

NTT BRL School 2005 – Lecture #1

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# Quantum Transport and Electron Dephasing in Diffusive Metal Wires:

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# Disorder and Interactions – The Big Picture

1.  $V = 0, U = 0$

Bloch's Thm. ⑨  $\rho(T=0) = 0$

◻ ☎ ⚡ ⓘ from electron-phonon

scattering

2.  $V$  small,  $U = 0$

$\rho(T=0) = \rho_{\square} \neq 0$ , impurity

scattering

3.  $V = 0, U$  small

Fermi liquid theory of metals (Landau)

4.  $V$  large,  $U = 0$

localization of 1-particles wavefunctions (Anderson)

5.  $V = 0, U$  large

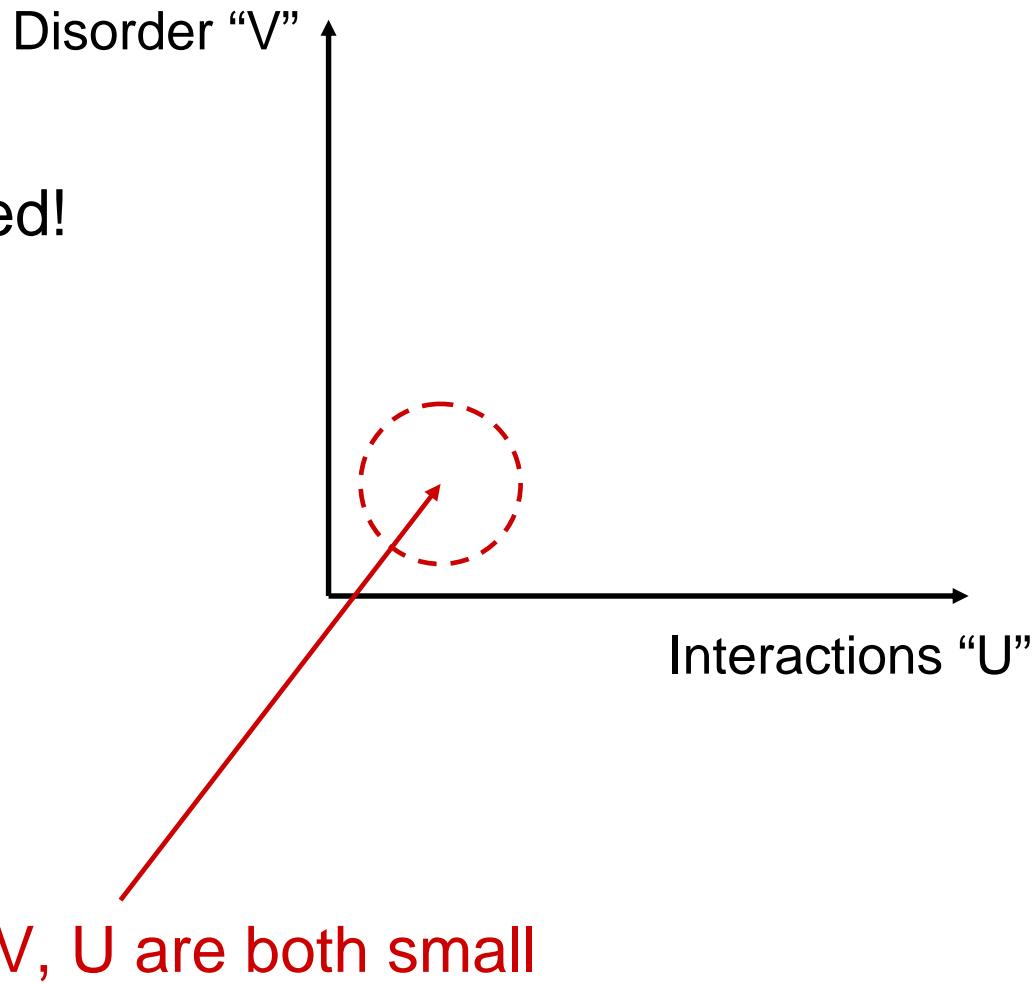
Wigner crystal – Coulomb repulsion

Disorder "V"

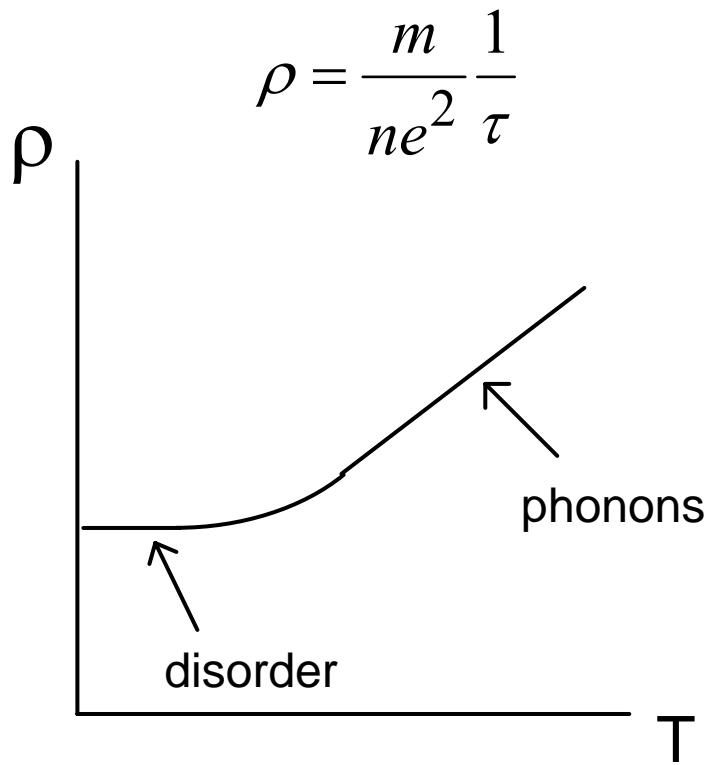
Interactions "U"

# Disorder and Interactions – The Big Picture

6.  $V \neq 0, U \neq 0$  complicated!



# Weakly-disordered metals ... 1980's



Matthiessen's rule: **IS WRONG!**

$$\frac{1}{\tau} = \frac{1}{\tau_{disorder}} + \frac{1}{\tau_{el-ph}} + \dots$$

elastic

preserves quantum phase coherence

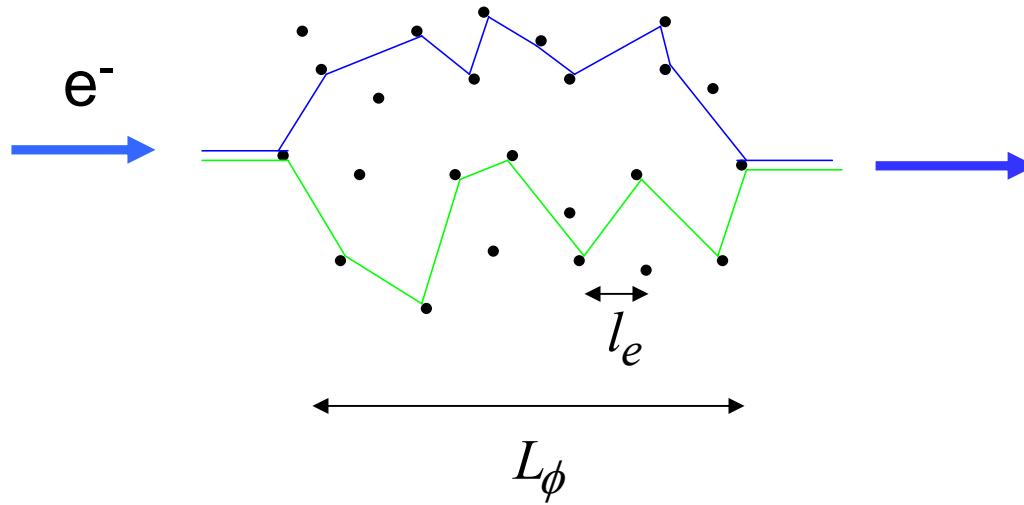
inelastic

destroys quantum phase coherence

Low  $T$ :  $\frac{1}{\tau_{inelastic}} \ll \frac{1}{\tau_{elastic}}$

Electrons maintain quantum phase coherence over distance  $L_\phi \gg l_e$

# Phase-coherent diffusive electron transport



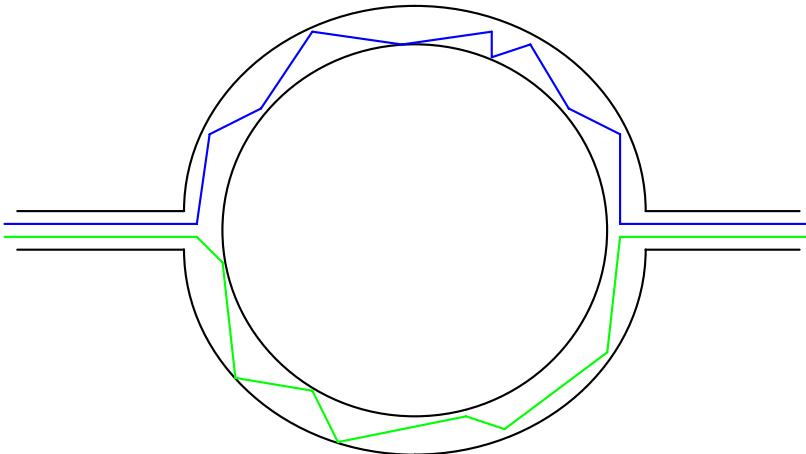
Diffusion:  $D = v_F l_e / 3$  (elastic)

Quantum interference over distance  $L_\phi$   
(inelastic)

# Interference effects and $L_\phi$

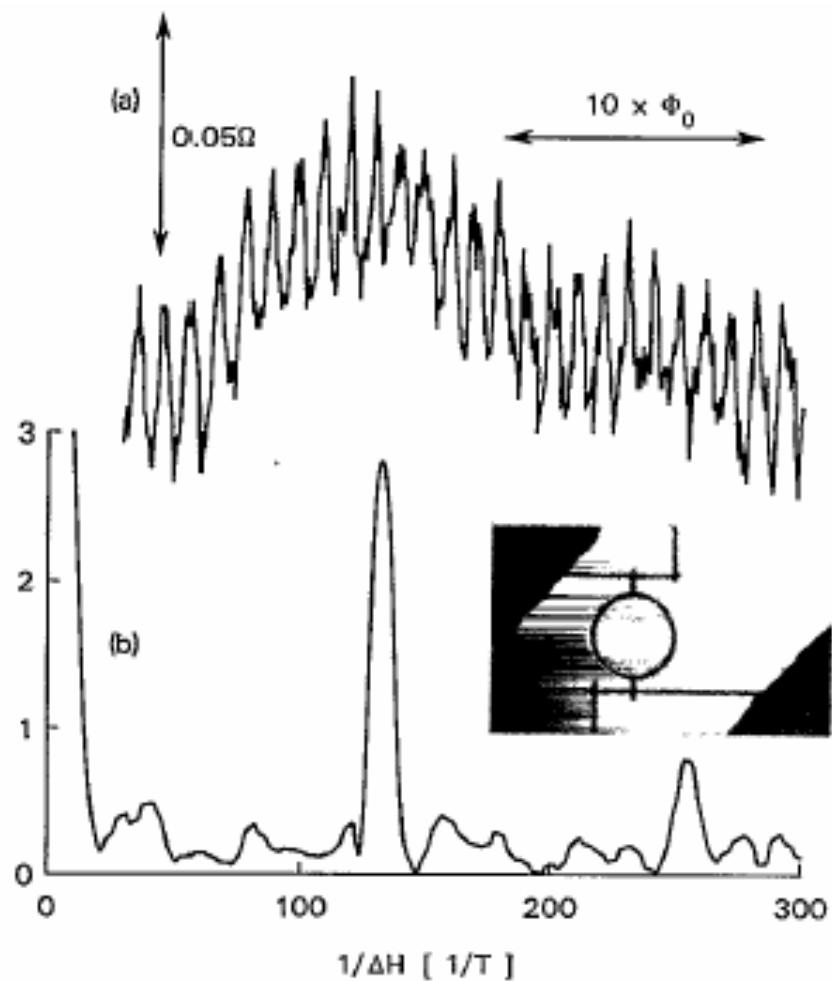
- Aharanov-Bohm effect

Washburn & Webb, 1984



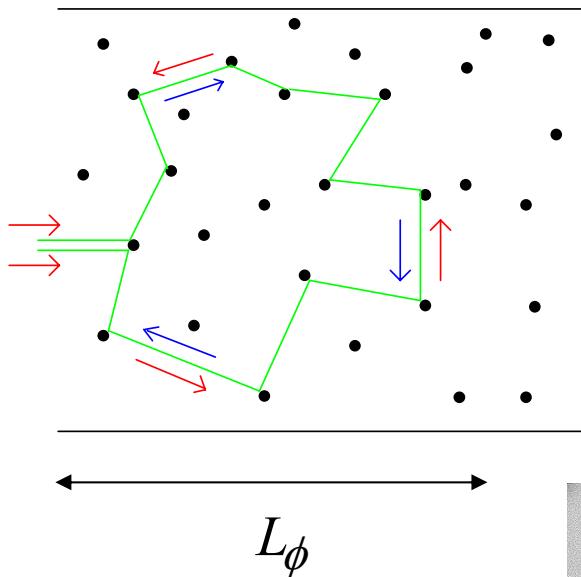
$$\begin{aligned} P_{trans} &= |A_{up} + A_{down}|^2 \\ &= P_{up} + P_{down} + 2 \operatorname{Re}(A_{up} A_{down}) \end{aligned}$$

$$\Delta G \propto \exp\left(-\frac{\pi r}{L_\phi}\right)$$

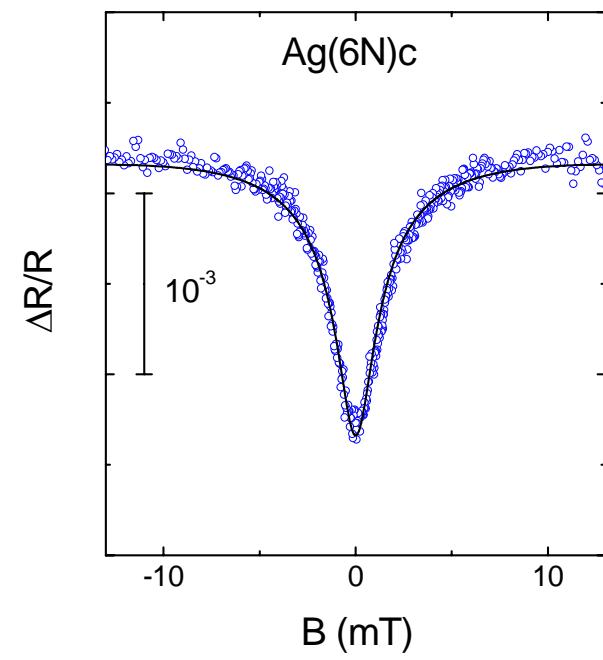
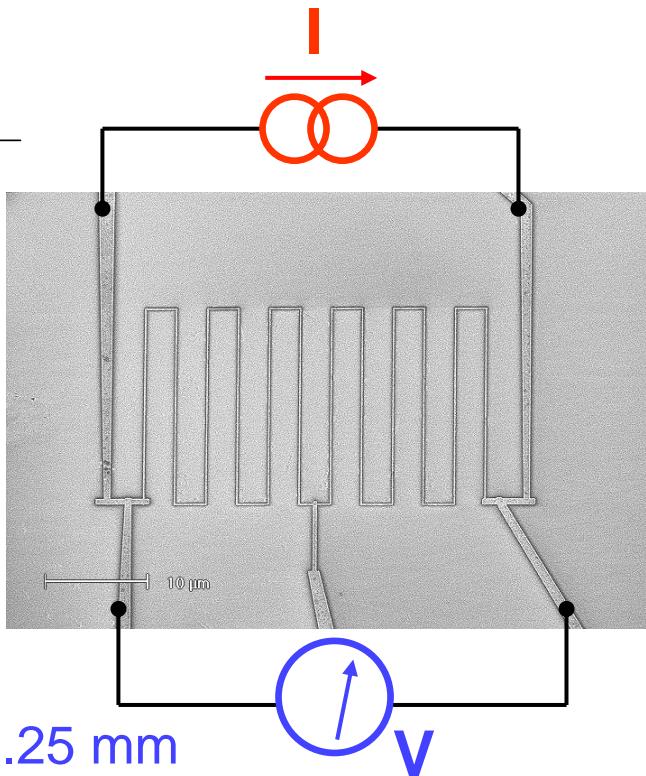


# Interference effects and $L_\phi$

- Aharonov-Bohm effect
- Weak localization

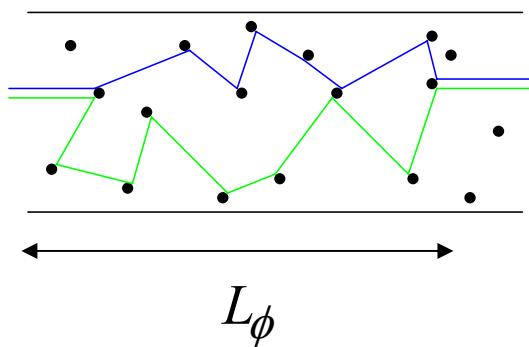


$$P_{\text{return}} = |A_{\rightarrow} + A_{\leftarrow}|^2 = \underbrace{P_{\rightarrow} + P_{\leftarrow}}_{\text{classical}} + \underbrace{2\text{Re}(A_{\rightarrow} A_{\leftarrow})}_{\text{quantum interference}}$$

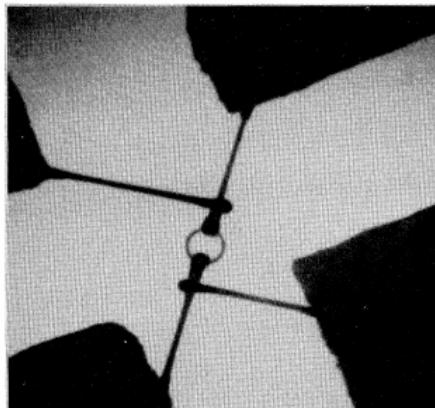


# Interference effects and $L_\phi$

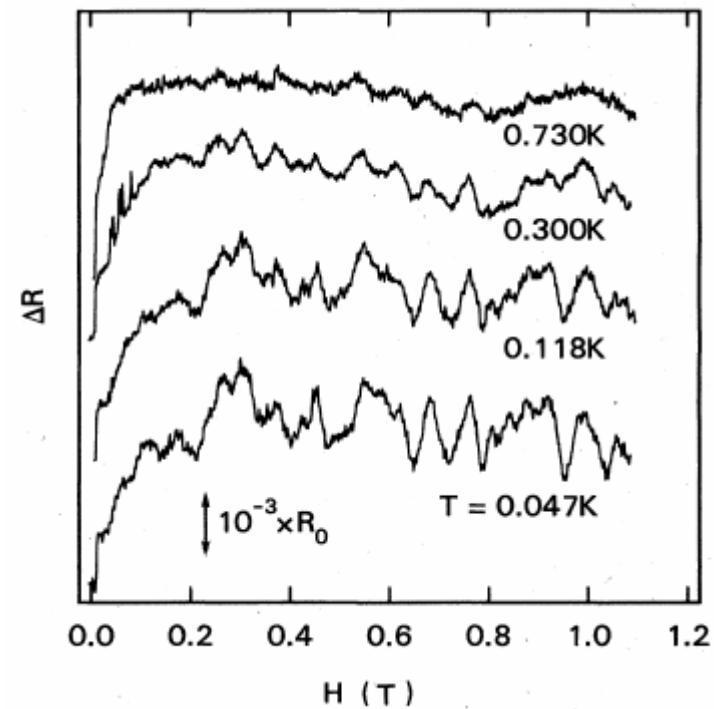
- Aharonov-Bohm effect
- Weak localization
- *Conductance Fluctuations*



$$P_{trans} = \left| \sum_{paths \alpha} A_\alpha \right|^2$$



Umbach, Washburn, Laibowitz,  
and Webb (1984)



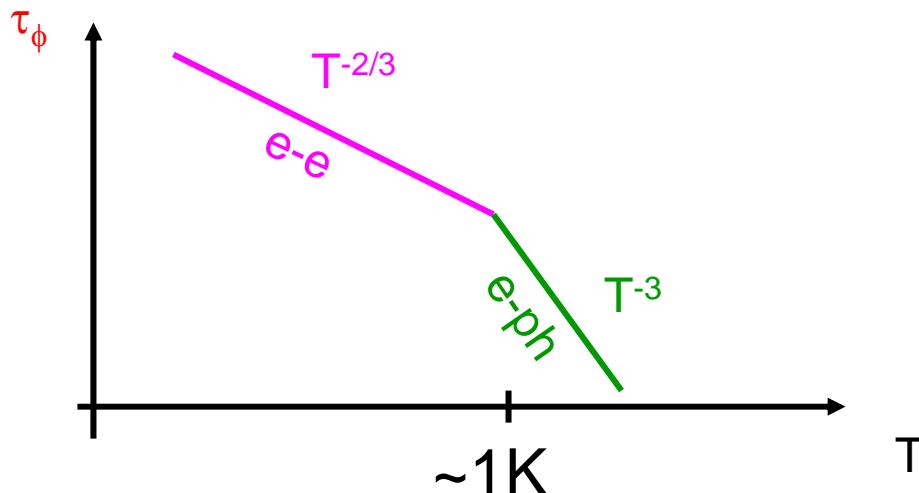
# Interference effects and $L_\phi$

- *Aharonov-Bohm effect*
- *Weak localization*
- *Conductance Fluctuations*
- *Persistent currents*
- *Superconducting proximity effect*
- ...

Size of the effects depends on  $L_\phi = \sqrt{D\tau_\phi}$

# Mechanisms of inelastic scattering: $\tau_\phi(T)$ in wires: theory

Altshuler, Aronov, Khmelnitskii, 1982



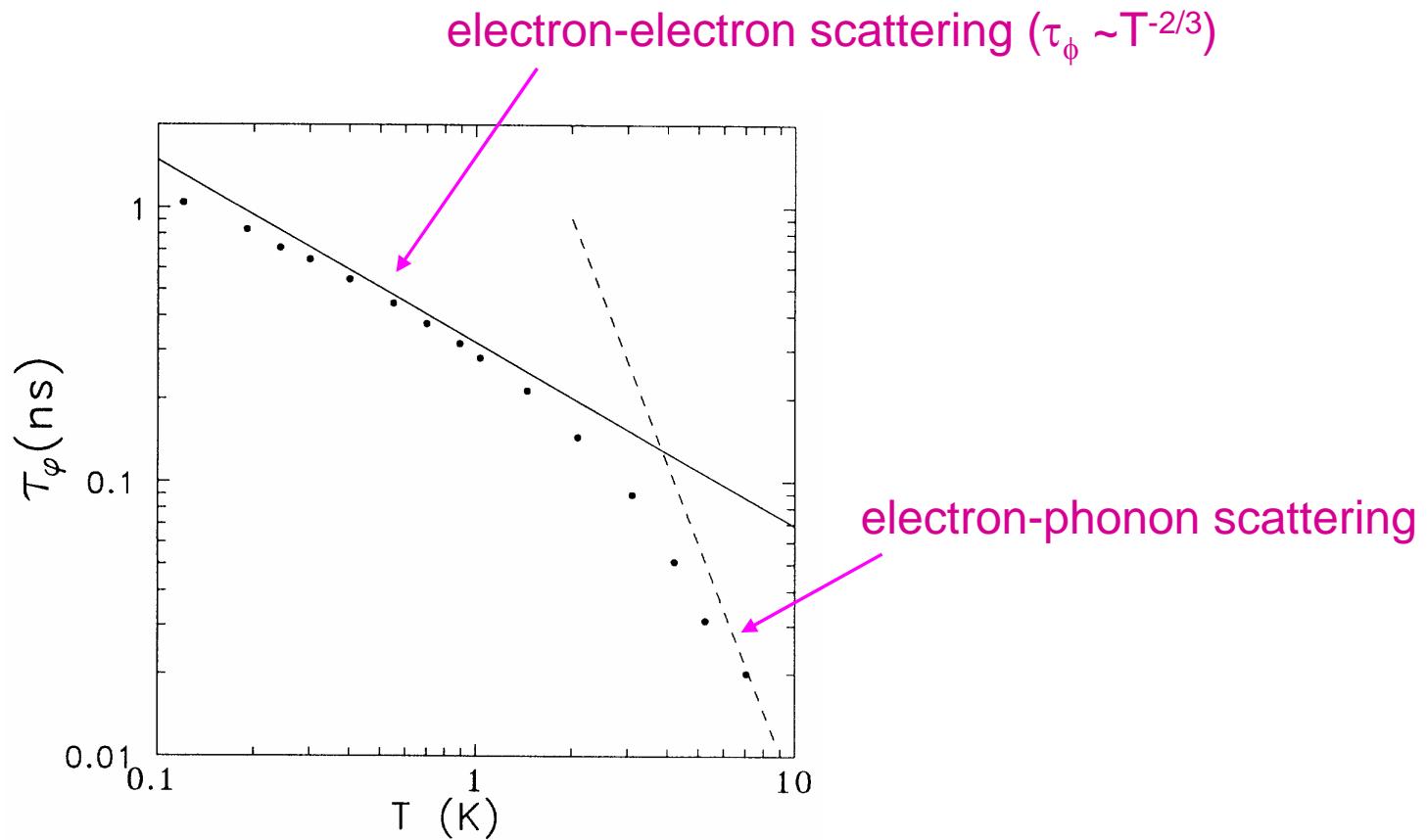
$$\tau_\phi = (A T^{2/3} + B T^3)^{-1}$$

$$A = \frac{1}{h} \left( \frac{\pi k_B^2}{4\nu_F L w t} \frac{R}{R_K} \right)^{1/3}$$

Screened Coulomb  
Interaction for  $d=1$

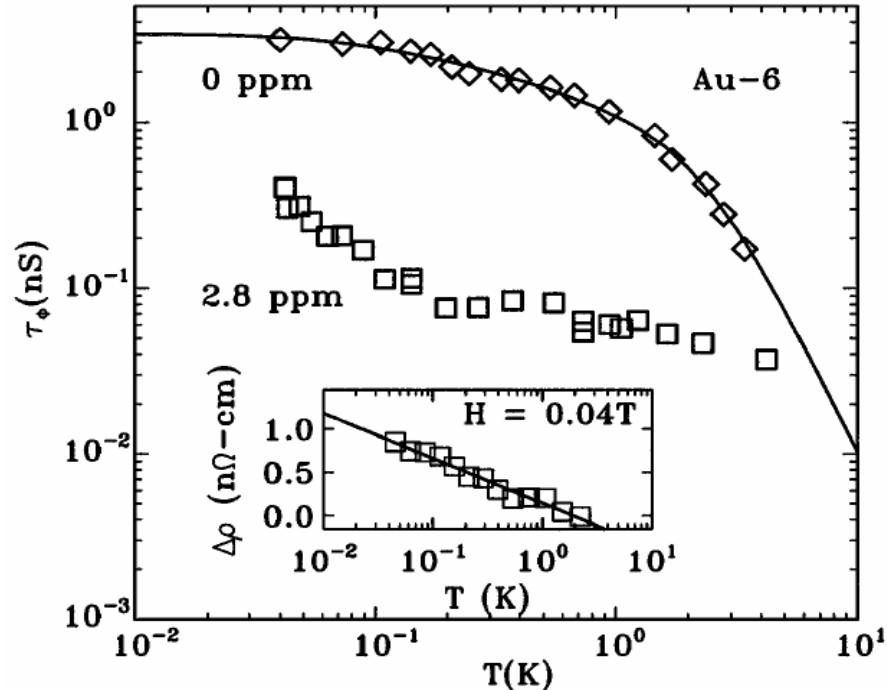
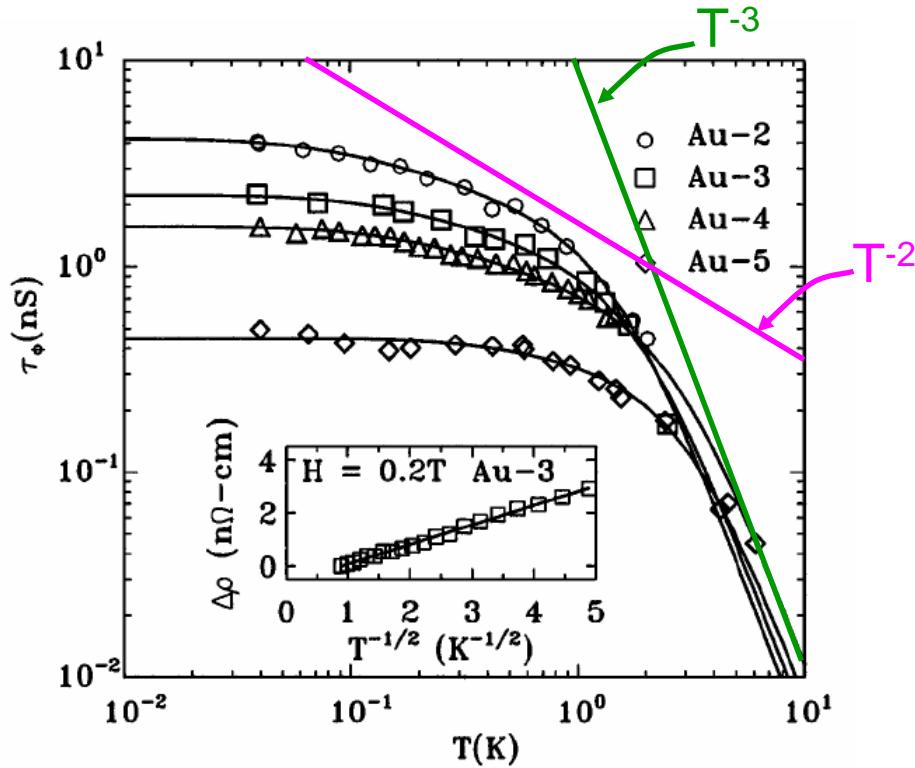
# $\tau_\phi(T)$ in wires: experiment

Echternach, Gershenson, Bozler, Bogdanov & Nilsson, PRB **48**, 11516 (1993)



# A few years later ... a puzzle

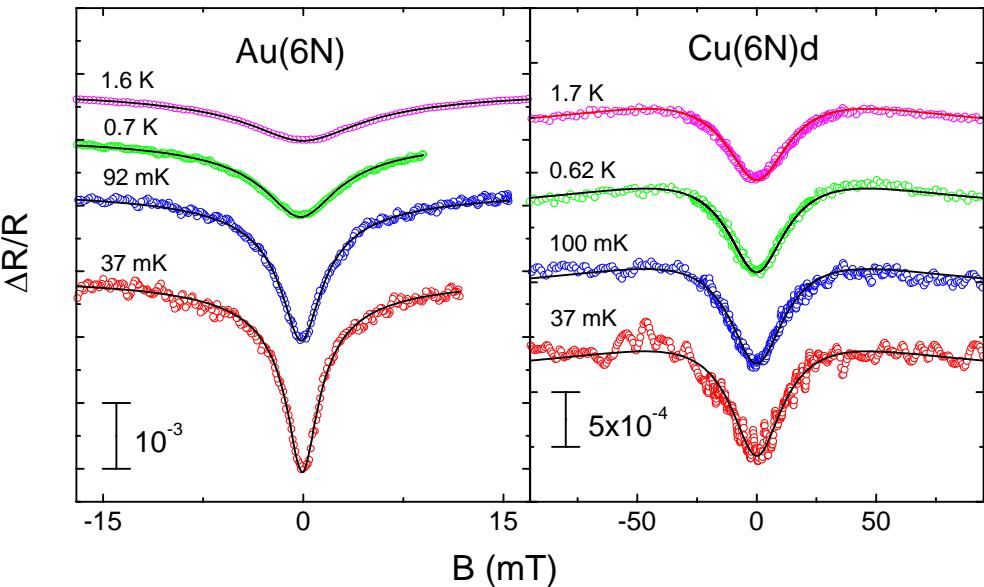
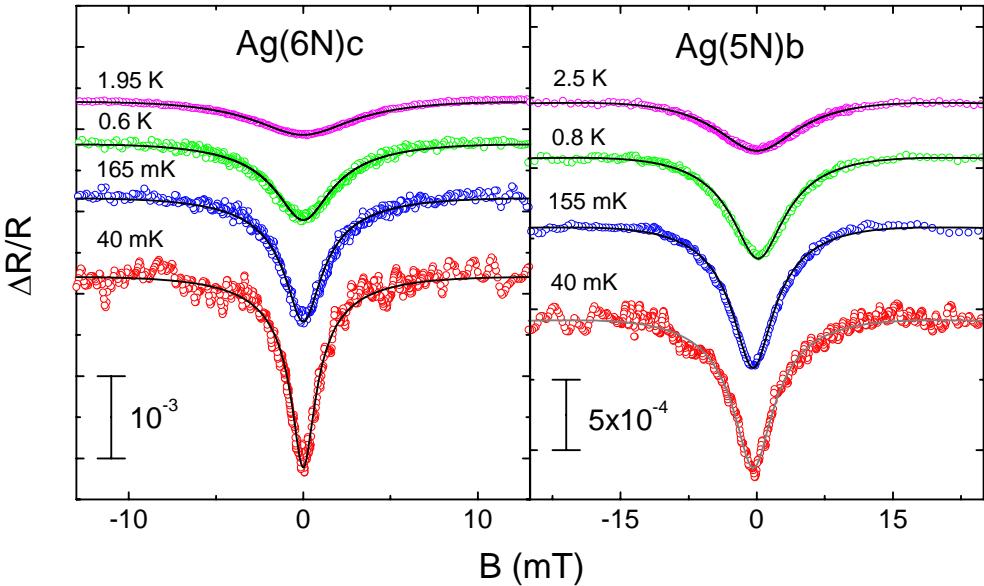
Mohanty, Jariwala and Webb, PRL **78**, 3366 (1997)



“Saturation” of  $\tau_\phi$ :

e-e interaction badly understood ?  
another process dominates ?

# Measuring $\tau_\phi(T)$ : raw data



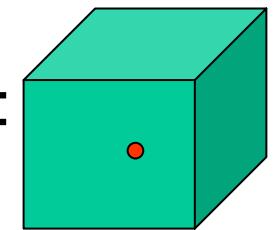
5N = 99.999 % source purity

6N = 99.9999 % “ “ “



1 ppm of

*impurities*:



100 atoms  $\sim$  25 nm

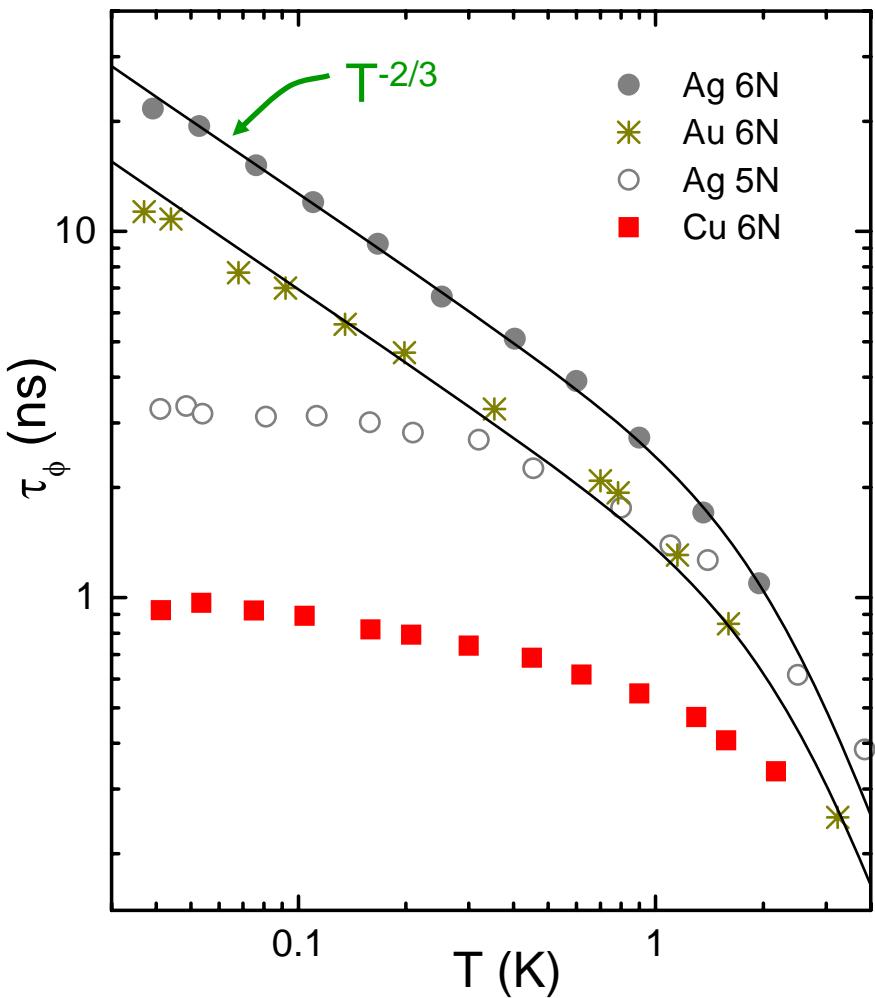
Ag(6N) & Au(6N):

$\Delta R$  grows as T decreases

Ag(5N) & Cu(6N):

$\Delta R$  saturates below  $\sim 100$ mK

# $\tau_\phi(T)$ in Ag, Au & Cu wires



5N = 99.999 % source material purity  
6N = 99.9999 % “ “ “ “

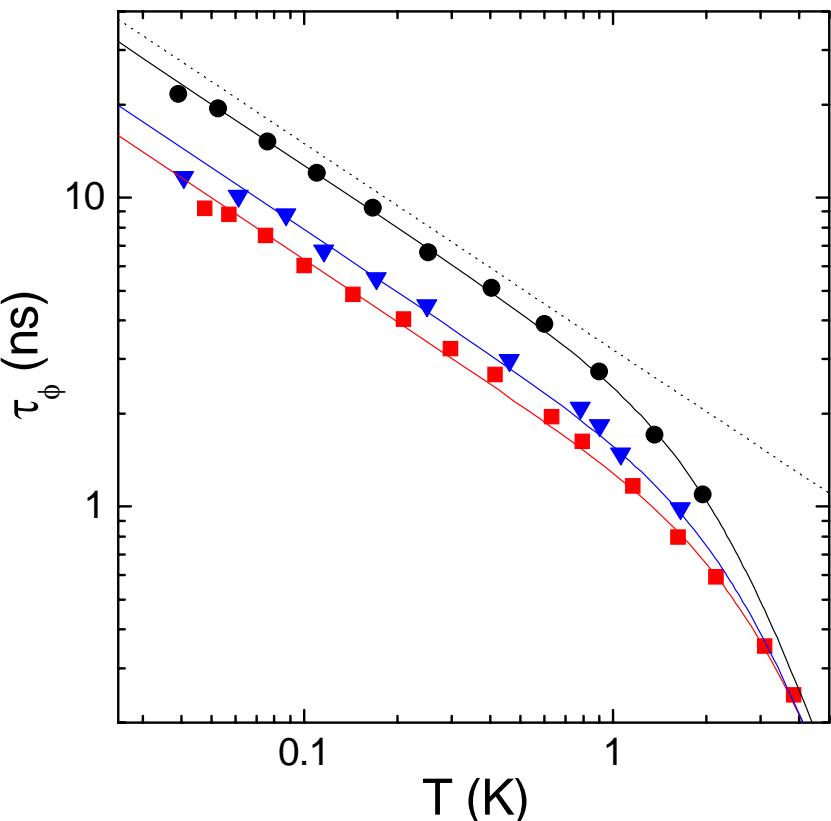
## Low T behavior vs. Purity:

- Ag 6N, Au 6N  
→ agreement with AAK theory

- Ag 5N, Cu 6N  
→ saturation of  $\tau_\phi(T)$

Saturation of  $\tau_\phi$  is sample dependent

# Quantitative comparison with AAK theory for high-purity samples



$$\tau_\phi = (A T^{2/3} + B T^3)^{-1}$$

| Sample  | $A_{thy}$<br>( $\text{ns}^{-1} \text{K}^{-2/3}$ ) | $A$<br>( $\text{ns}^{-1} \text{K}^{-2/3}$ ) |
|---------|---|---|
| Ag(6N)a | 0.55  | 0.73  |
| Ag(6N)b | 0.51  | 0.59  |
| Ag(6N)c | 0.31  | 0.37  |
| Ag(6N)d | 0.47  | 0.56  |
| Au(6N)  | 0.40  | 0.67  |

F. Pierre *et al.*,  
PRB **68**, 0854213 (2003)

$$A_{thy} = \frac{1}{h} \left( \frac{\pi k_B^2}{4\nu_F L w t} \frac{R}{R_K} \right)^{1/3}$$

# Detour: The Kondo Effect

## Resistivity of metals

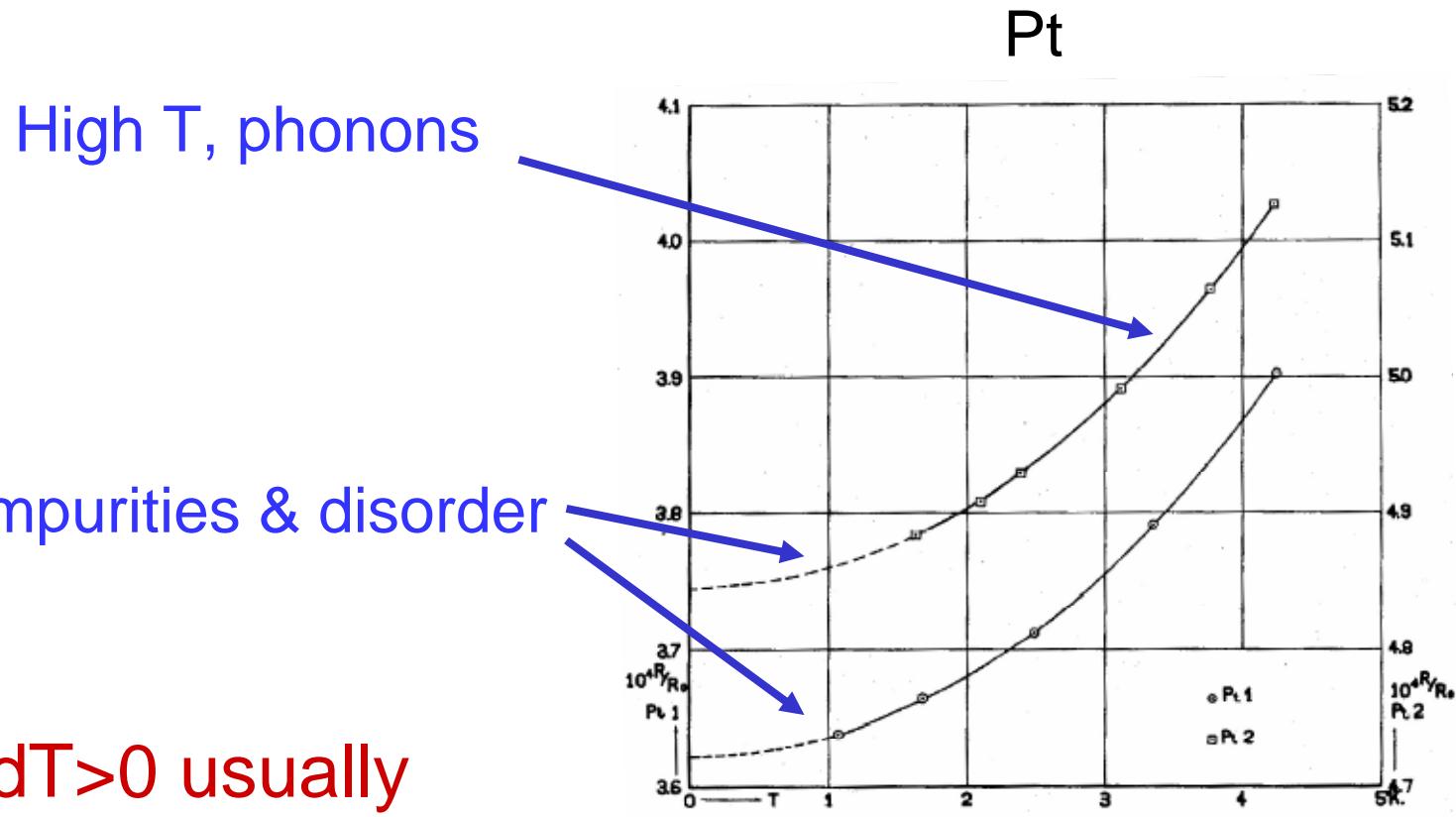


Fig. 1. Electrical resistance of Pt between 0°K. and 4.2°K.

De Haas & de Boer, 1934

# But $dR/dT < 0$ in some samples!

Au

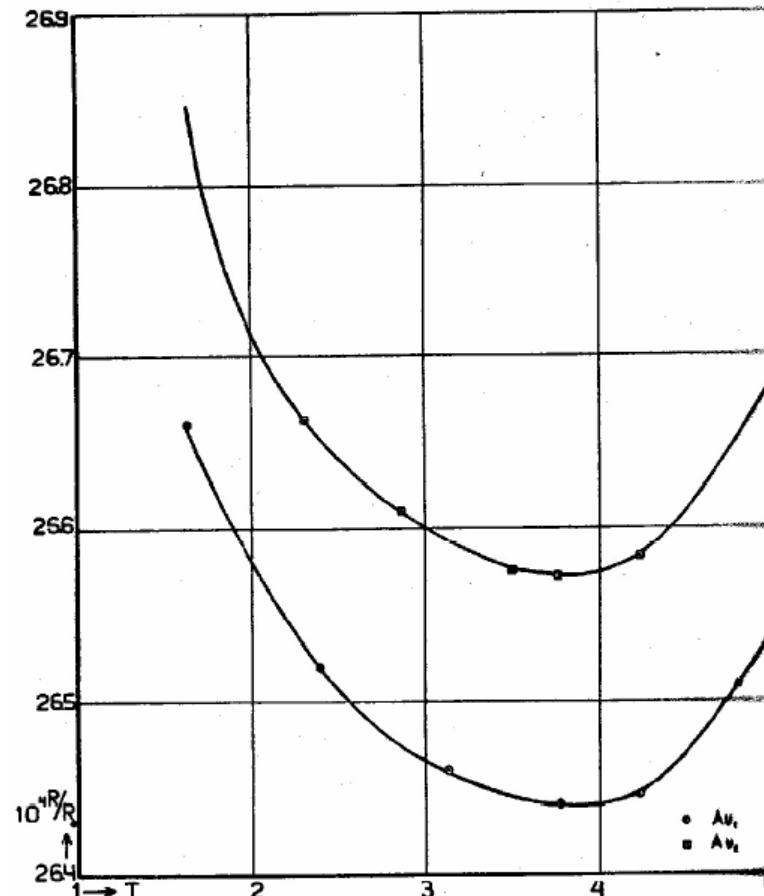


Fig. 1. Resistance of  $Au$  between 1°K. and 5°K.

De Haas, de Boer, & van den Berg, 1934

# Suspect magnetic impurities

Fe in Cu:

J.P. Franck, Manchester, Martin (1961)

But how do they work?

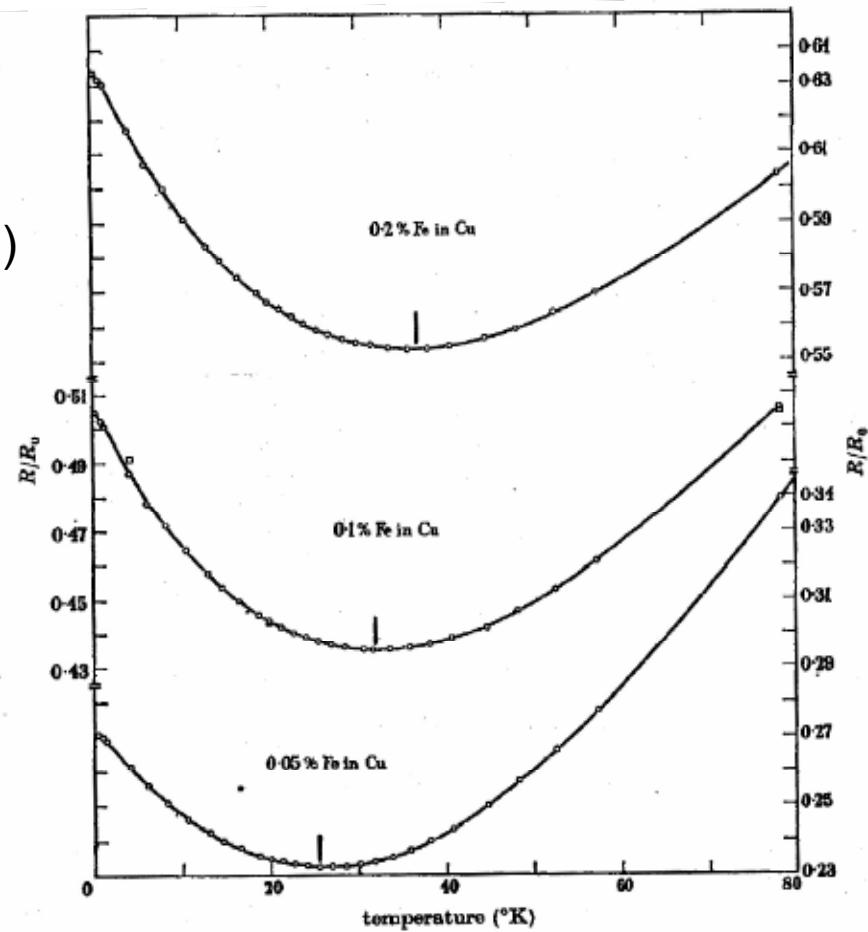
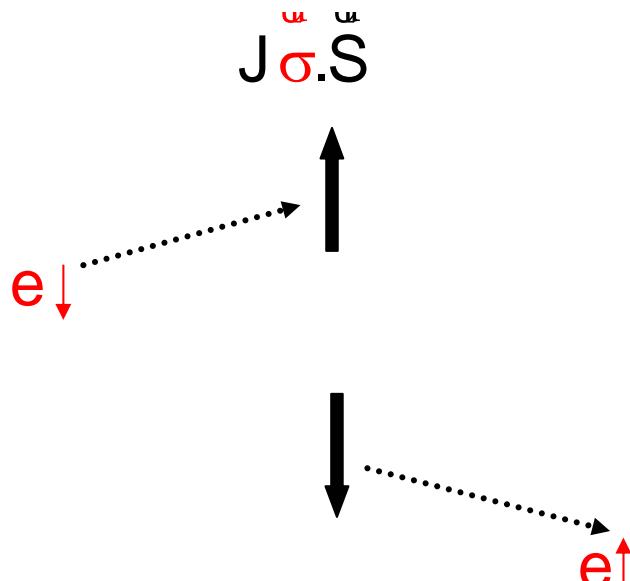


FIGURE 3. The electrical resistance of dilute copper + iron alloys. The bars indicate the point of minimum resistance. The points shown □ were taken after re-annealing the 0.1% alloy.

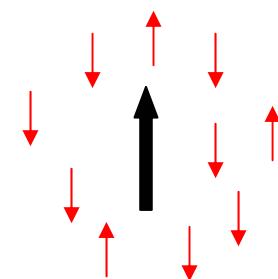
Kondo (1964):  $H = \sum_i J \vec{s}_i \cdot \vec{\xi}$



Spin-flip scattering

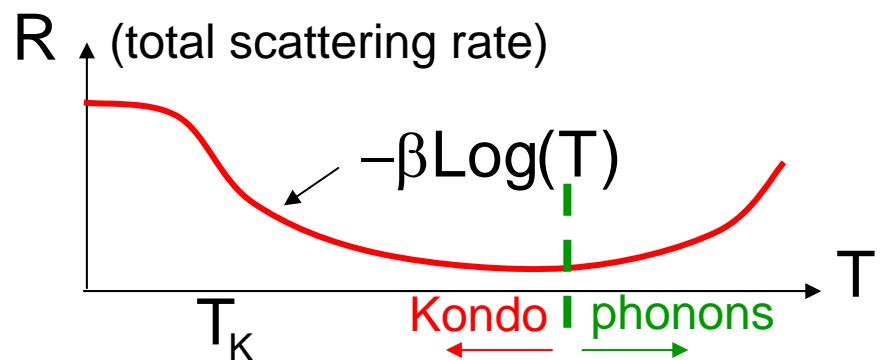
⇒ increased resistivity

Collective effect:

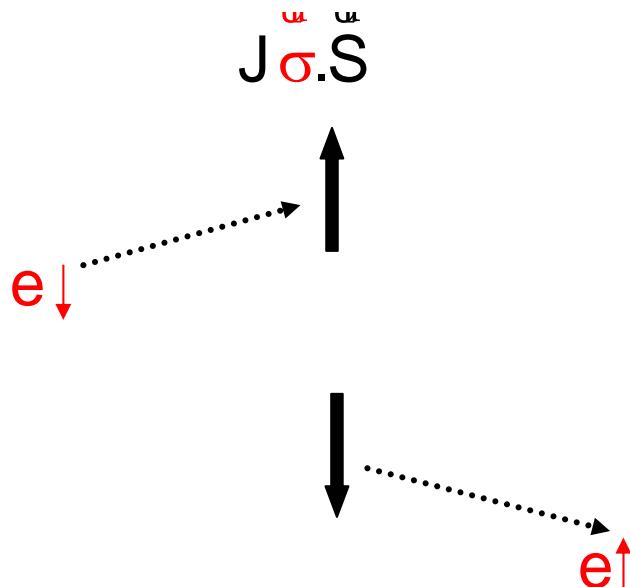


Formation of a singlet spin state

$$k_B T_K \approx E_F e^{-1/\nu J}$$



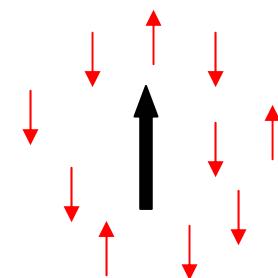
# The Kondo effect and $\tau_\phi(T)$



Spin-flip scattering

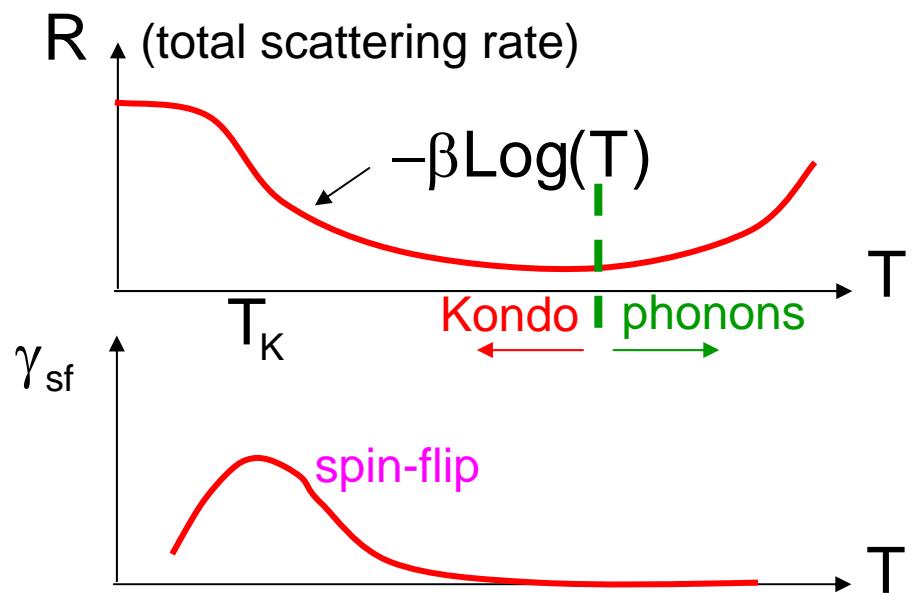
- ⇒ increased resistivity
- ⇒ reduction of  $\tau_\phi$

Collective effect:

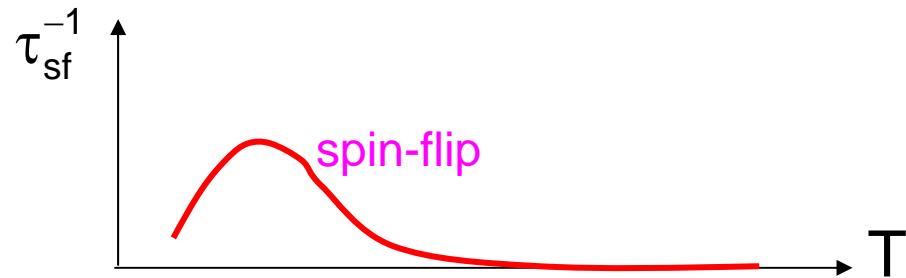


Formation of a singlet spin state

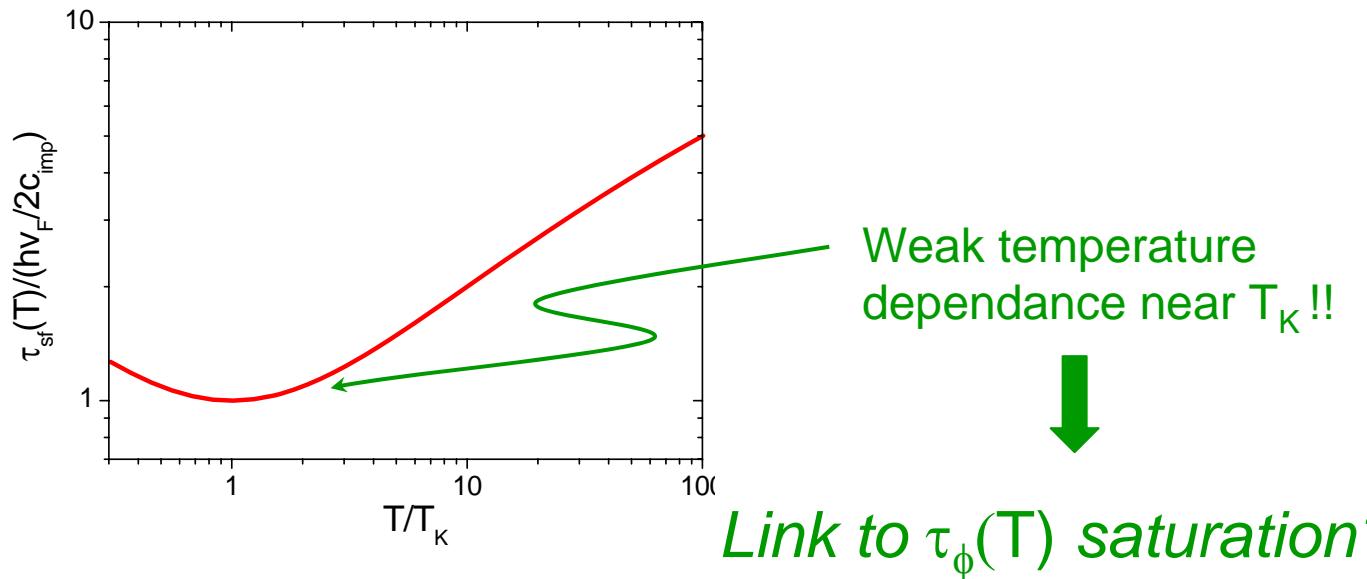
$$k_B T_K \approx E_F e^{-1/\nu J}$$



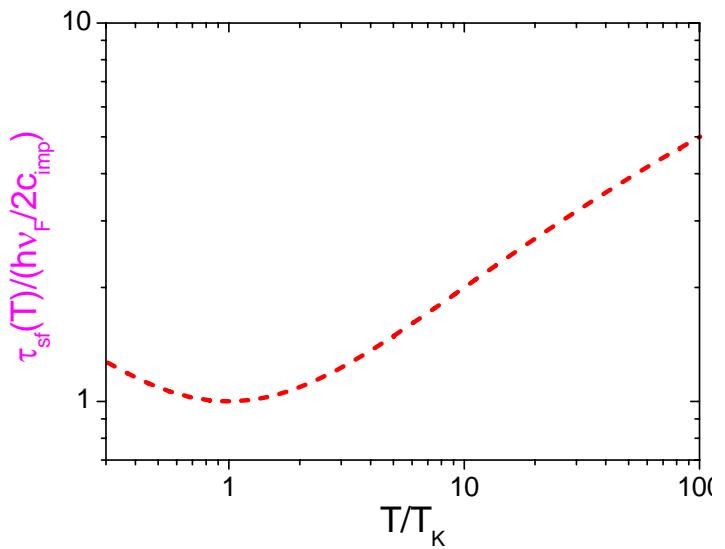
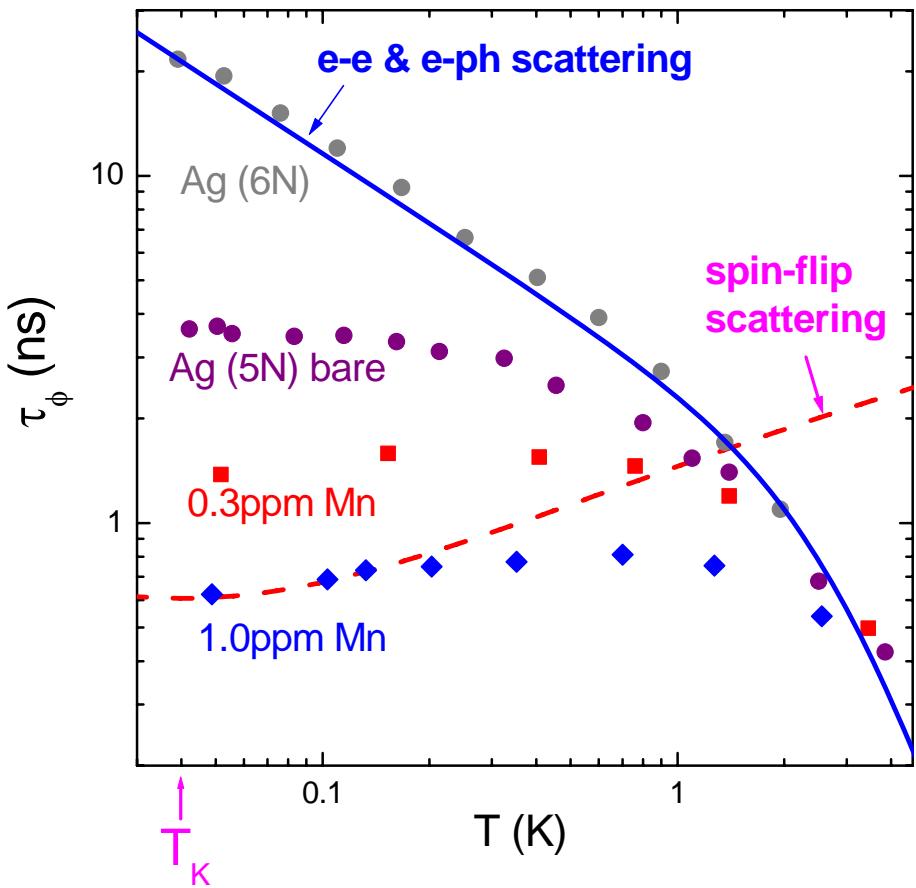
# Nagaoka-Suhl expression of the spin-flip scattering rate near $T_K$



$$\frac{1}{\tau_{sf}} = \frac{c_{\text{mag}}}{\pi \hbar \nu_F} \frac{\pi^2 S(S+1)}{\pi^2 S(S+1) + \ln^2(T/T_K)}$$



# Effect of magnetic impurities on $\tau_\phi$

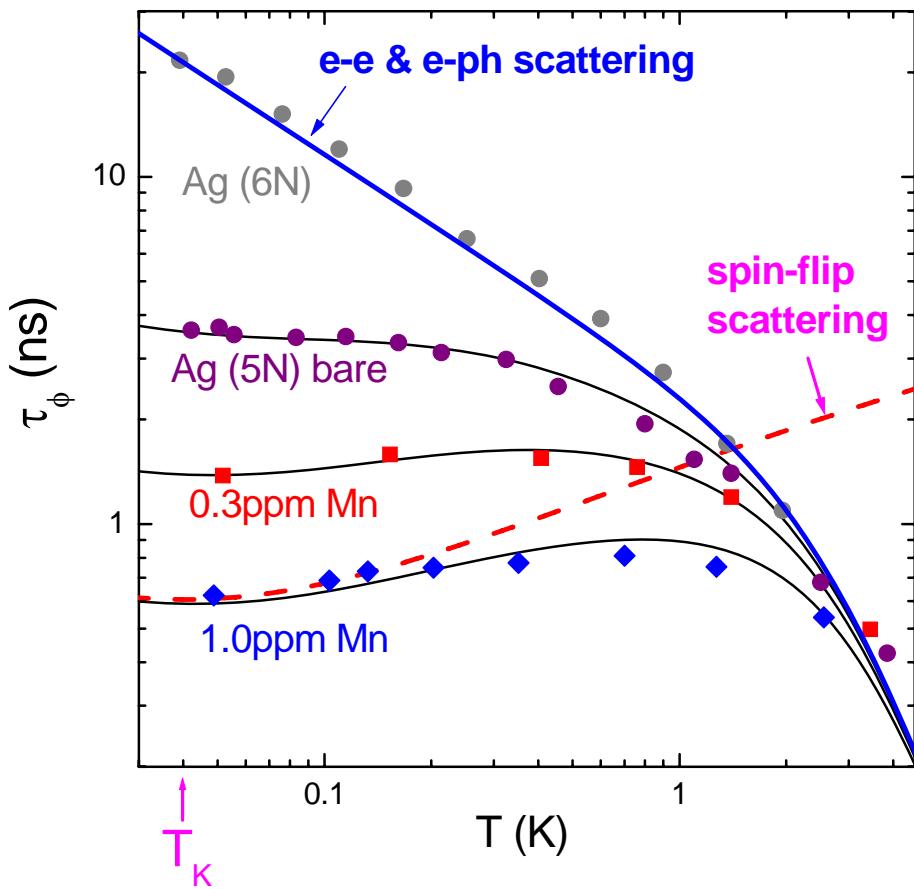


Spin-flip rate peaks at  $T_K$ :

$$\tau_\phi(T_K) = \frac{0.6 \text{ ns}}{c_{\text{imp}} (\text{ppm})}$$

$$\frac{1}{\tau_\phi} = \frac{1}{\tau_{ee}} + \frac{1}{\tau_{e-ph}} + \frac{1}{\tau_{sf}}$$

# Effect of magnetic impurities on $\tau_\phi$



F. Pierre *et al.*,  
PRB **68**, 0854213 (2003)

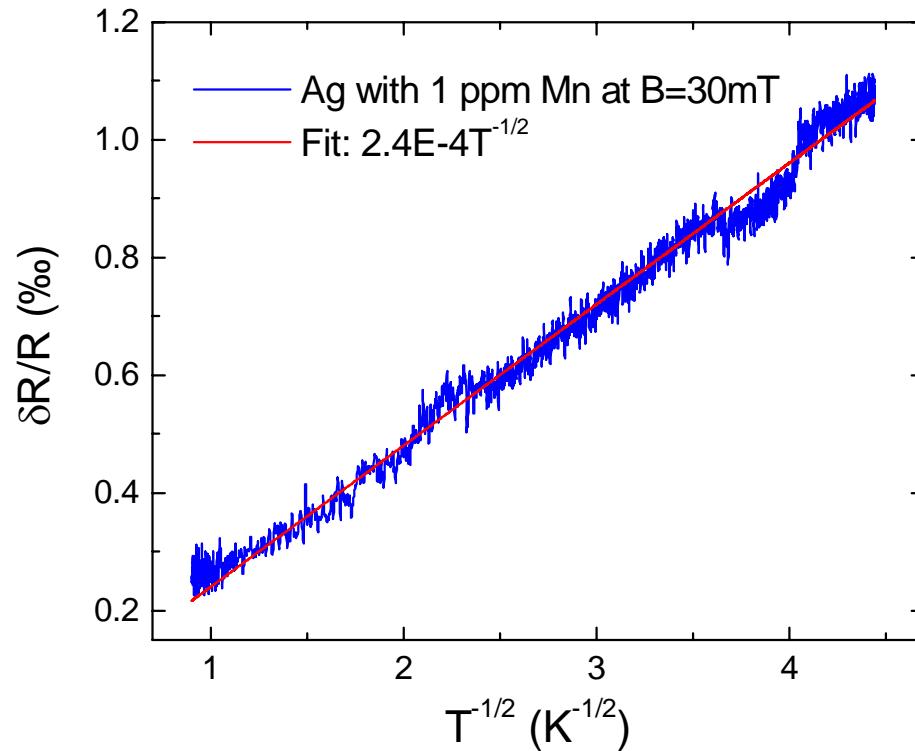
## Fit parameters:

|              |                     |
|--------------|---------------------|
| Ag(5N) bare: | $0.13\text{ ppm}$   |
| + 0.3 ppm    | $: 0.40\text{ ppm}$ |
| + 1 ppm      | $: 0.96\text{ ppm}$ |

Above  $T_K$  : partial compensation of e-e and s-f

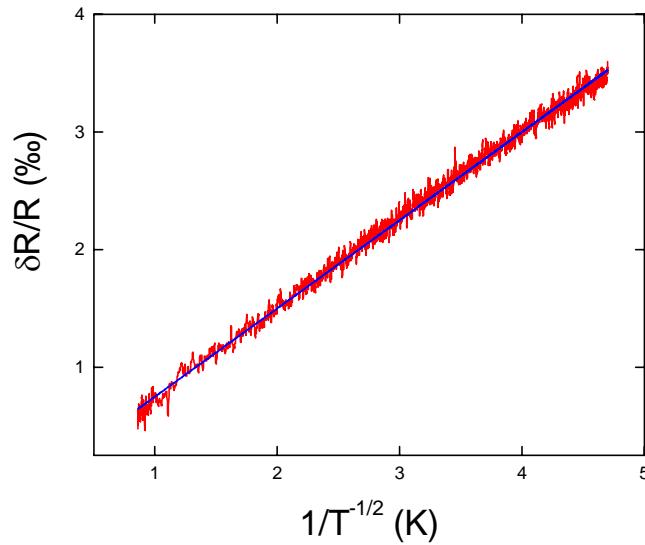
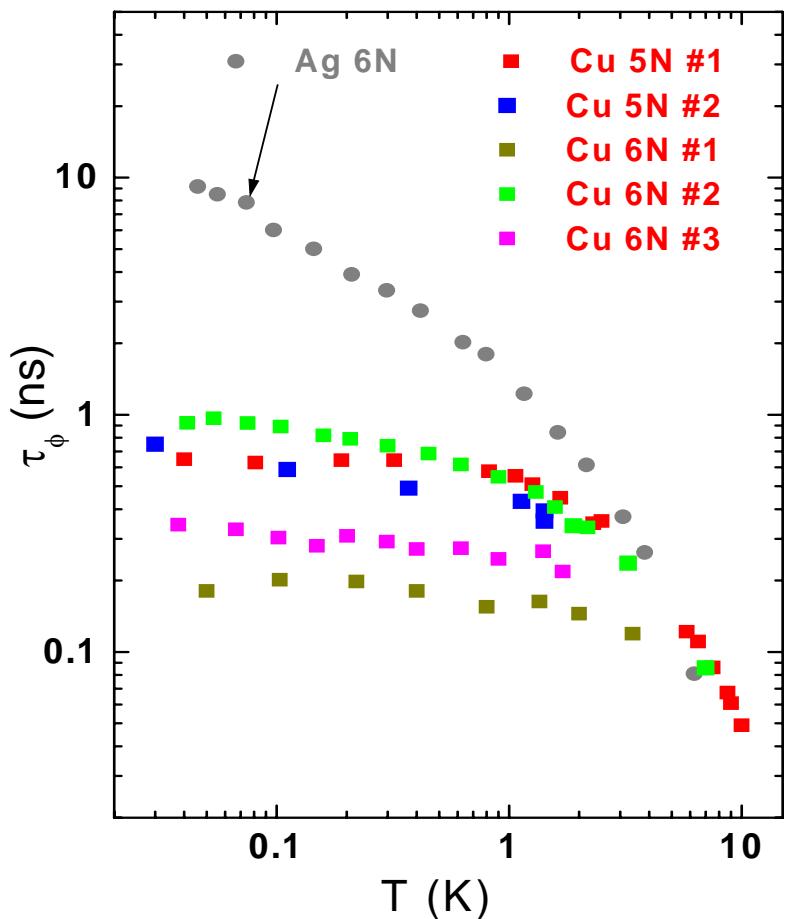
→ apparent saturation

# Why can't we just detect magnetic impurities with $R(T)$ (the original Kondo effect)?



1 ppm of Mn is invisible in  $R(T)$   
(hidden by e-e interactions)

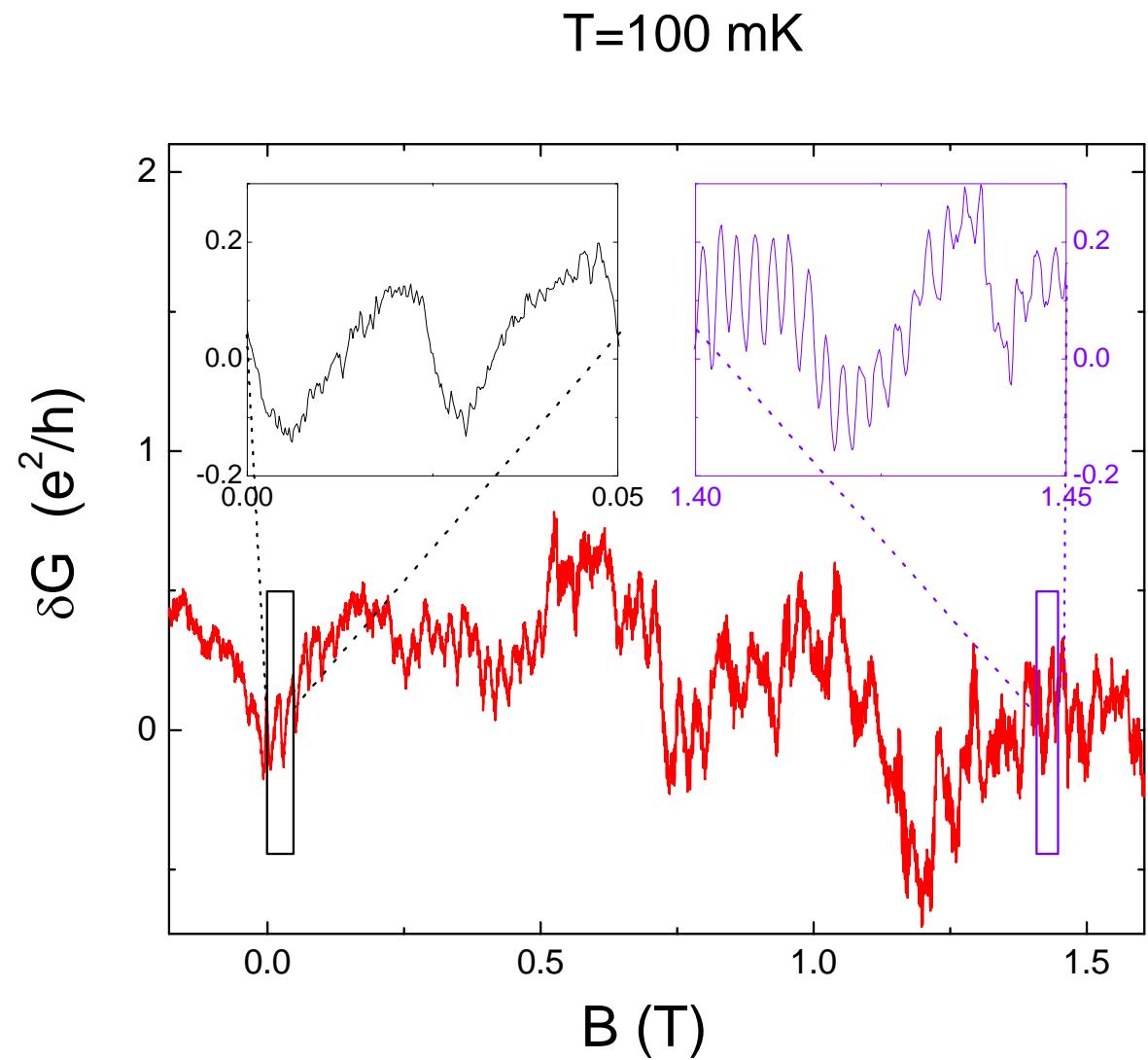
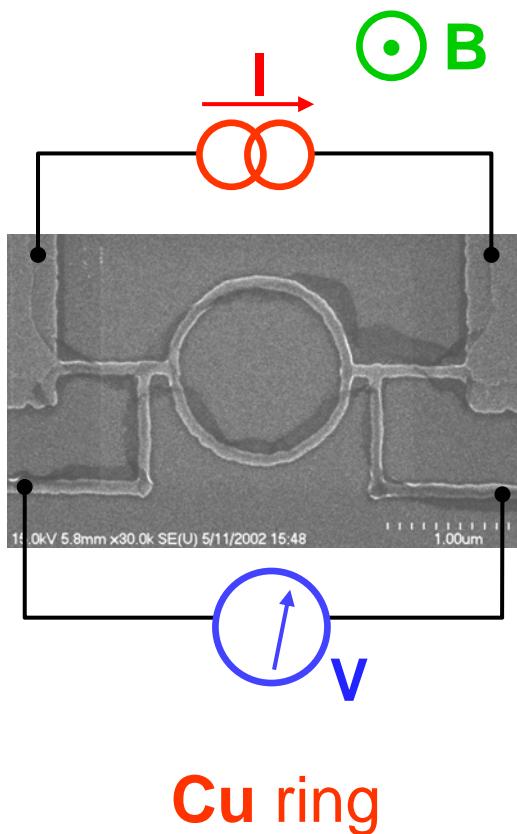
# Source material purity vs. sample purity: Cu samples



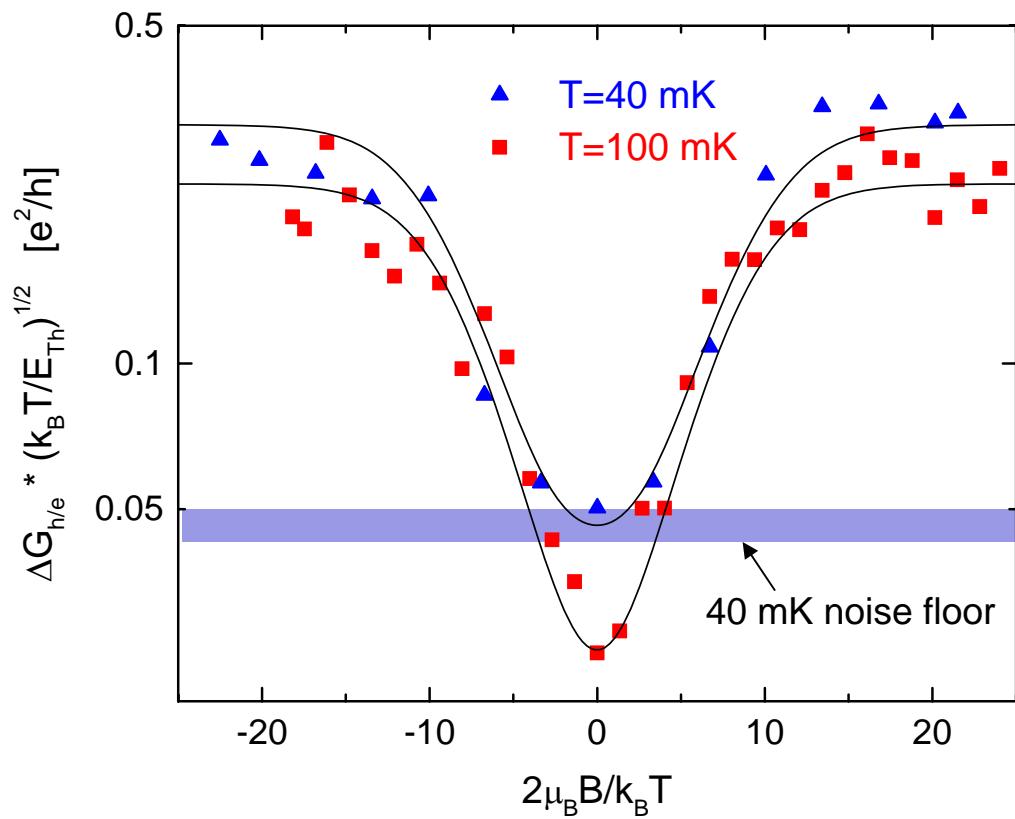
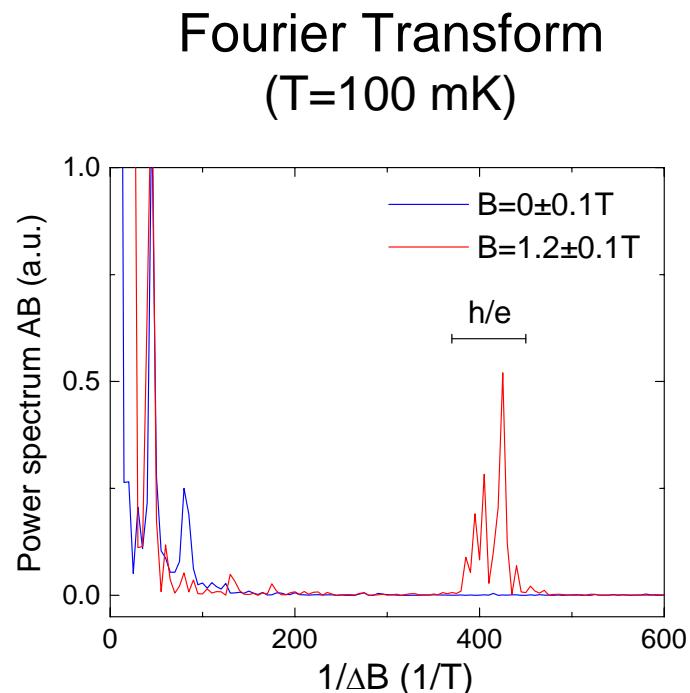
Magnetic impurities  
are invisible in  $R(T)$

- In all Cu samples  $\tau_\phi(T)$  saturates at low  $T$
- $\tau_\phi(T)$  is strongly reduced but shows no dip

# Measure $\tau_\phi(B)$ from Aharonov-Bohm oscillations



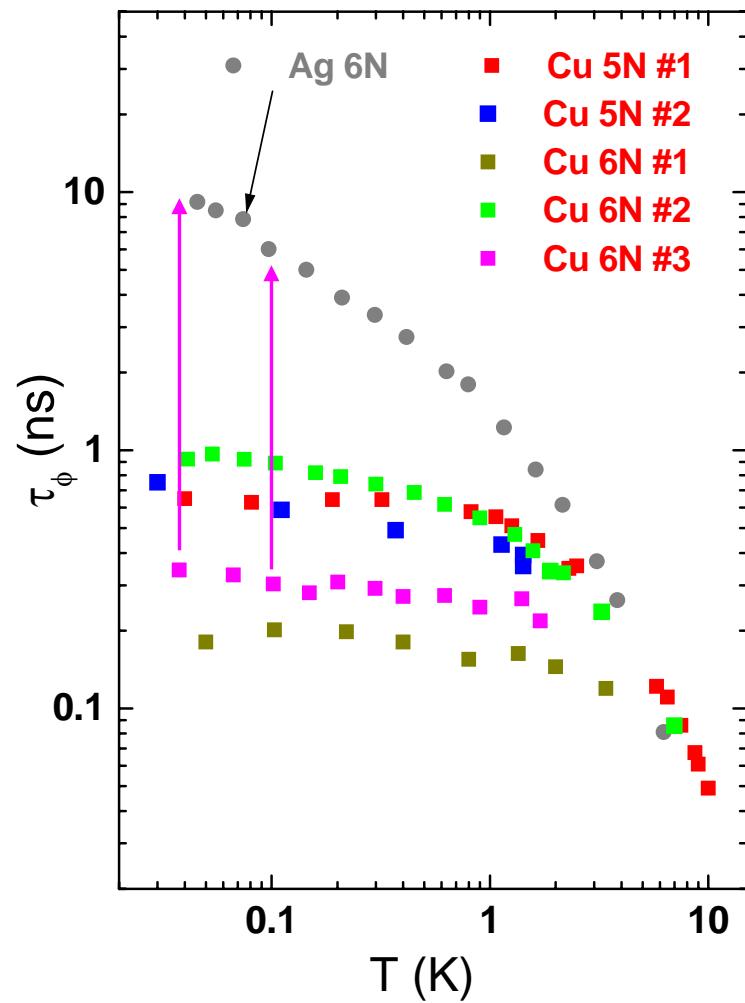
# Aharonov-Bohm oscillations vs. magnetic field



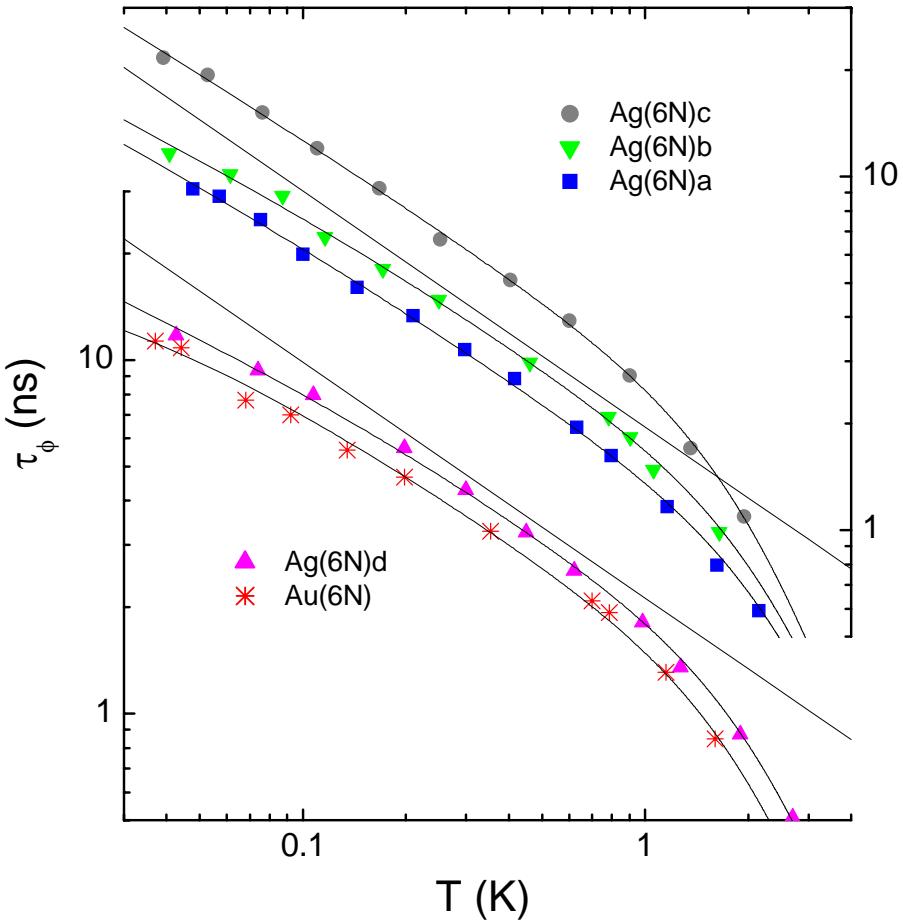
AB oscillations increase with B  
⇒ presence of magnetic “impurities” !

In Cu,  $\tau_\phi(B > B_c) \gg \tau_\phi(B=0)$

Apply B

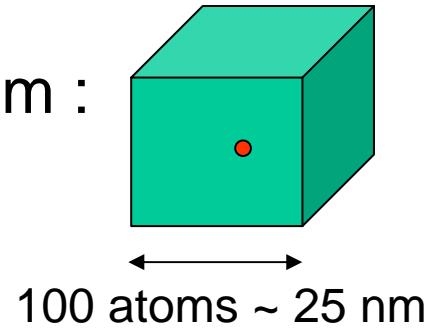


# Evidence for extremely dilute magnetic impurities even in purest samples



| Sample  | Imp. | $T_K(K)$ | c (ppm) |
|---------|------|----------|---------|
| Ag(6N)a | Mn   | 0.04     | 0.009   |
| " b     | "    | "        | 0.011   |
| " c     | "    | "        | 0.0024  |
| " d     | "    | "        | 0.012   |
| Au(6N)  | Cr   | 0.01     | 0.02    |

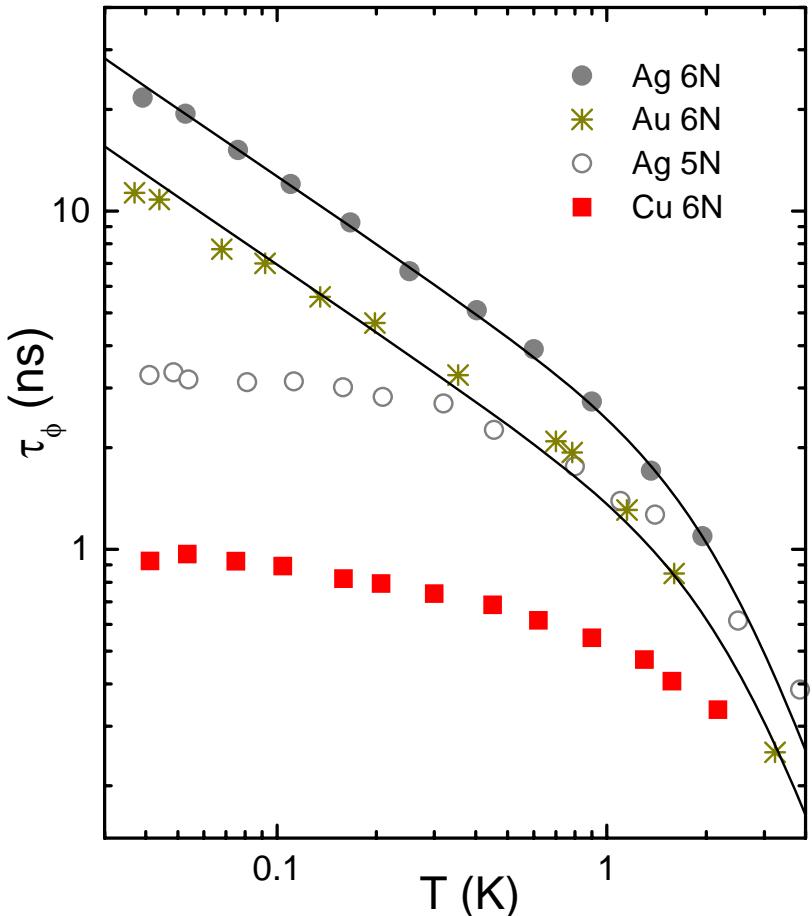
1 ppm :



100 atoms ~ 25 nm

In the wire, 0.01 ppm = 3 impurities/ $\mu\text{m}$

# Conclusions



Moral of the story: even at concentrations as low as 1 ppm and below, magnetic impurities dominate electron decoherence in metals at low temperature.

# Compare $\tau_\phi$ data with AAK and GZS theories

