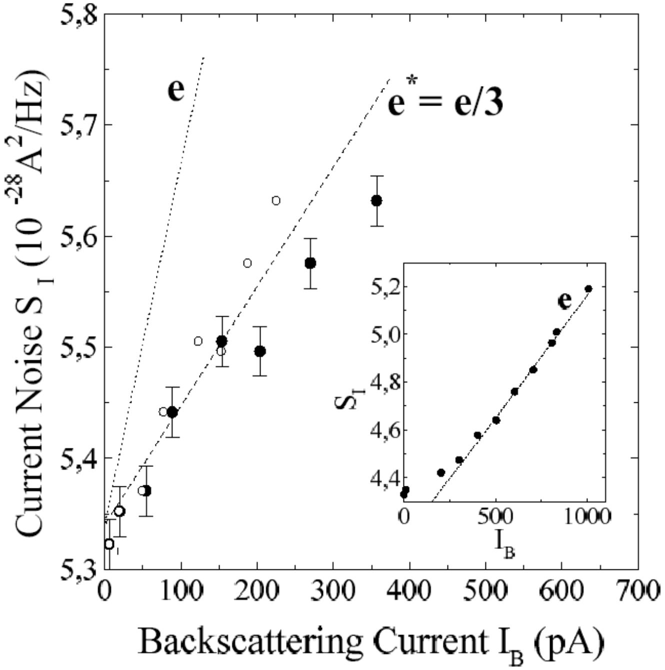
$$\langle (\Delta I)^2 \rangle = 2(e/3) I_R \Delta f$$

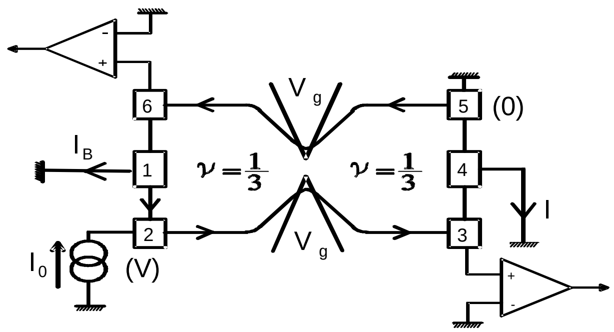
charge q=e/3



15 Tesla superconducting coil





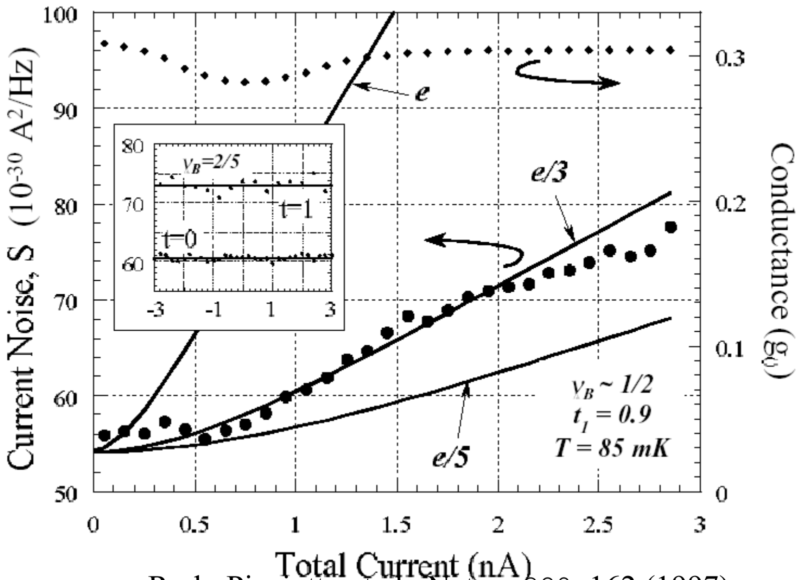


e / 3 fractional charges are observed

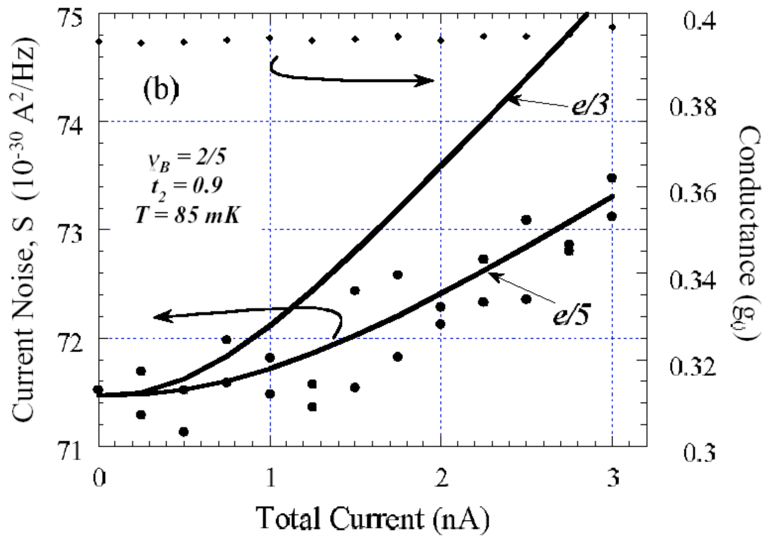
at filling factor 1/3 (Weizmann and Saclay 97) and

e / 5 charges at filling 2/5 (Weizmann 99).

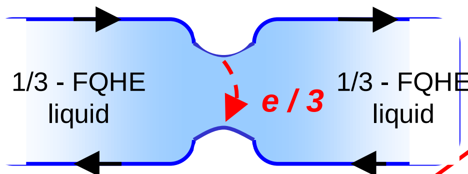
L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne,
Phys. Rev. Lett. 79, 2526 (1997).

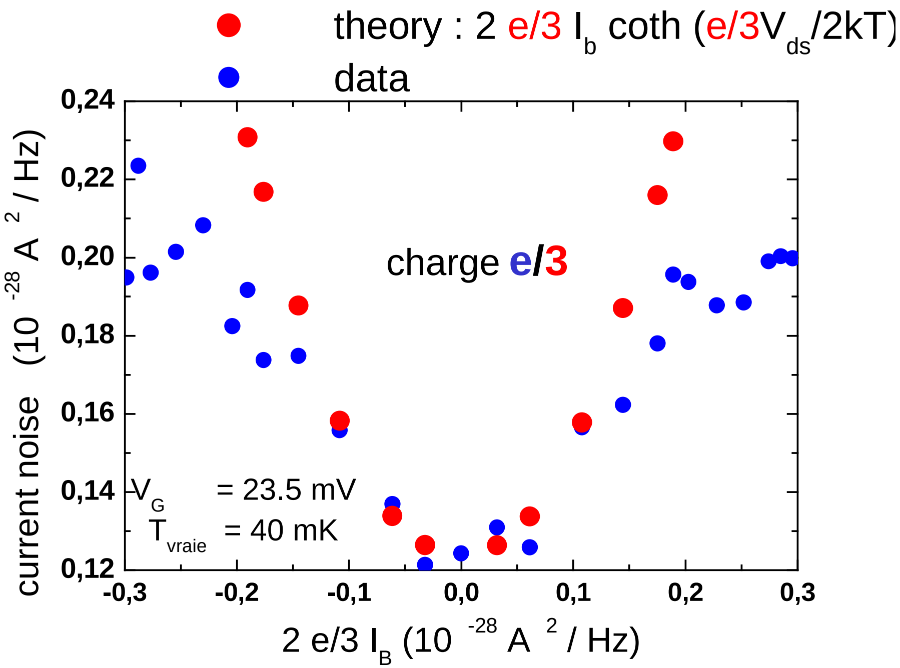


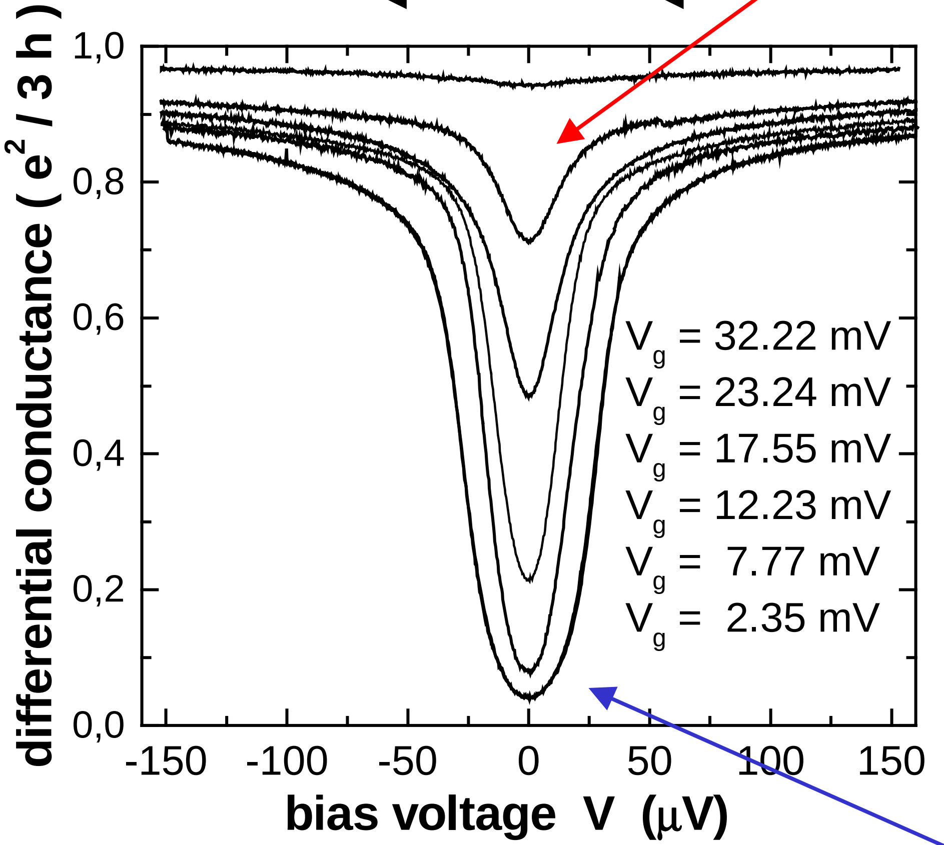
R. de-Picciotto et al., Nature 389, 162 (1997).



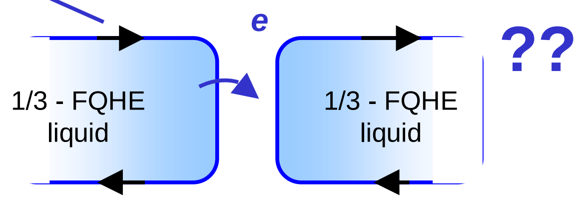
M. Reznikov, R. de-Picciotto, T. G. Griths,
M. Heiblum, and V. Umansky, Nature 399, 238 (1999).





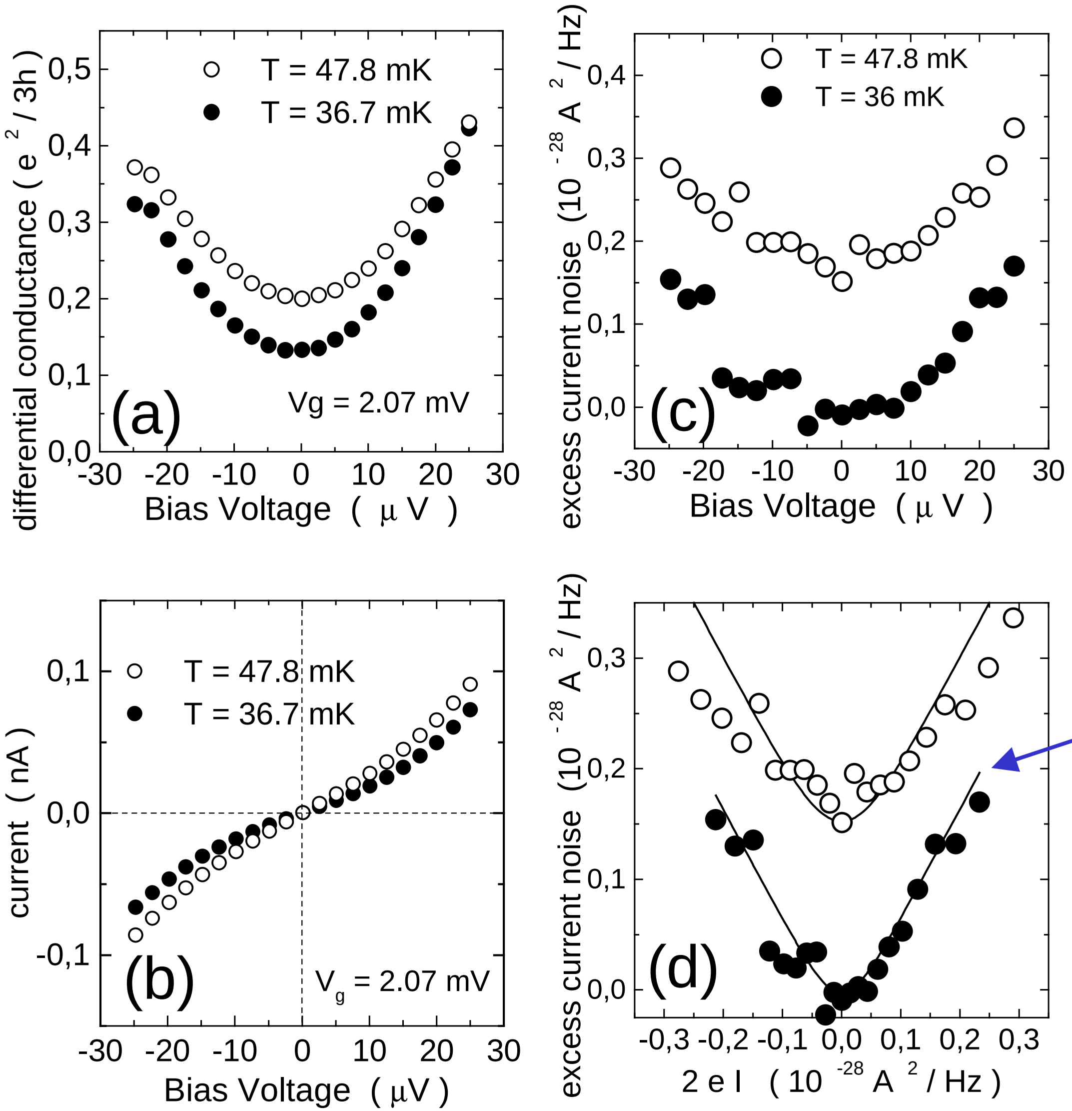


charge **e**



V. Rodriguez et al (2000)

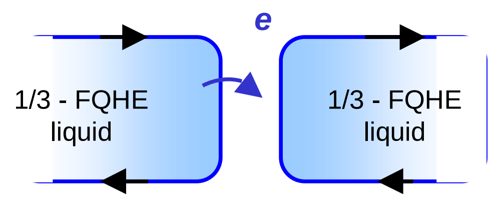
integer charge in the strong backscattering regime



$$S_I = 2\,e\,|I|$$

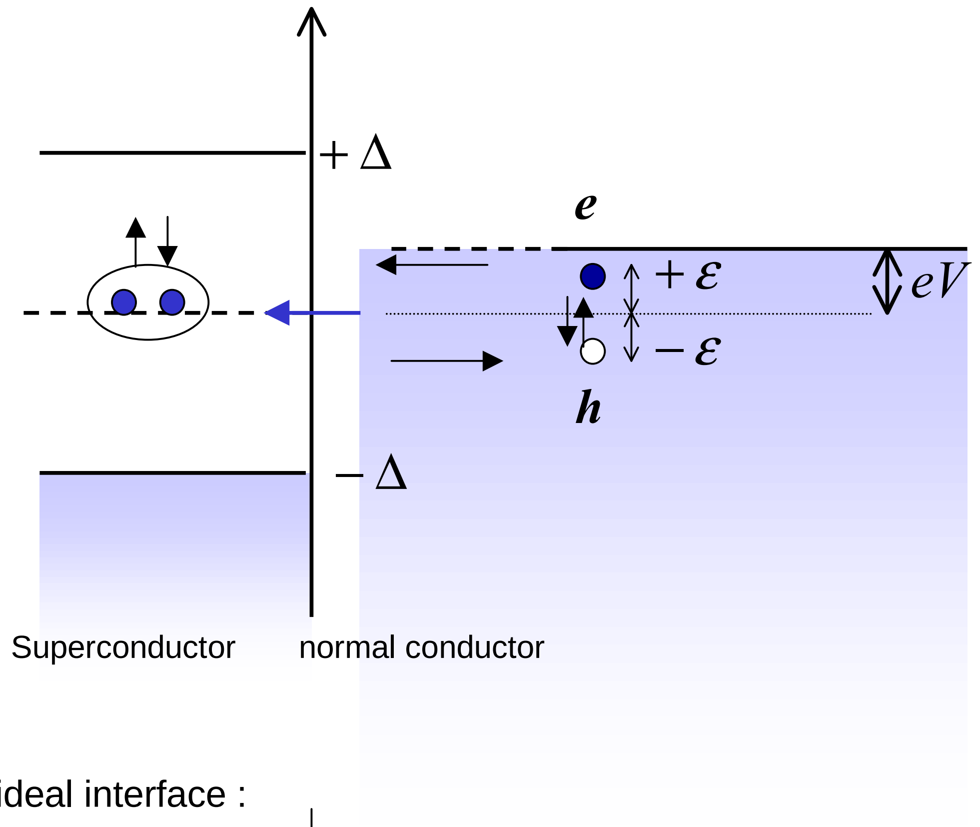
Schottky noise

for charge e



In the same sample, same quantum point contact, one can go from the e/3 regime to the e regime just by changing coupling

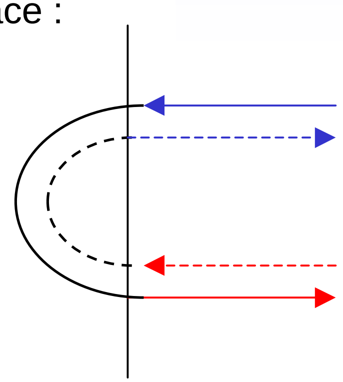
III . 2. normal / superconductor interface:



no single particle current between superconducting and normal metal expected for sub-gap energies.

second order process involving two quasi-particle allows for finite current: Andreev reflection.

ideal interface :

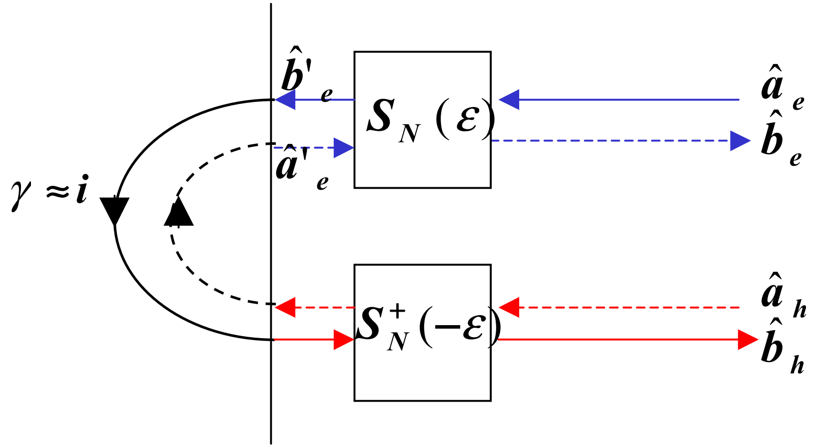


incoming electron (energy $+\varepsilon$) $\rightarrow$ outgoing hole (energy - ε)

plus phase $\gamma \equiv \exp(-i\cos^{-1}(\varepsilon/\Delta))$

incoming hole (energy $-\varepsilon$) $\rightarrow$ outgoing electron (energy + ε)

plus phase $\gamma \equiv \exp(-i\cos^{-1}(\varepsilon/\Delta))$



$$S_N(\varepsilon) = \begin{pmatrix} r_{11} & t_{12} \\ t_{21} & r_{22} \end{pmatrix}$$

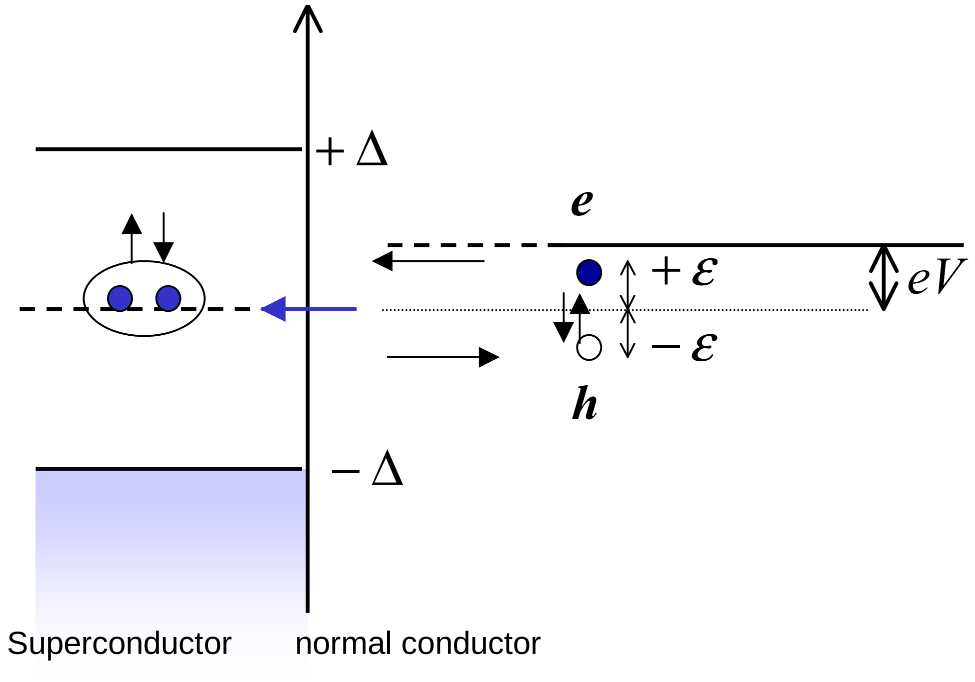
scattering matrix in the normal lead

the complete scattering matrix including Andreev reflection and normal scattering is :

$$\begin{pmatrix} \hat{b}_e \\ \hat{b}_h \end{pmatrix} = S \begin{pmatrix} \hat{a}_e \\ \hat{a}_h \end{pmatrix} \qquad S = \begin{pmatrix} s_{ee} & s_{eh} \\ s_{he} & s_{hh} \end{pmatrix}$$

$$s_{he}(\varepsilon) = t_{21}(\varepsilon)\gamma t_{12}^{*}(-\varepsilon) \quad + \quad t_{21}(\varepsilon)\gamma r_{22}^{*}(-\varepsilon)\ \gamma r_{22}(\varepsilon) t_{12}^{*}(-\varepsilon) \quad + \quad t_{21}(\varepsilon)\left(\gamma r_{22}^{*}(-\varepsilon)\ \gamma r_{22}(\varepsilon)\right)^2 t_{12}^{*}(-\varepsilon) \quad + \ldots$$
$$= \frac{t_{21}(\varepsilon)\gamma t_{12}^{*}(-\varepsilon)}{1-\gamma r_{22}^{*}(-\varepsilon)\ \gamma r_{22}(\varepsilon)}$$

(Fabry-Pérot like multiple interferences)



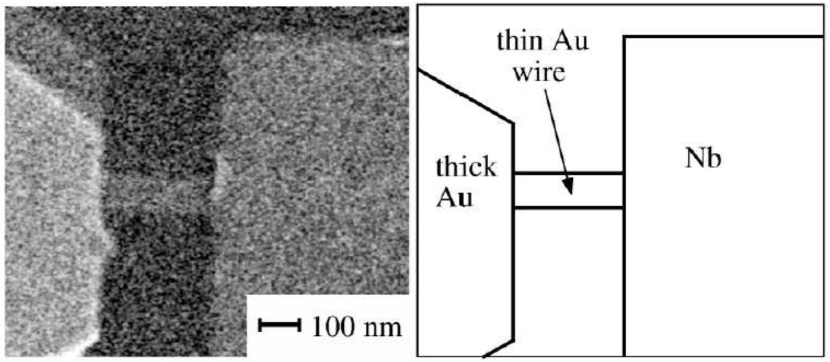
$$G = \frac{(2e)^2}{h}\left|s_{he}\right|^2 = \frac{(2e)^2}{h}\frac{D^2}{(1+R)^2} \qquad R = 1 - D$$

‘doubled’ shot noise :

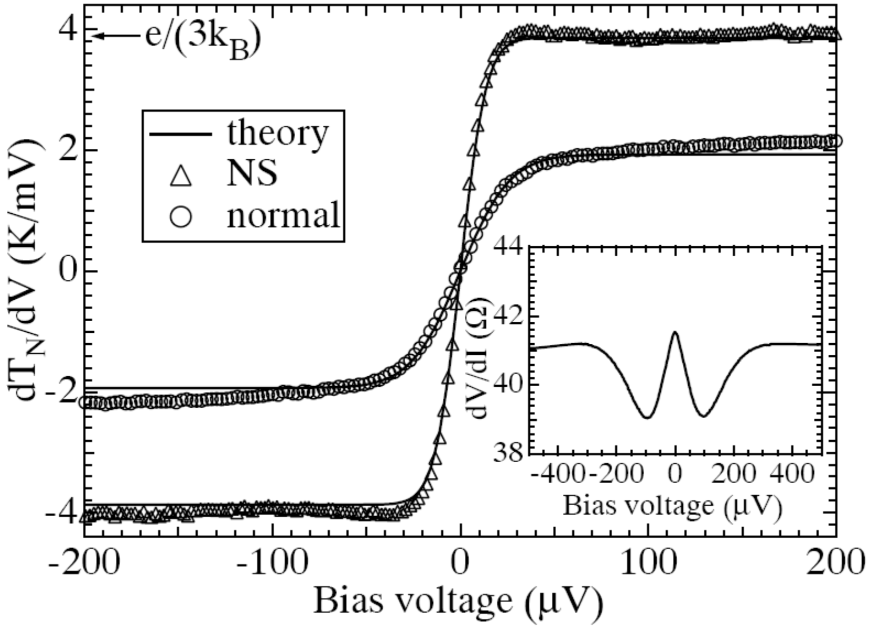
$$S_I = 2\,(2e)\,\frac{e^2}{h}V\left|s_{he}\right|^2(1-\left|s_{he}\right|^2)$$

- twice the electron charge
- binomial law of quantum partitioning
- noiseless property of the Fermi sea

doubling of the shot-noise
for a diffusive S-N junction



A. A. Kozhevnikov, R. J. Schoelkopf, and D. E. Prober,
Phys.Rev.Lett. 84, 3398 (2000)



$$S_I = \frac{1}{3} 2.(2e) I$$

Fano for
diffusive
regime

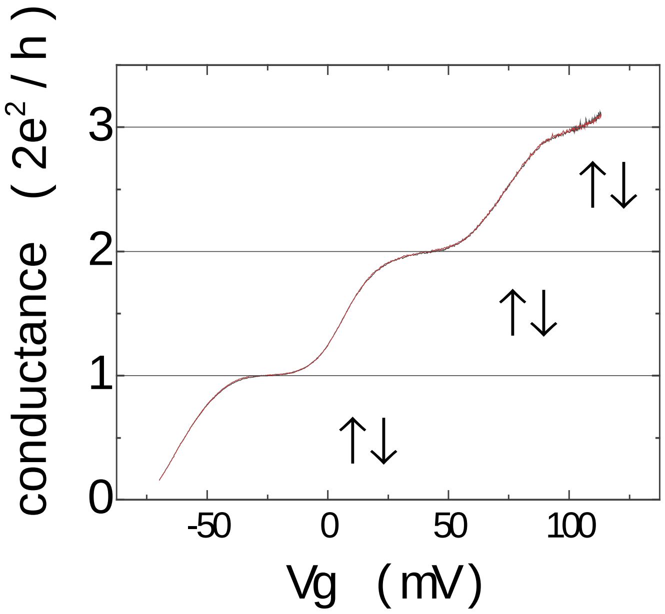
Doubled
charge

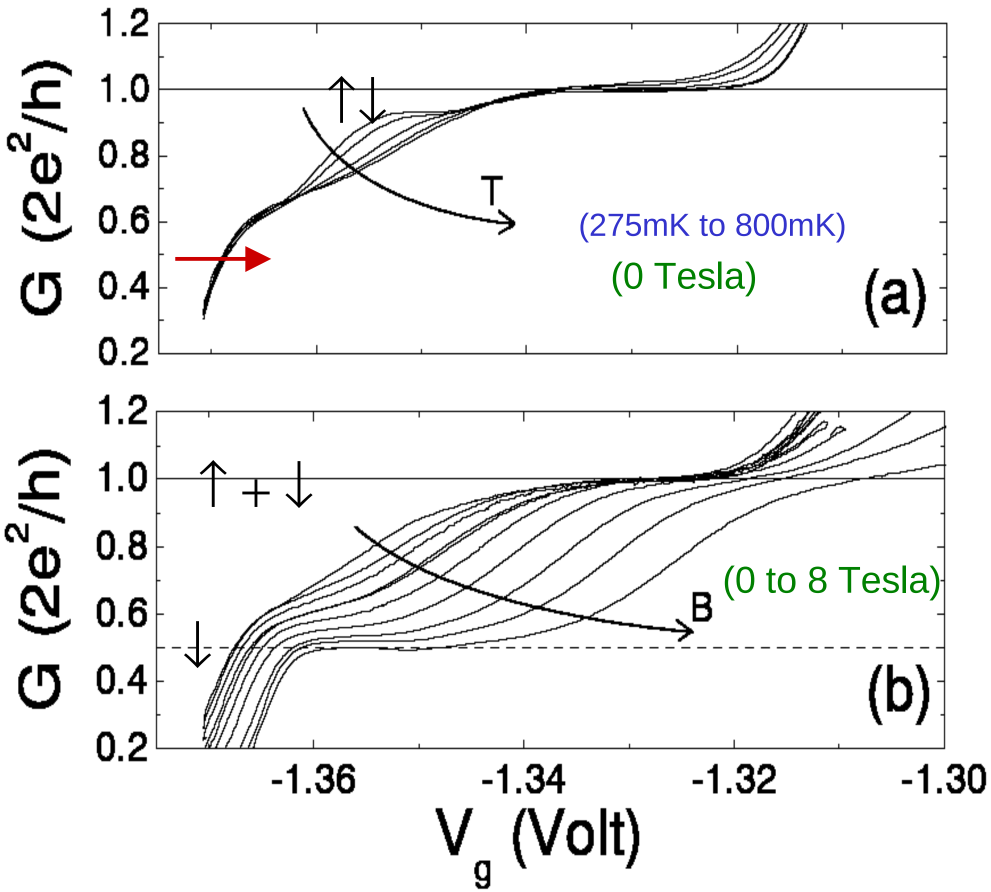
$$T_{Noise} = \frac{S_I}{4G\, k_B}$$

III. 3. interaction effects in a Quantum Point Contact : the '0.7x2e²/h' structure

ultra-short QPC: no '0.7'

long QPC: conductance plateau around 0.7X2e²/h





plateaus at:

$$G = (\text{integer}) \times 2\frac{e^2}{h}$$

$(1+1)$

$\uparrow + \downarrow$

- resonance in transmission? (**single** spin degenerate mode)

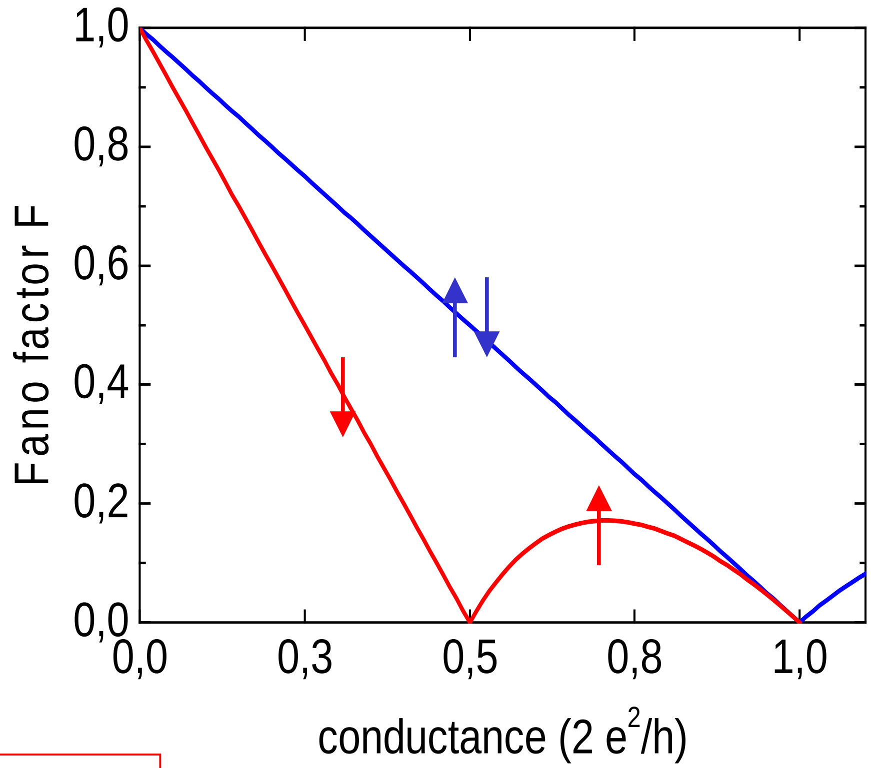
- or spin degeneracy lifted by interaction? (**two** distinct modes)

↑↓

↑↑

spin degenerate case

$$S_I = 2eI\,(1 - D_{\uparrow\downarrow}) = 2eI\,.\,F_{\uparrow\downarrow}$$



spin degeneracy fully lifted:

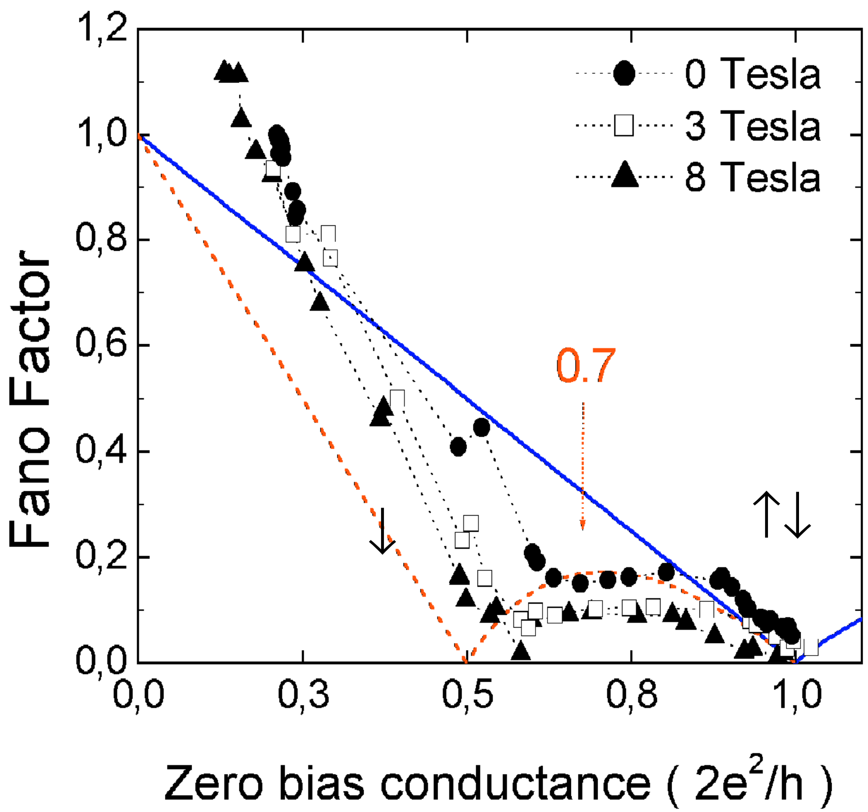
$$S_I = 2eI\,\frac{D_{\downarrow}(1 - D_{\downarrow}) + D_{\uparrow}(1 - D_{\uparrow})}{D_{\downarrow} + D_{\uparrow}} = 2eI\,.\,F_{\uparrow+\downarrow}$$

conductance can not distinguish between one or two modes

but shot noise can

Here: the Fano factor shows direct signature of **two** modes

$\rightarrow$ lifted spin degeneracy scenario.



P. Roche, J. Segala, and D. C. Glattli,
J. T. Nicholls, M. Pepper, A. C. Graham,
M. Y. Simmons, and D. A. Ritchie,
Phys. Rev. Letters 2004