NTT BRL School 2005 – Lecture #2

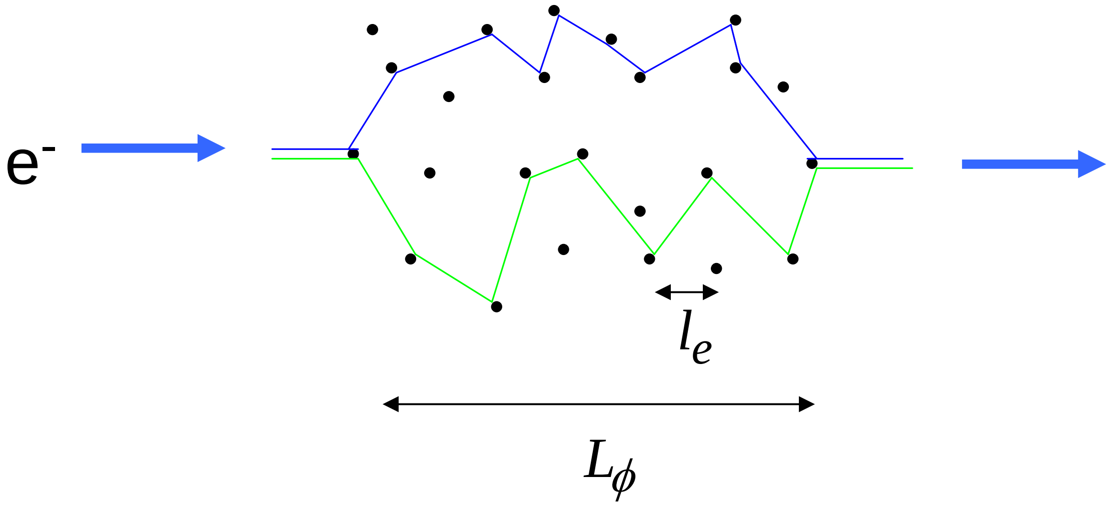
Norman Birge, Michigan State University

Electron Energy Exchange in Diffusive Metal Wires:

Collaborators (CEA Saclay, France):

S. Guéron, F. Pierre, A. Anthore, B. Huard, H. Pothier, D. Esteve, M. H. Devoret

Electron transport in diffusive regime



1. Elastic scattering (film boundaries, impurities) $l_e = v_F \tau_e$

   ⇨ diffusive states $D = \frac{1}{3} v_F l_e$

2. Inelastic scattering (phonons, other electrons, spins)

   ⇨ loss of phase coherence $L_\phi = \sqrt{D\tau_\phi}$

   ⇨ energy exchange between electrons

Background: Shot noise in diffusive metal wires

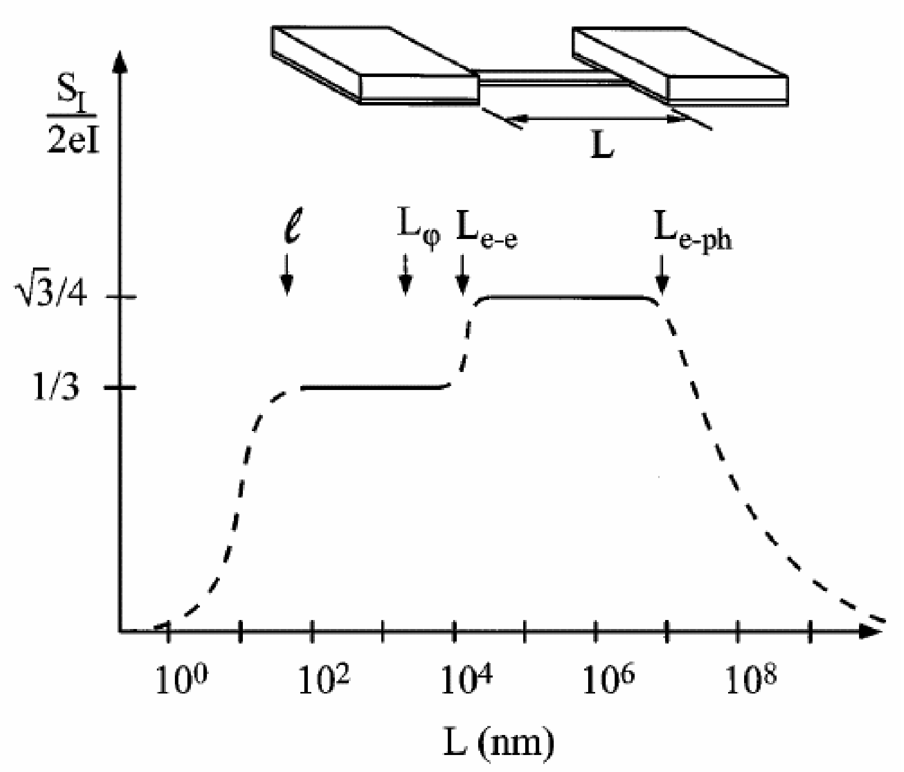
Steinbach, Martinis and Devoret, PRL 76, 3806 (1996)

Theory:

Nagaev 1992,1995

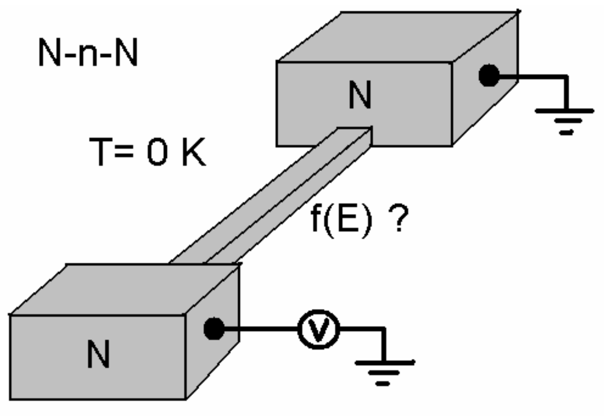
Kozub & Rudin, 1995

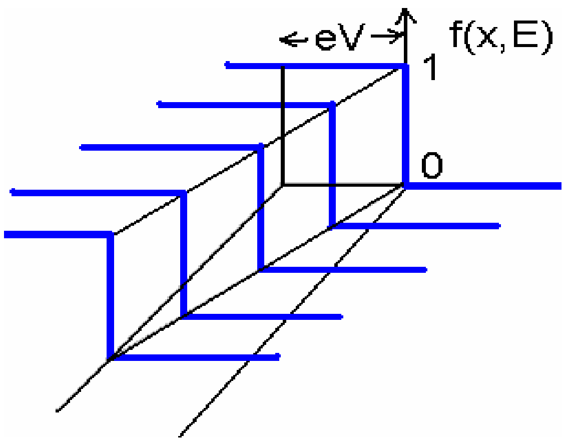
$$S_I = \frac{4}{RL} \iint dxdE\ f(x,E)(1 - f(x,E))$$



What does f(x,E) look like?

Distribution function -- textbook case (no shot noise)



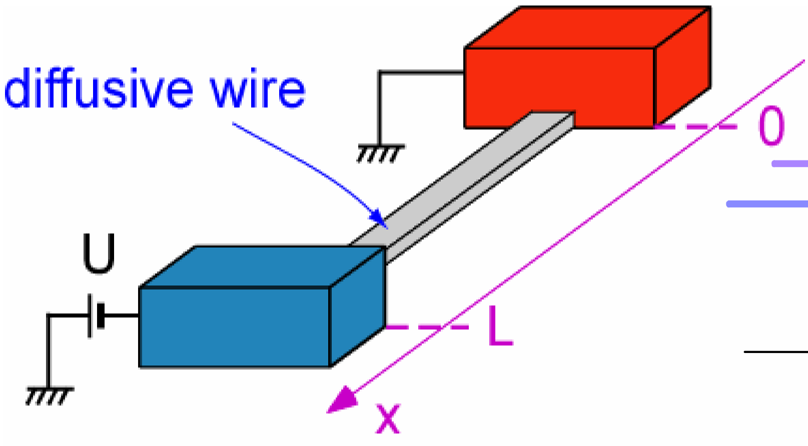


$\tau_D = L^2/D$

Assumes complete thermalization -- $t_D >> t_{electron\text{-}phonon}$

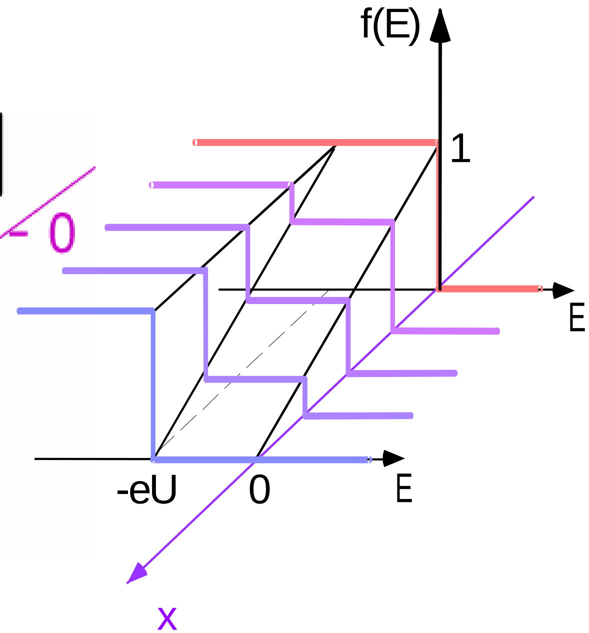
Never true in mesoscopic metal samples at low T!

Distribution function for $\tau_D << \tau_{electron\text{-}phonon}$



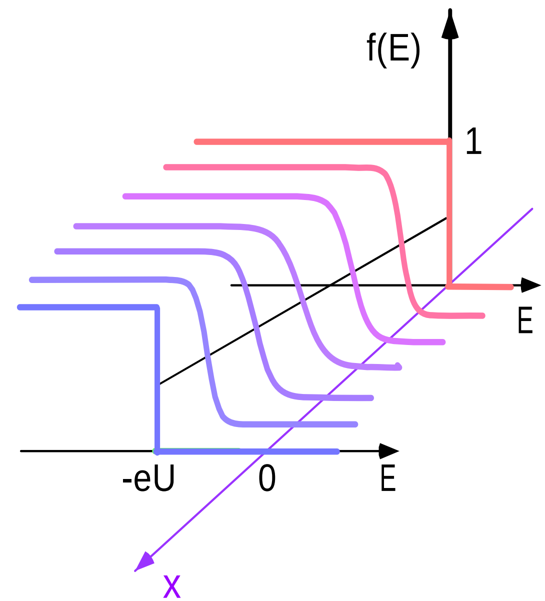
$\boxed{\tau_D = L^2/D}$

free electrons



$\tau_D << \tau_{interaction}$

"hot" electrons



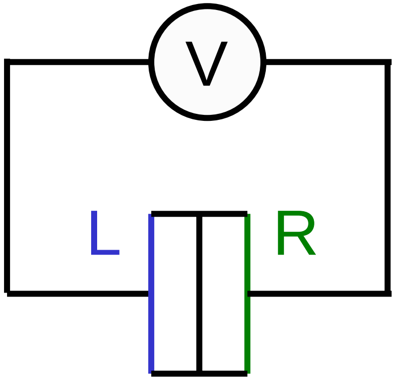
$\tau_D >> \tau_{interaction}$

f(x,E) shaped by energy exchange

How to measure f(x,E)?

(local electron energy distribution function)

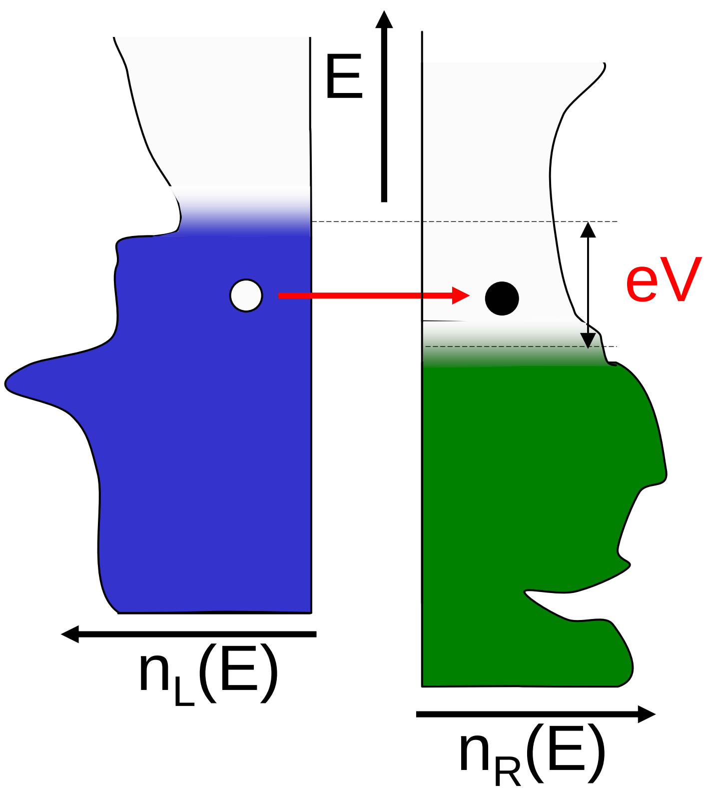
Aside 1: Current through a tunnel junction



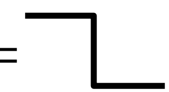
$$I = e\left(\Gamma_{\rightarrow} - \Gamma_{\leftarrow}\right)$$

$$\Gamma_{\rightarrow} = \frac{2\pi\nu_F^2}{h}\int dE \left|\langle M\rangle\right|^2 n_L(E)\, n_R(E+eV)\, f_L(E)\left(1 - f_R(E+eV)\right)$$

$$\Gamma_{\leftarrow} = \qquad \left(1 - f_L(E)\right) f_R(E+eV)$$

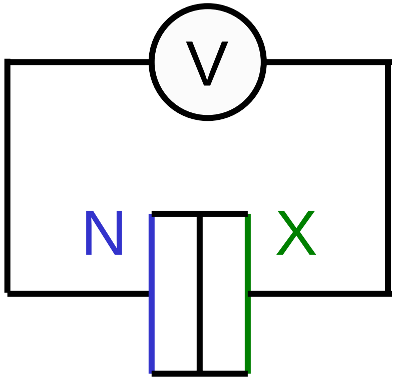


$$I = \frac{1}{eR_T}\int dE\, n_L(E)\, n_R(E+eV)$$
$$\text{x}\,\left(f_L(E) - f_R(E+eV)\right)$$

NN junction: $n(E) = 1 \quad f(E) =$ 

$$\Rightarrow I = \frac{V}{R_T}$$

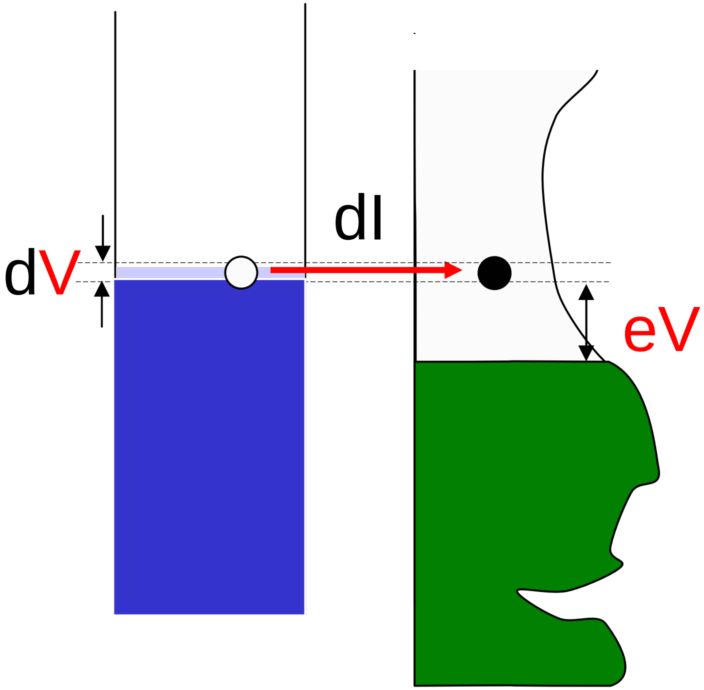
Conductance of an N-X junction at T=0



$$I = \frac{1}{eR_T}\int dE\, n_L(E)\, n_R(E+eV) \times \left(f_L(E) - f_R(E+eV)\right)$$

$$= \frac{1}{eR_T}\int_{-eV}^{0} dE\, n_X(E+eV)$$

$$= \frac{1}{eR_T}\int_{0}^{eV} dE\, n_X(E)$$



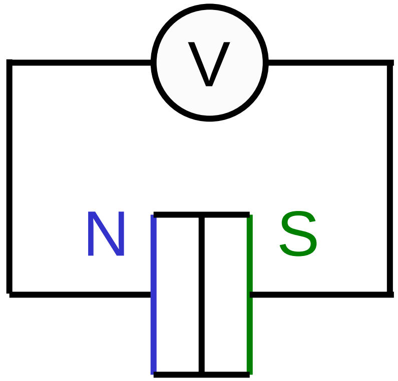
$$\boxed{\frac{dI}{dV} = \frac{1}{R_T}\, n_X(eV)}$$

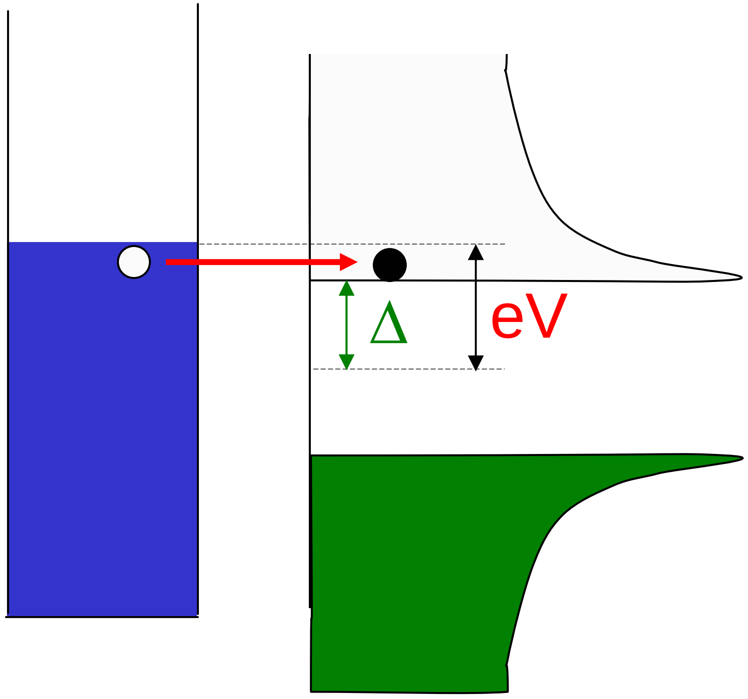
Spectroscopy of n_X

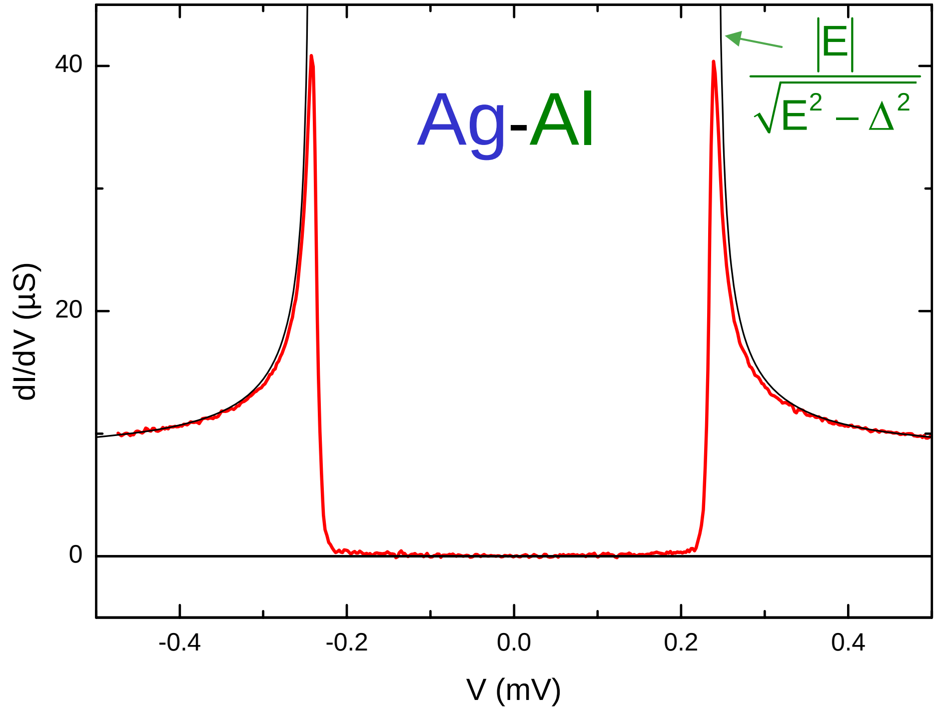
How to measure f(E):

tunnel spectroscopy using an N-S junction

$$I = \frac{1}{eR_T}\int dE\, n_S(E)\left(f_N(E-eV) - f_S(E)\right)$$

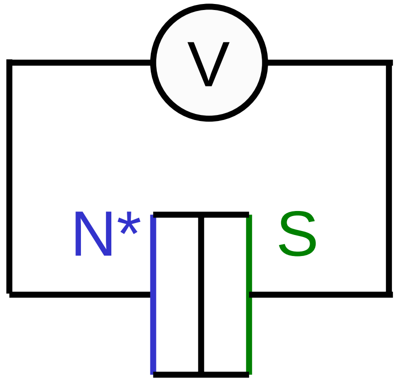






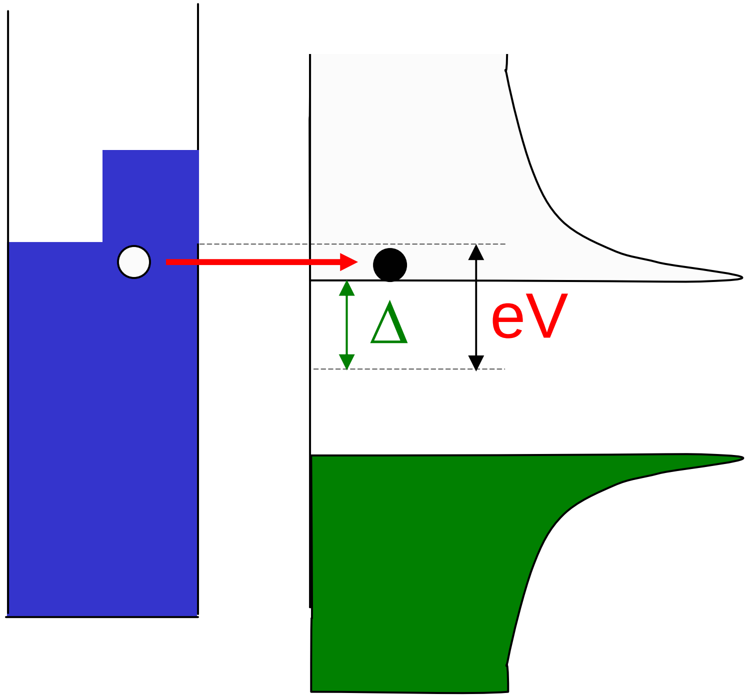
N out of equilibrium:

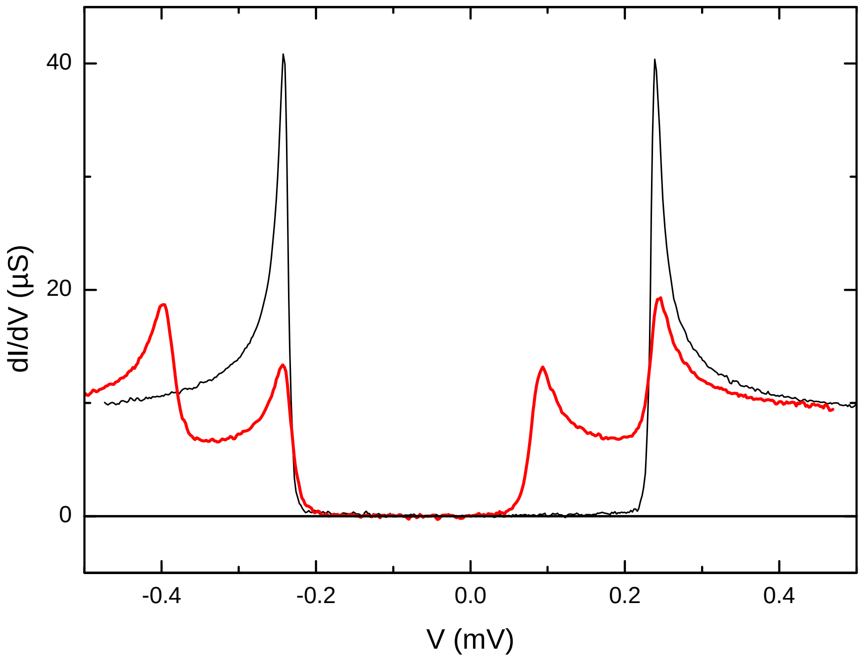
spectroscopy of f(E)



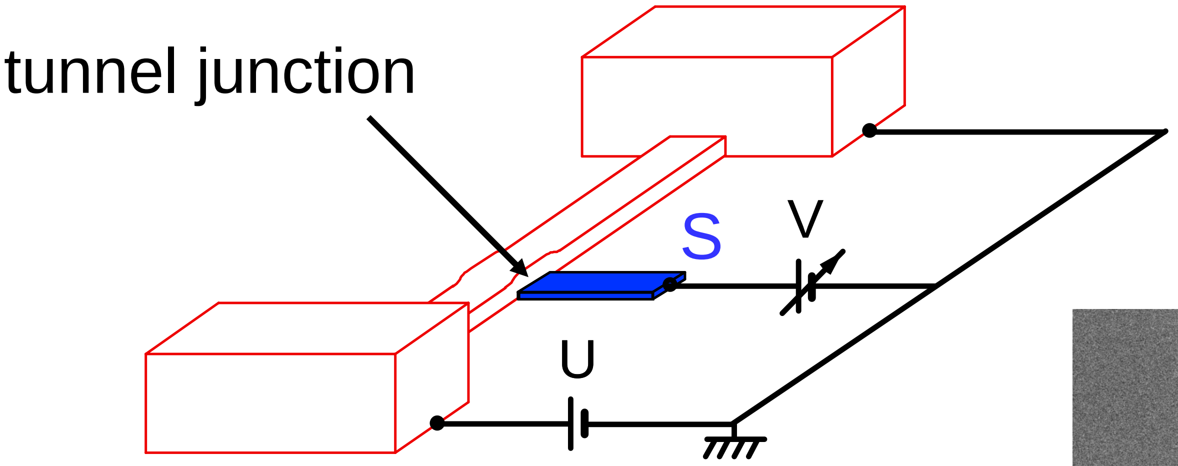
$$I = \frac{1}{eR_T}\int dE\, n_S(E)\left(f_N(E-eV)-f_S(E)\right)$$

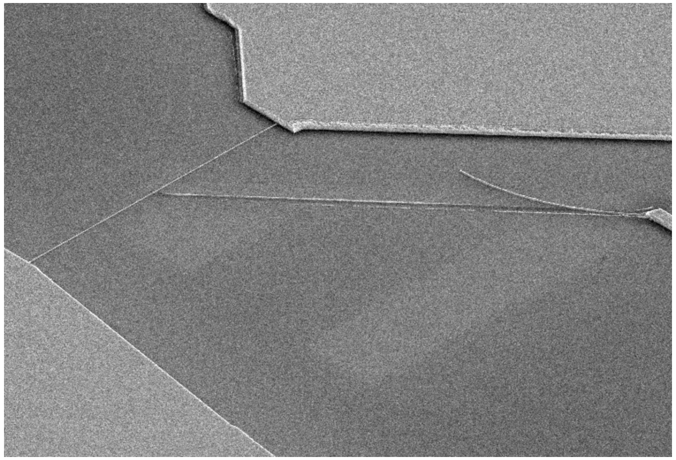
$$\frac{dI}{dV} = \frac{-1}{R_T}\int dE\, n_S(E)\, f_N'(E-eV)$$





Experimental setup



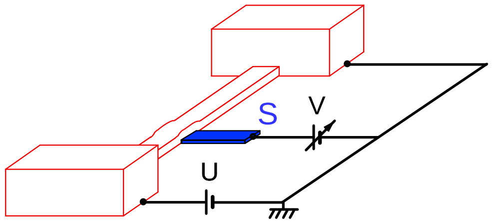


L=5 to 40 µm

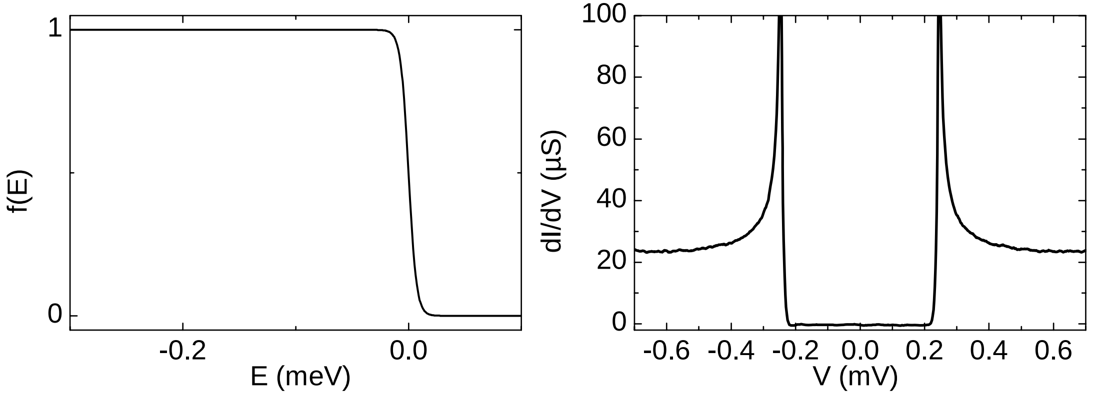
Diffusion time: $\tau_D = \frac{L^2}{D} = 1$ to 60 ns

$$\frac{dI}{dV}(V) \xrightarrow[\textit{deconvolution}]{\textit{numerical}} f(E)$$

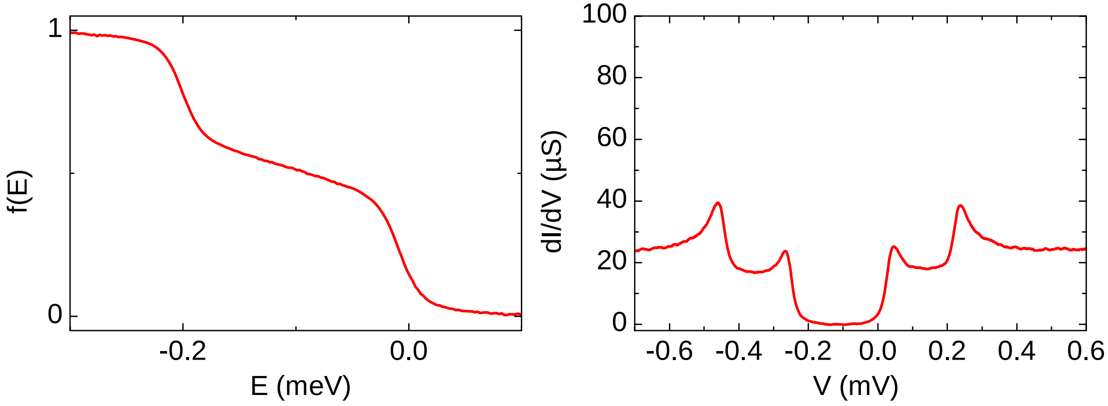
Summarize how to measure f(E):



U=0 mV

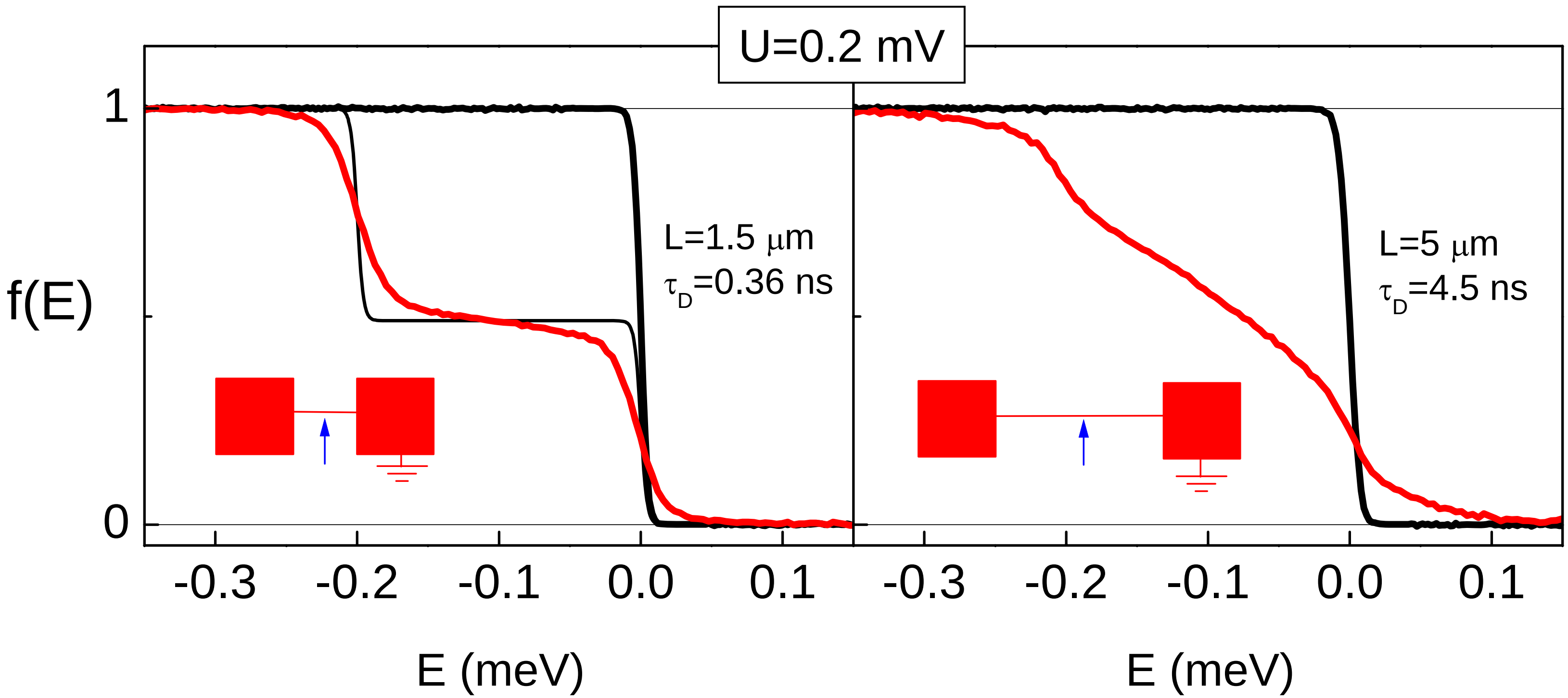


U=0.2 mV



f(E) dI/dV

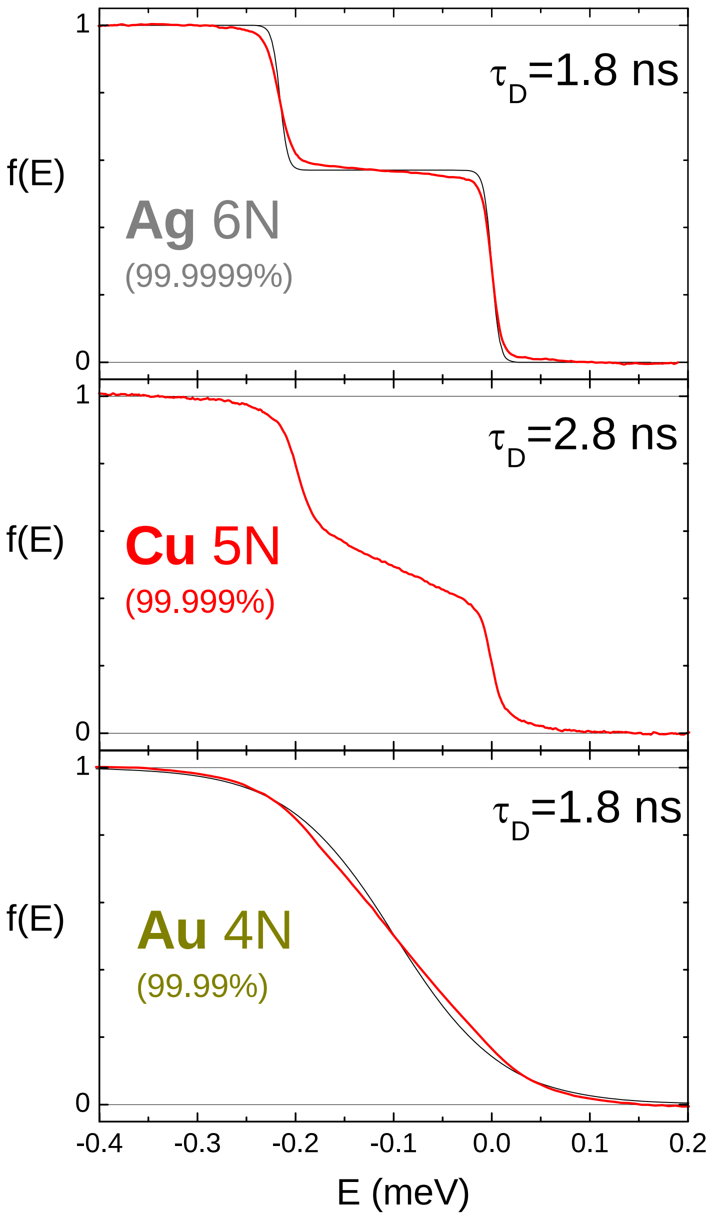
Effect of the diffusion time τ_D on f(E)



longer interaction time $\Rightarrow$ more rounding

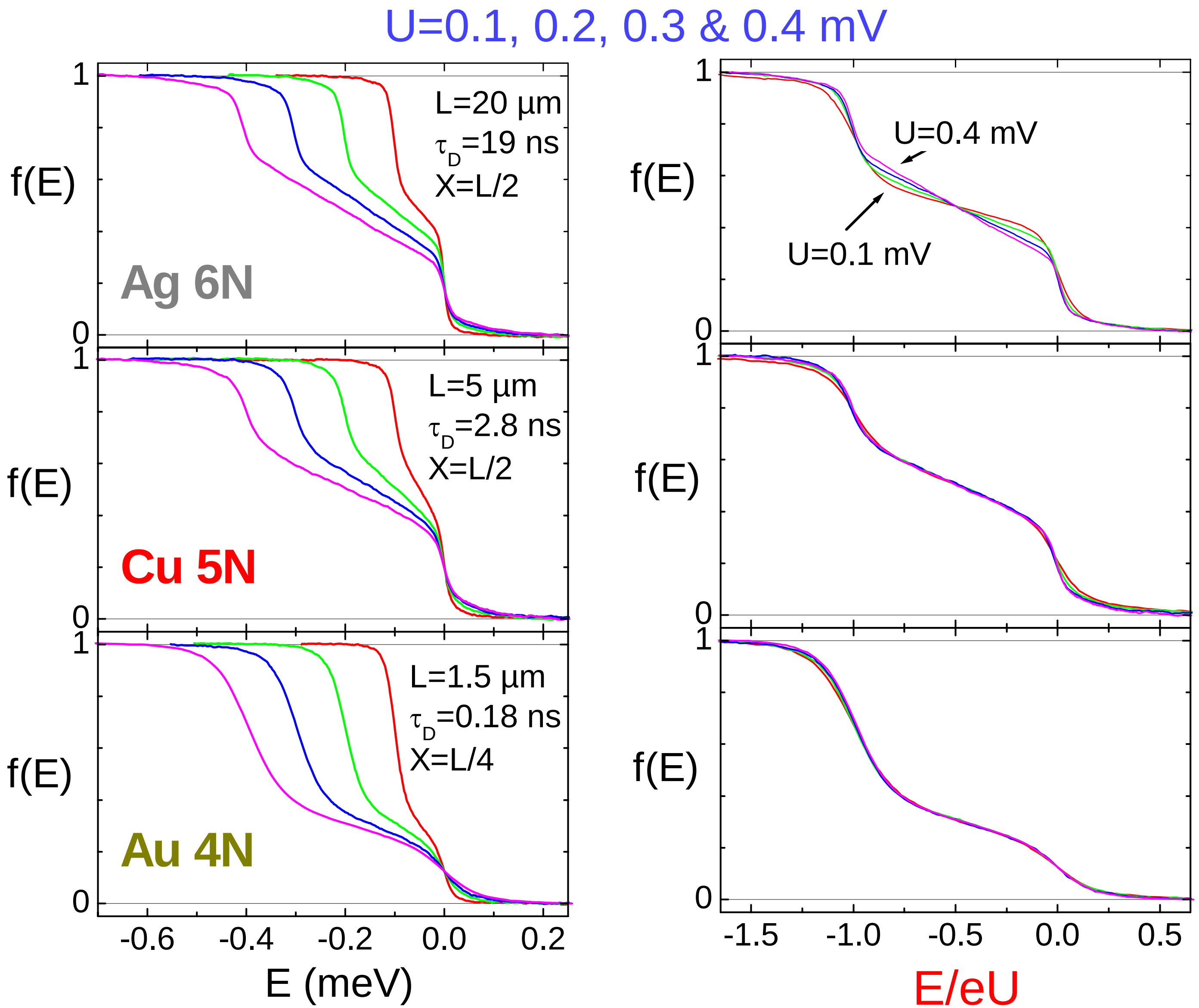
H. Pothier et al., PRL 79, 3490 (1997)

Compare strength of interactions



effect of material ?
effect of purity ?

Compare Dependence on U

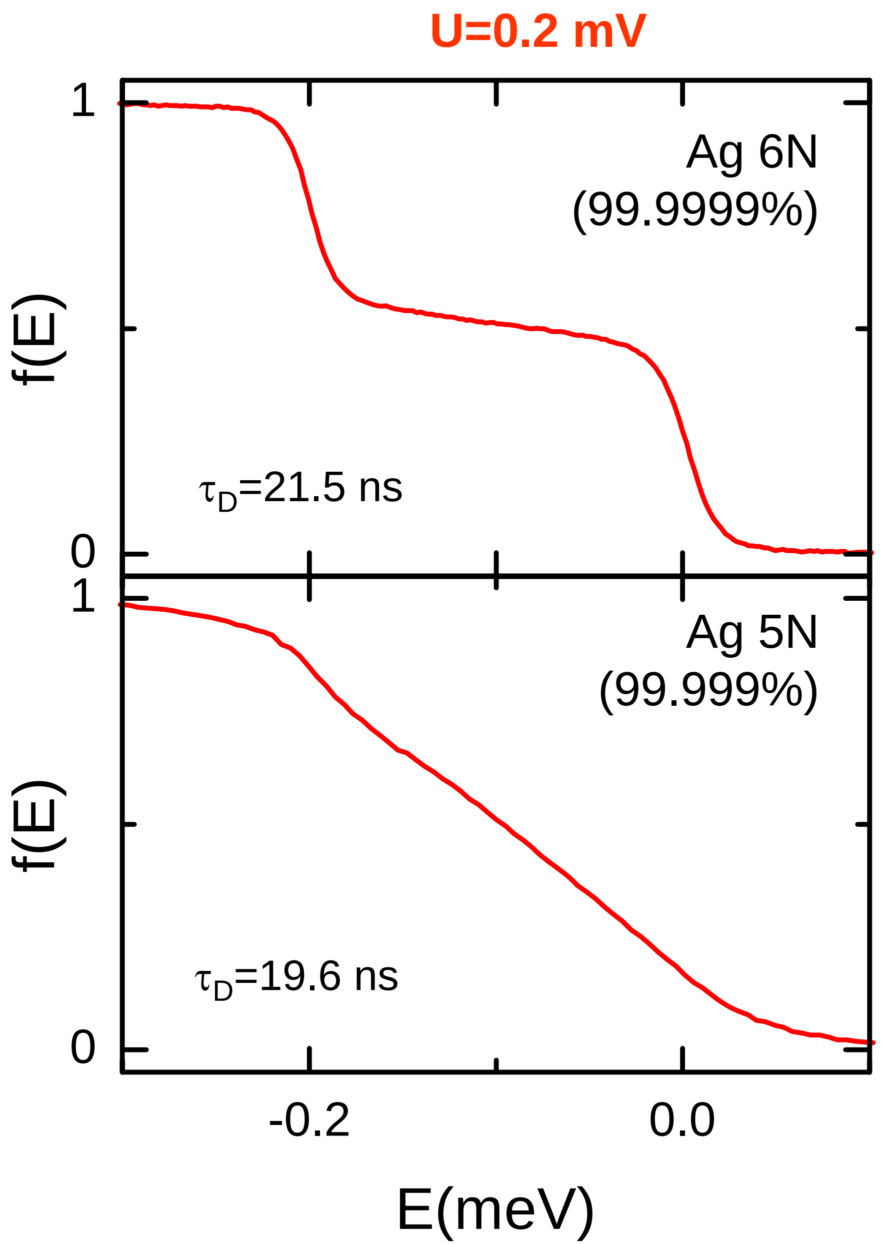


Observe scaling law in **Au 4N** & **Cu 5N** but <u>not</u> in **Ag 6N**

Energy Exchange Rate vs. Sample Purity

T=30 mK

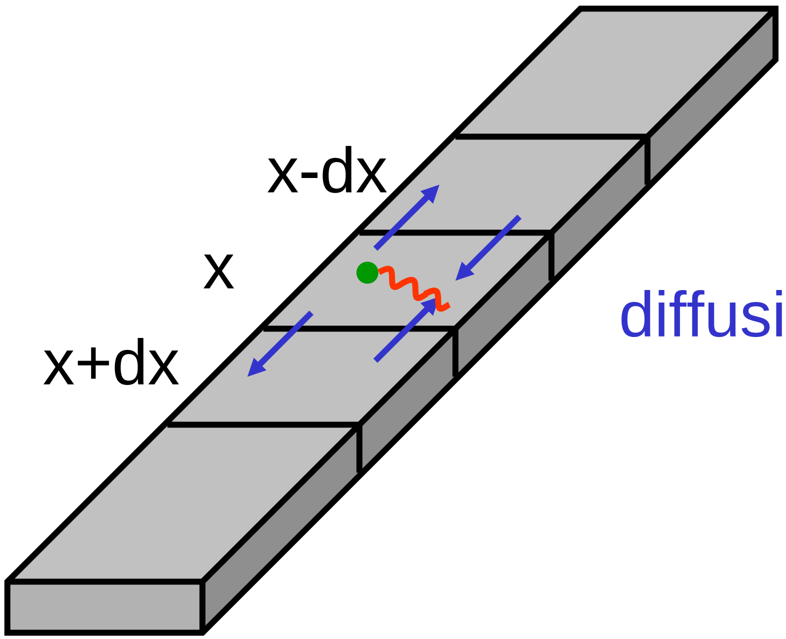
S probe



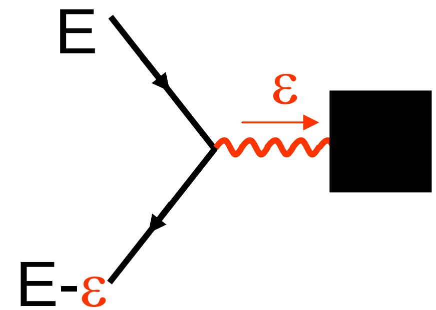
Calculation of f(x,E)

Boltzmann equation in the diffusive regime (Nagaev, Phys. Lett. A, 1992):

$$\cancel{\frac{\partial f(E)}{\partial t}} + D\frac{\partial^2 f(E)}{\partial x^2} = I_{in}\left(x,E,\{f\}\right) - I_{out}\left(x,E,\{f\}\right)$$



diffusion in real space



transfers of population
in the energy space

Boundary conditions :

$$f_{x=0}(E) = f_{x=L}(E) = \text{Fermi function}$$

Calculation of f(x,E)

Boltzmann equation in the diffusive regime (Nagaev, Phys. Lett. A, 1992):

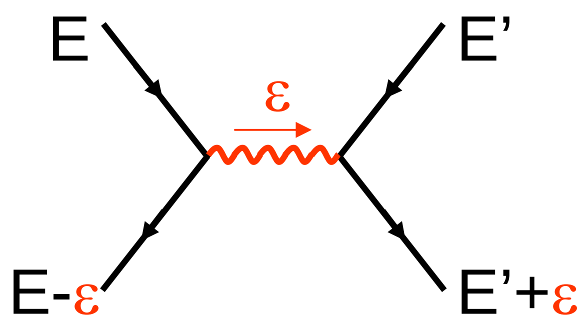
$$D\frac{\partial^2 f(E)}{\partial x^2} = I_{in}\left(x,E,\{f\}\right) - I_{out}\left(x,E,\{f\}\right)$$

e-e interactions :

$$\boxed{\frac{\kappa}{\varepsilon^{3/2}}}$$

(Altshuler, Aronov, Khmelnitskii, 1982)

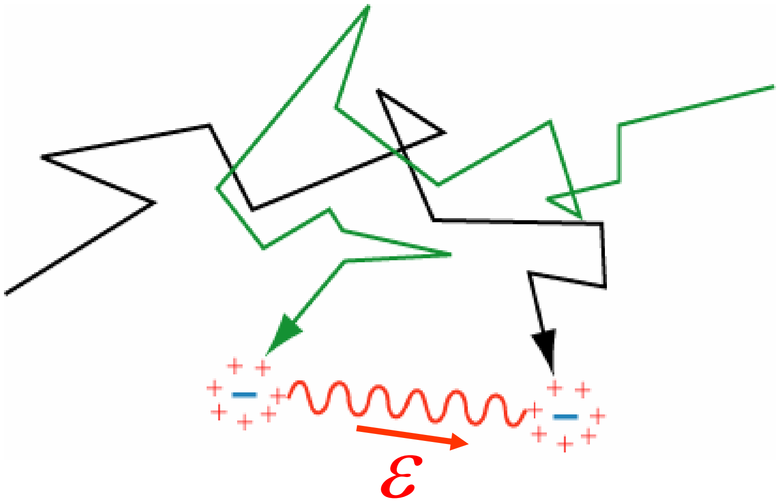
$$I_{out}\left(x,E,\{f\}\right) = \int dE'd\varepsilon\; K(\varepsilon)\; f(E)\left[1-f(E-\varepsilon)\right] f(E')\left[1-f(E'+\varepsilon)\right]$$



fn3D

Theory of screened Coulomb interaction in the diffusive regime

Altshuler, Aronov, Khmelnitskii, 1982



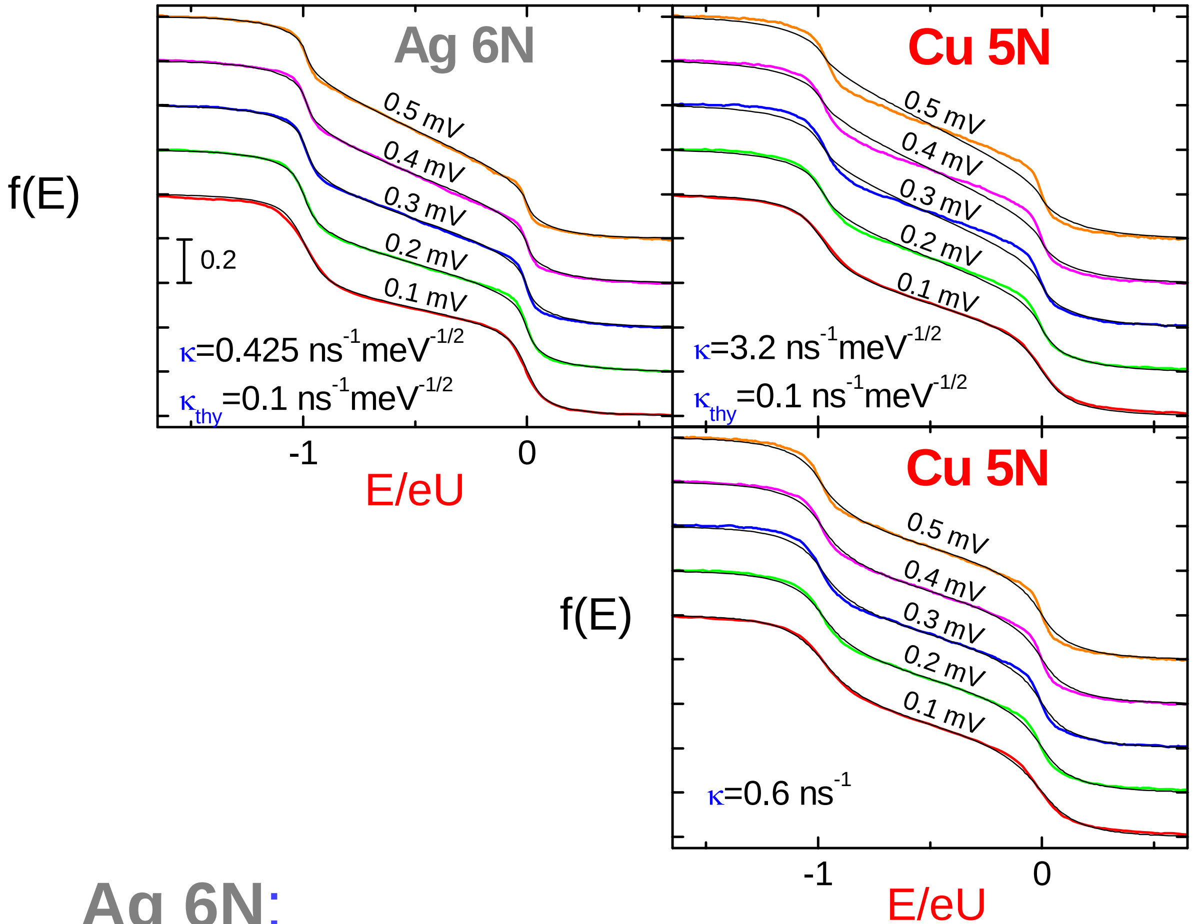
ingredients:
- polarisability ↘
- overlap ↗

Prediction for 1D wire :

$$\boxed{K(\varepsilon) = \frac{\kappa}{\varepsilon^{3/2}}}$$

$$\kappa = \left(\sqrt{2D}\ \pi\ \hbar^{3/2}\ \nu_F\ S_e\right)^{-1}$$

Experiment vs. Theory



$K(\varepsilon)=\kappa\,\varepsilon^{-3/2}$

$K(\varepsilon)=\kappa\,\varepsilon^{-2}$

Ag 6N:

experiment agrees with theory

Cu 5N, **Au 4N**, **Ag 5N**:

- energy exchange stronger than predicted
- $K(\varepsilon)=\kappa\,\varepsilon^{-2}$ fits data

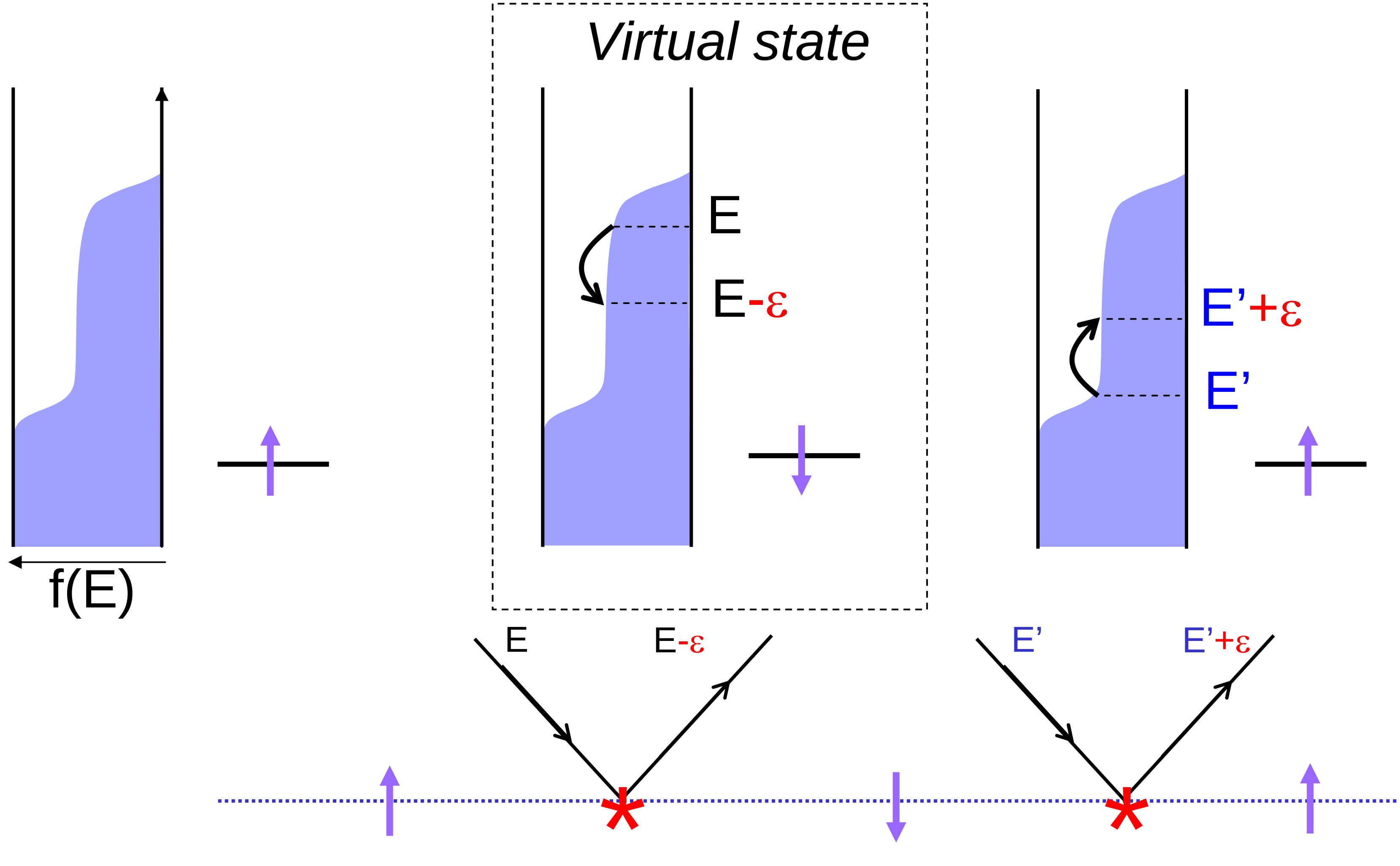
Recall from Lecture #1: τ_ϕ (T)

Comparison of the results of the two methods

	$\tau_\phi(T)$	f(E)
Ag_{6N}	$\propto T^{-2/3}$	$K(\varepsilon) = \dfrac{\kappa}{\varepsilon^{3/2}}$ (intensity?)
Ag_{5N} $Cu_{6N,5N}$ Au_{4N}	saturation	fast relaxation rates $K(\varepsilon) \cancel{\propto} \dfrac{1}{\varepsilon^{3/2}}$

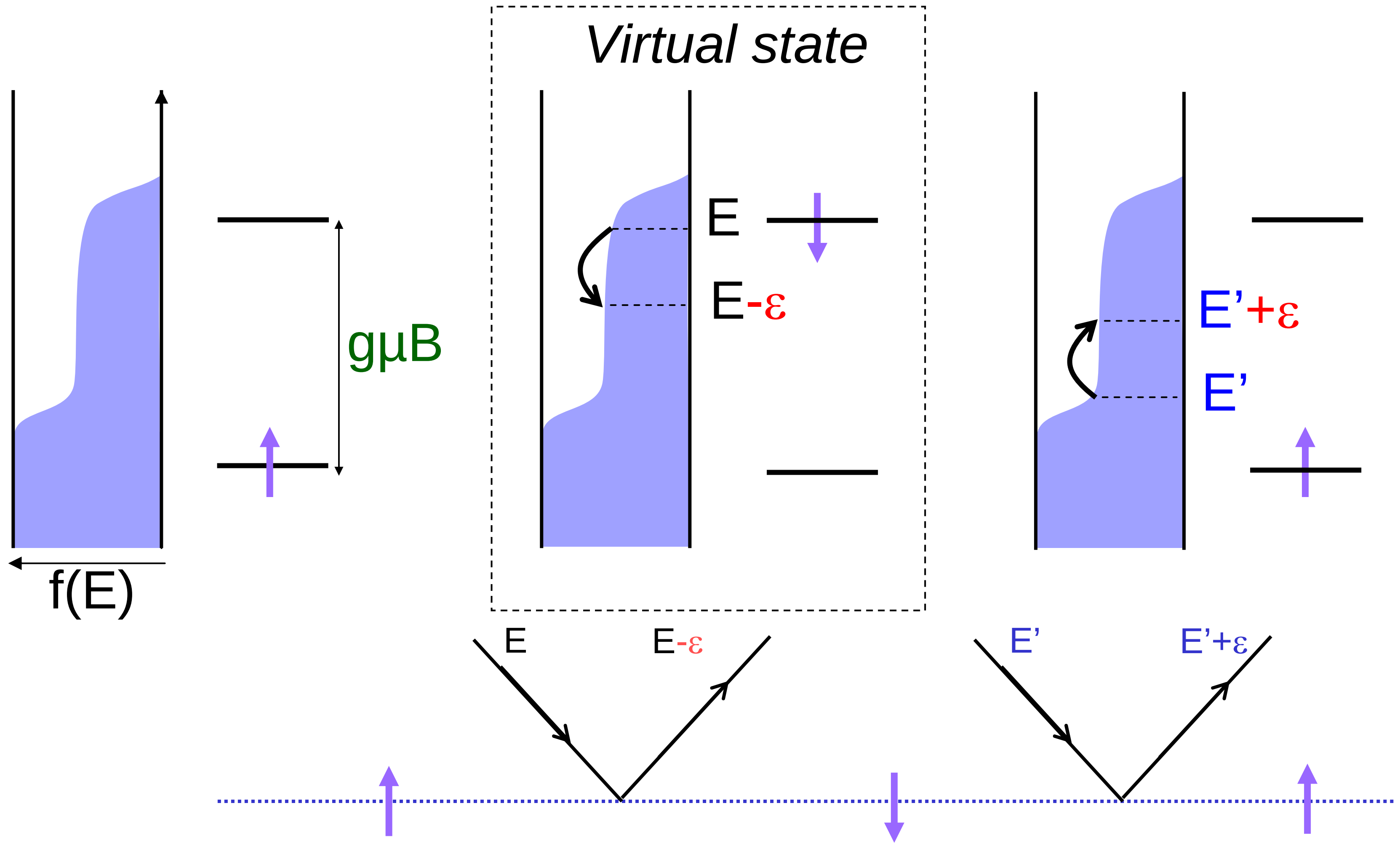
Energy exchange mediated by magnetic impurities

Kaminski and Glazman, PRL **86**, 2400 (2001)



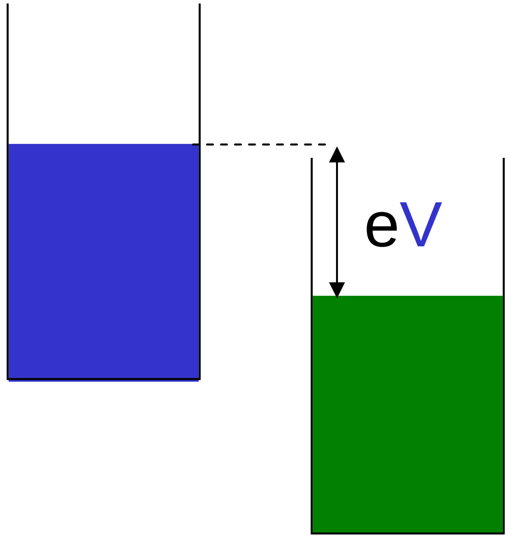
Reinforced by Kondo effect

Energy exchange mediated by magnetic impurities vanishes when $g\mu_B B >> eU$



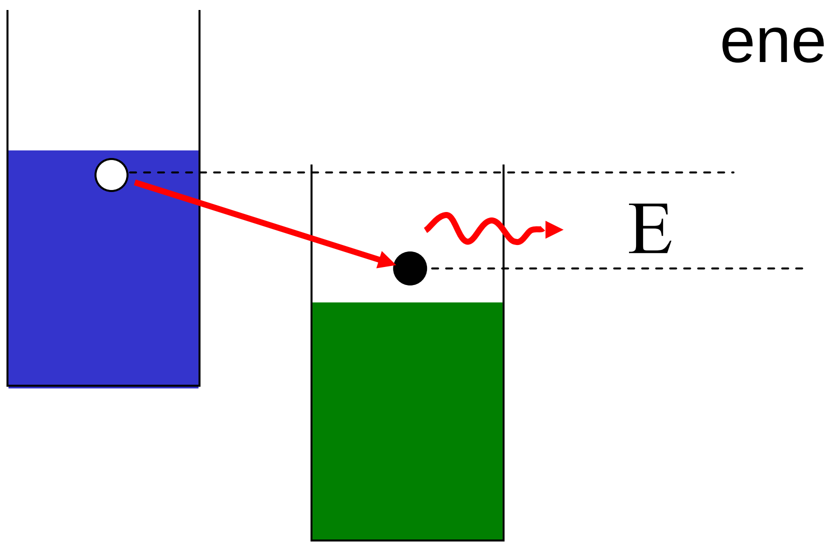
Aside 2: Inelastic tunneling

initial state:

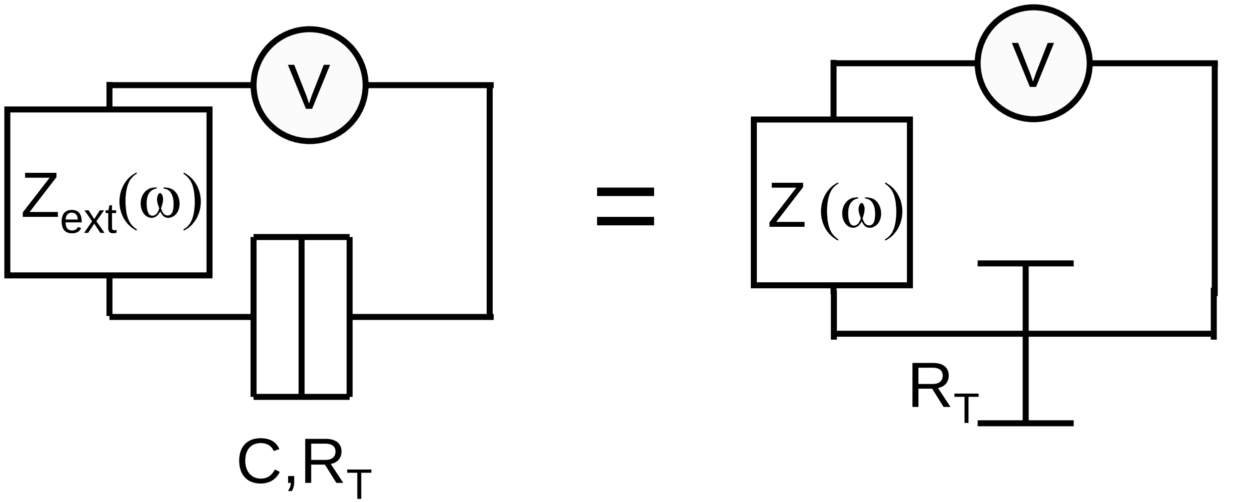


P(E)=probability to give energy E to environment

final state:



P(E) depends on environmental impedance



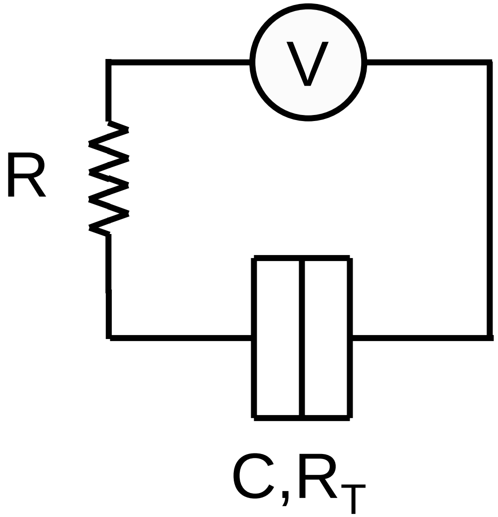
At T=0, one obtains :

$$\frac{dI}{dV} = \frac{1}{R_t}\int_0^{eV} P(E)\, dE$$

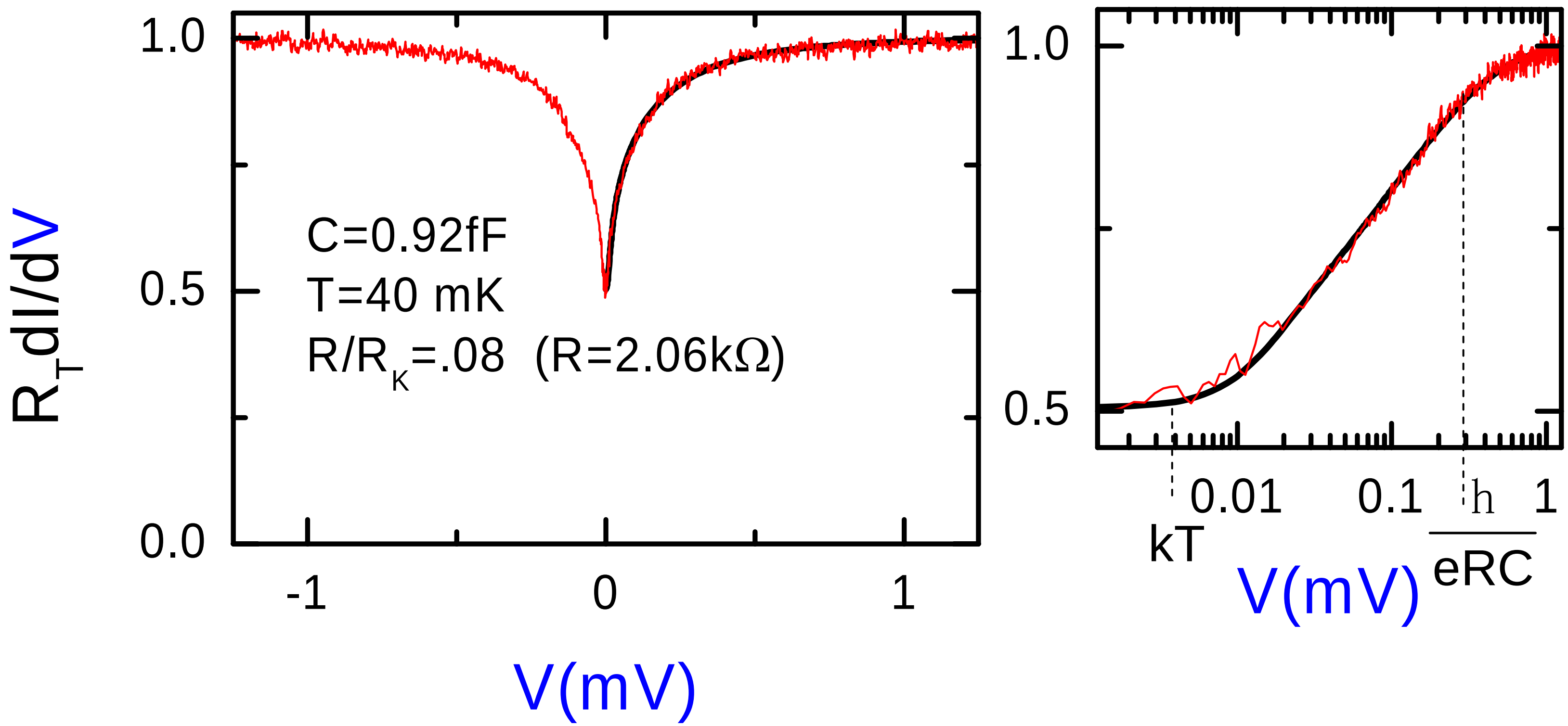
$$P(E) = \frac{1}{2\pi\hbar}\int e^{iEt/\hbar + J(t)}\, dt$$

$$J(t) = 2\int_0^{+\infty} \frac{d\omega}{\omega}\frac{\mathrm{Re}\left[Z(\omega)\right]}{R_K}\left(e^{-i\omega t} - 1\right)$$

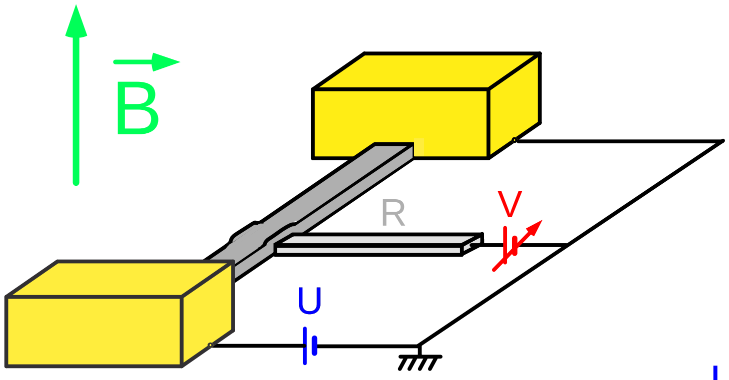
Resistive environment



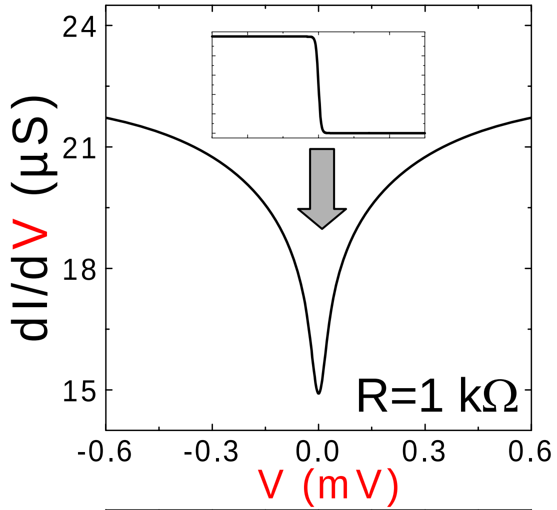
For $eV < \frac{\eta}{RC}$ $\qquad \frac{dI}{dV} \propto \left(V^{\frac{2R}{R_K}} + \text{cst.} \right)$



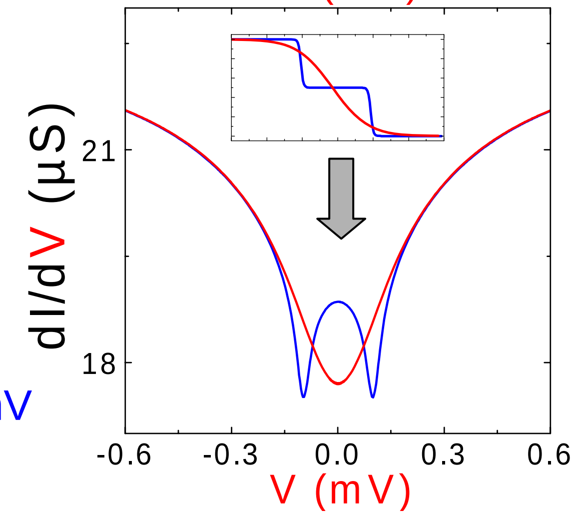
Measure f(E) at B $\neq$ 0 using Zero-Bias Anomaly
(Dynamical Coulomb Blockade)



U=0 mV

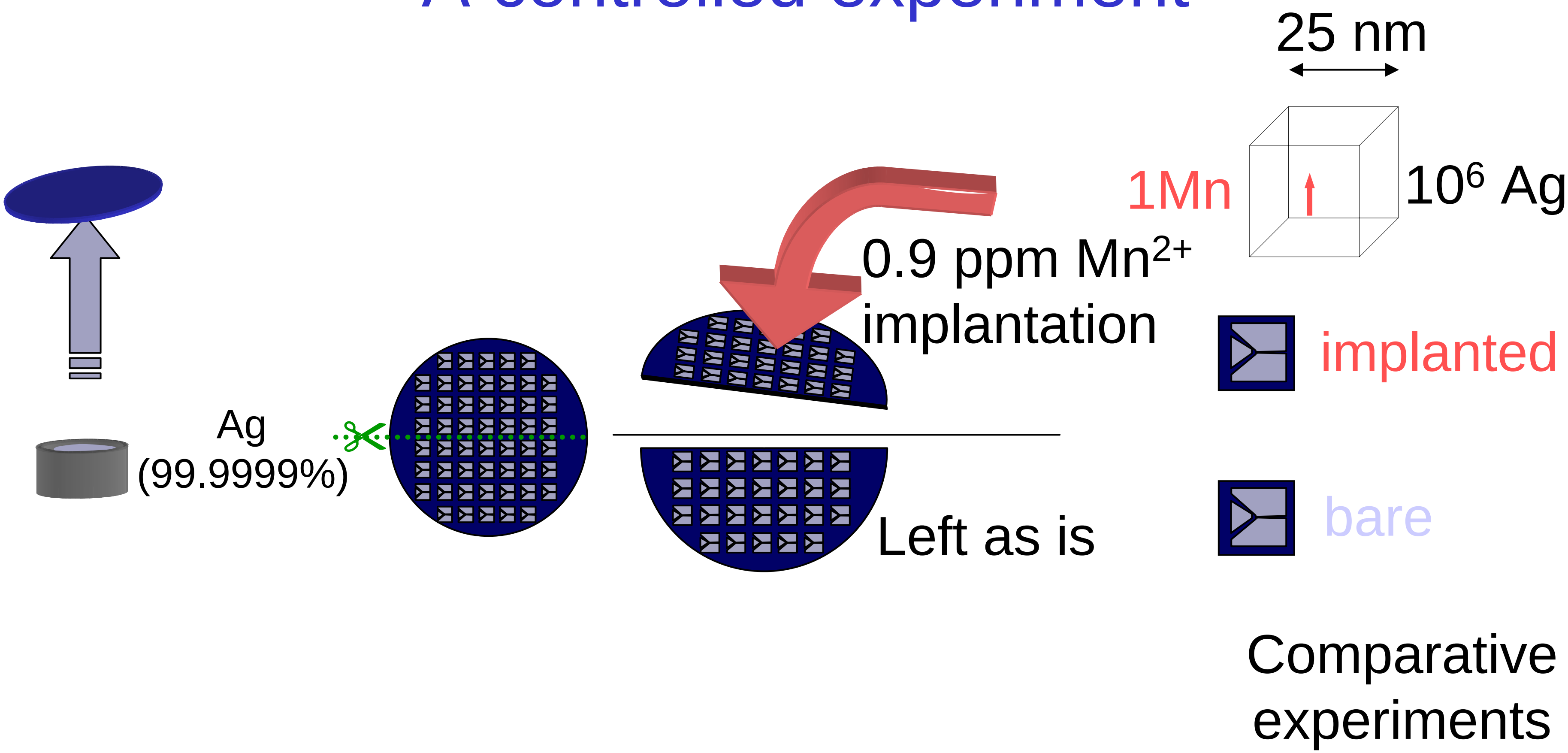


U=0.2 mV



dI/dV $\rightarrow$ f(E) $\rightarrow$ electron-electron interactions

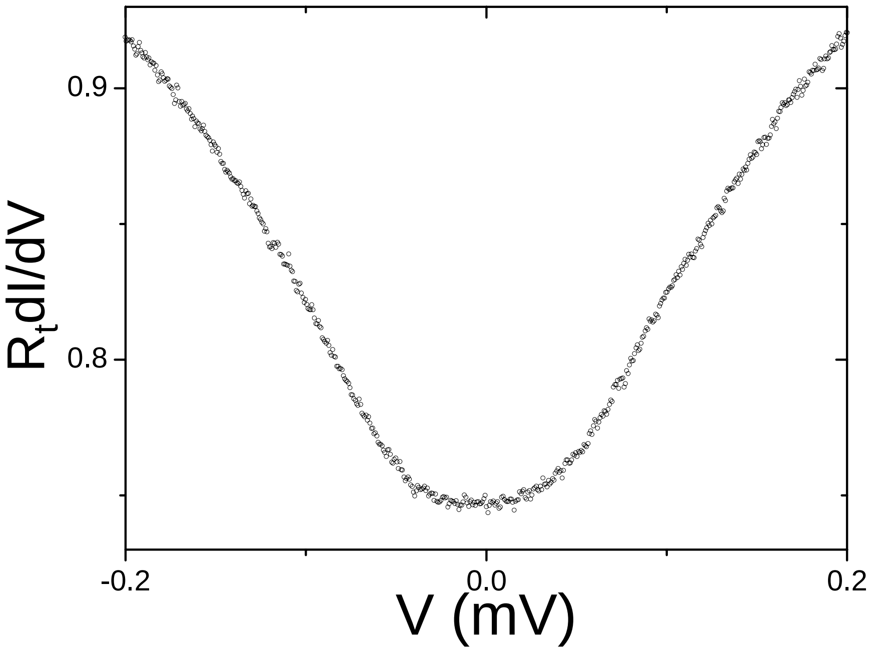
A controlled experiment

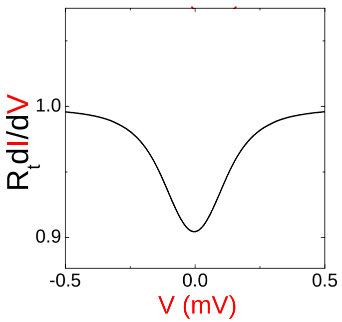


Effect of 1 ppm Mn on interactions ?

Experimental data at weak B

implanted

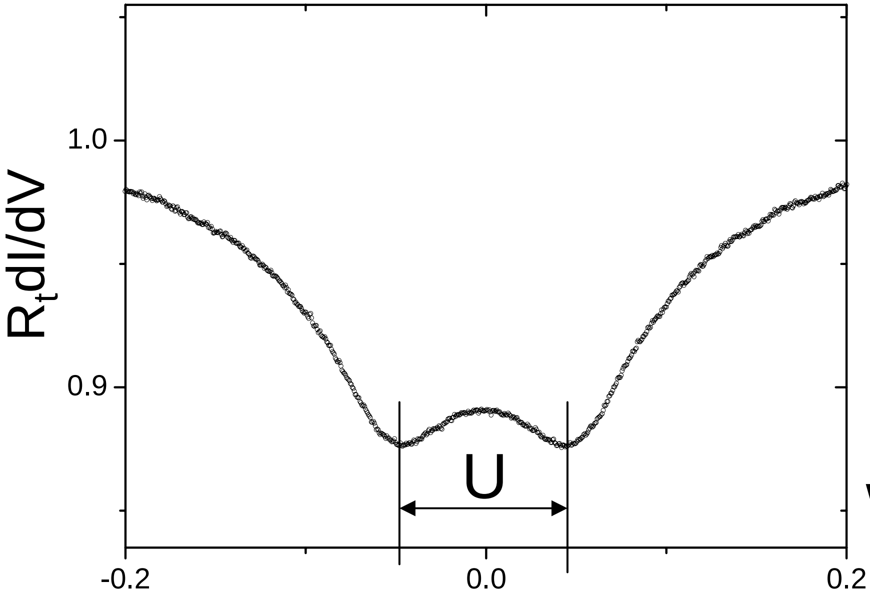


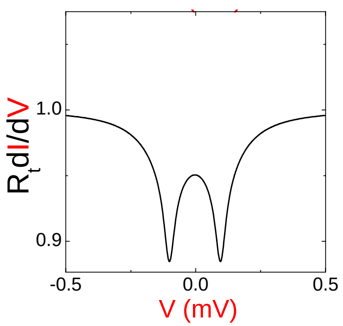


strong interaction

U = 0.1 mV
B = 0.3 T

bare





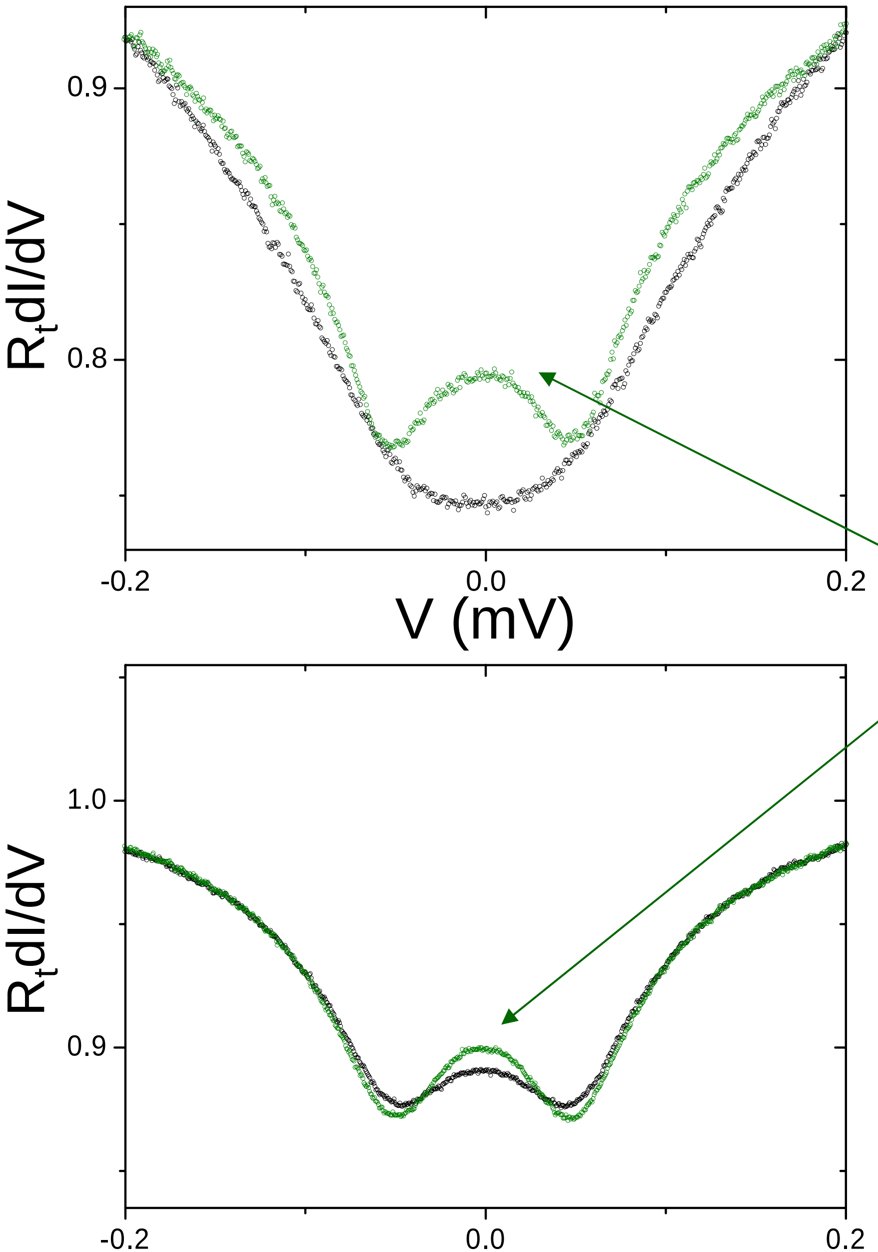
weak interaction

Experimental data at weak and at strong B

implanted

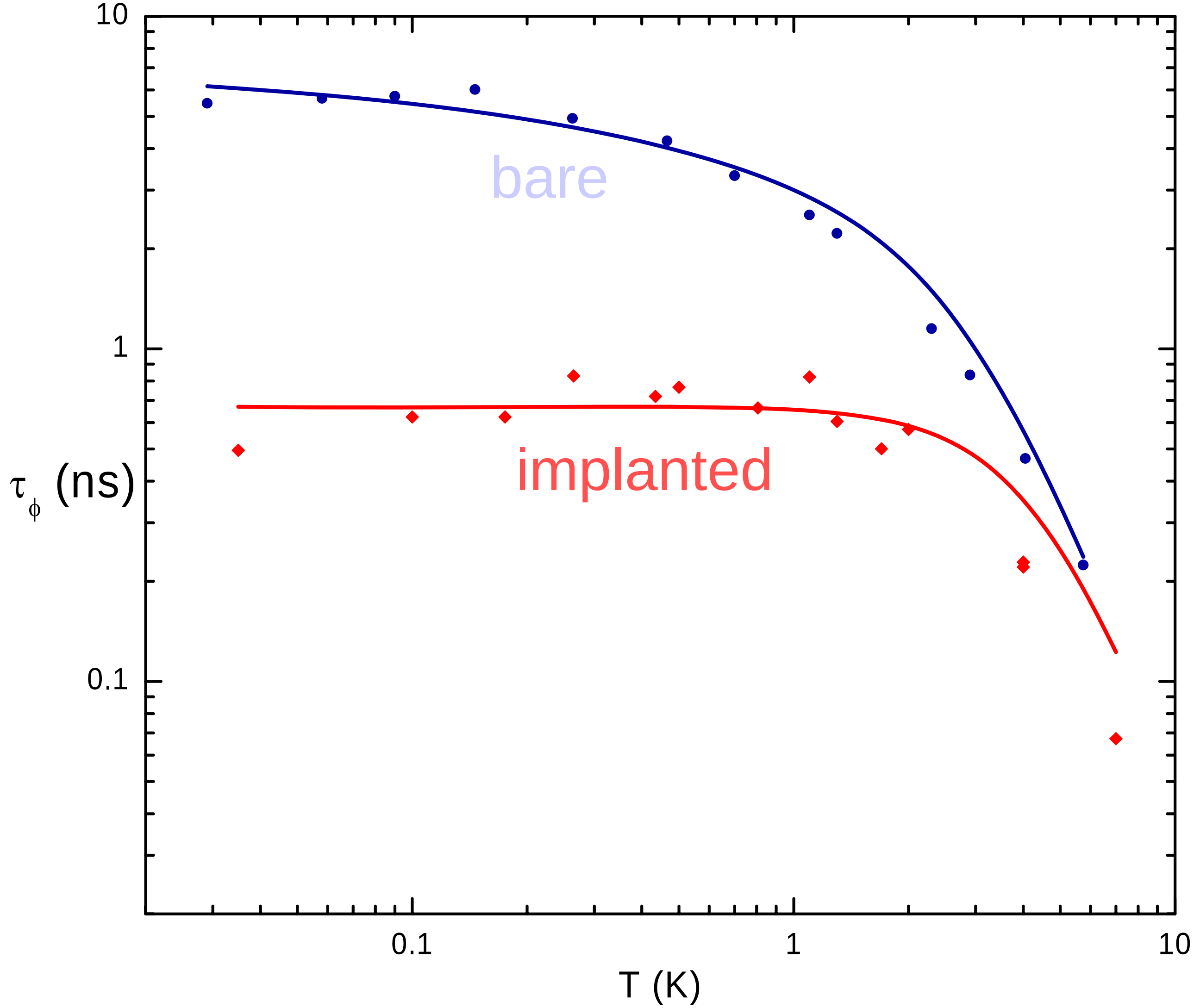
U = 0.1 mV
B = 0.3 T
B = 2.1 T

bare



Very weak interaction

Coherence time measurements on the same 2 samples

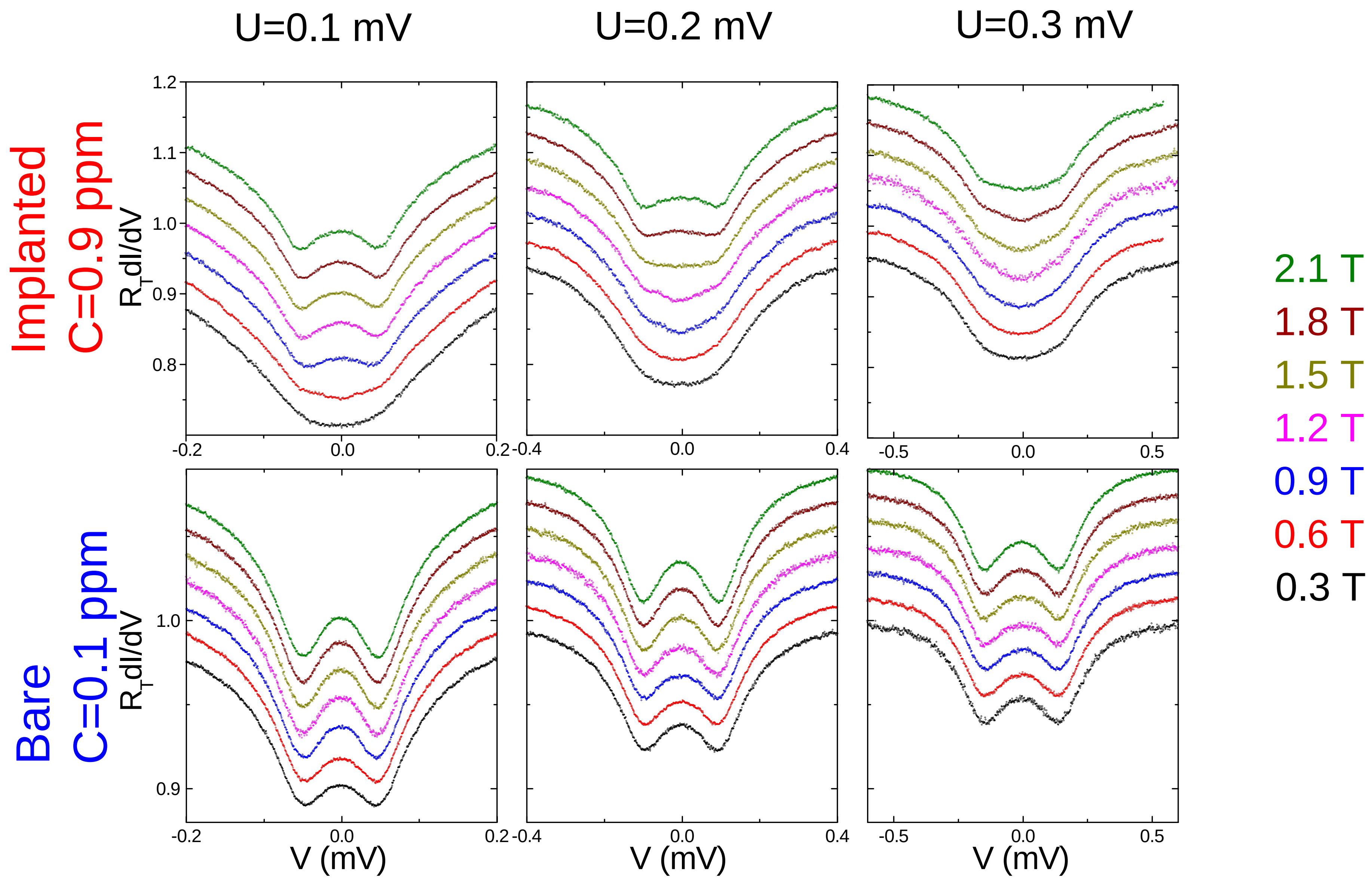


Fits:

C_{bare} =0.1 ppm

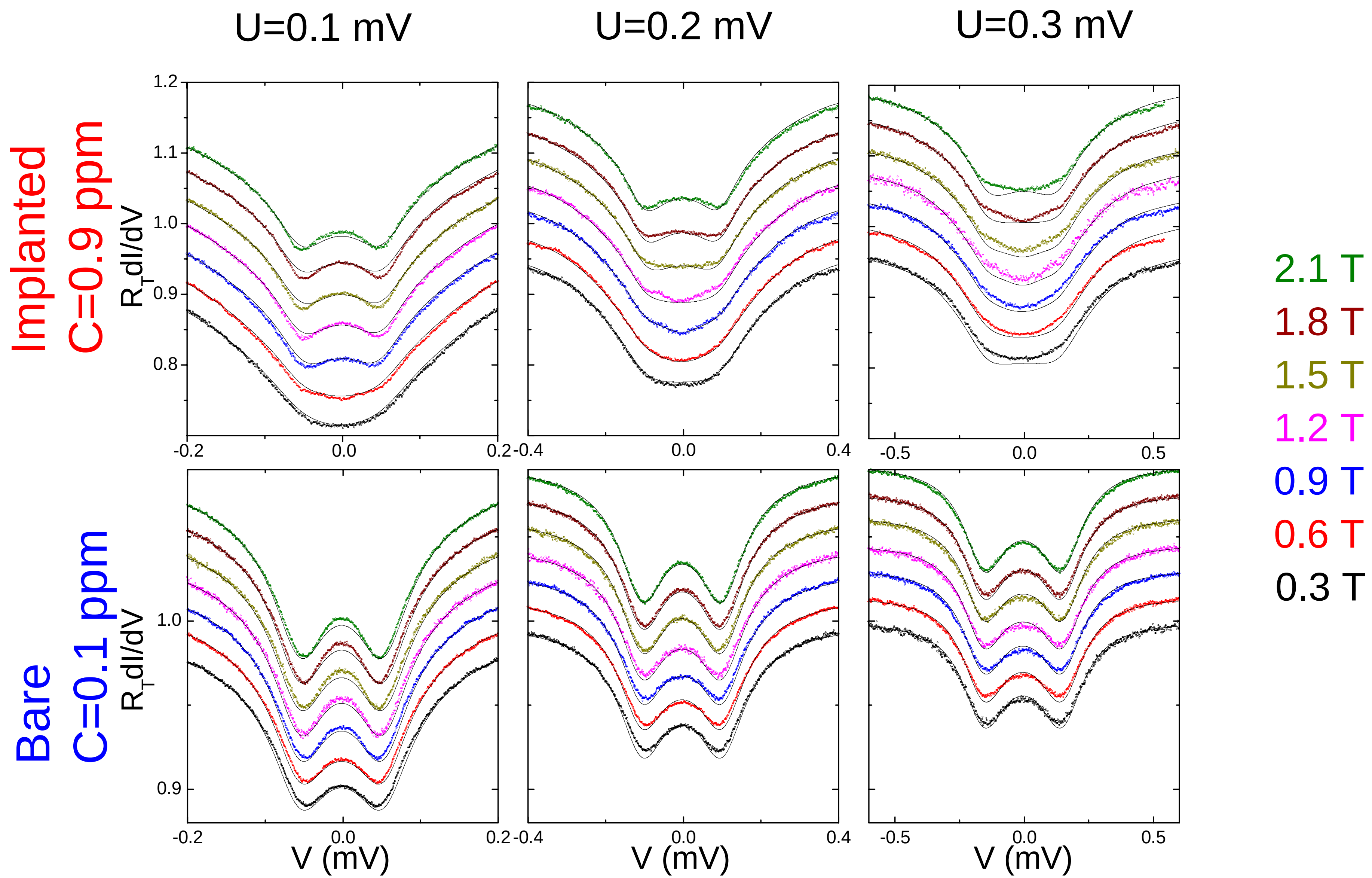
$C_{implanted}$=0.9 ppm

Full U,B dependence



Comparison with theory $\left(S = \frac{1}{2}\right)$

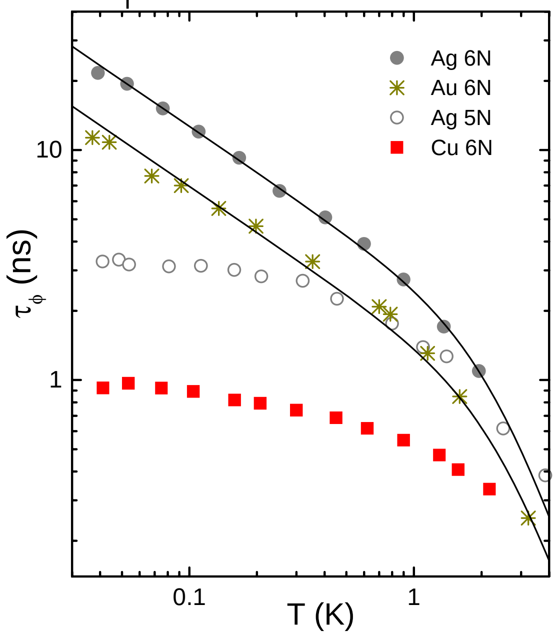
Goeppert, Galperin, Altshuler and Grabert, PRB 64, 033301 (2001)



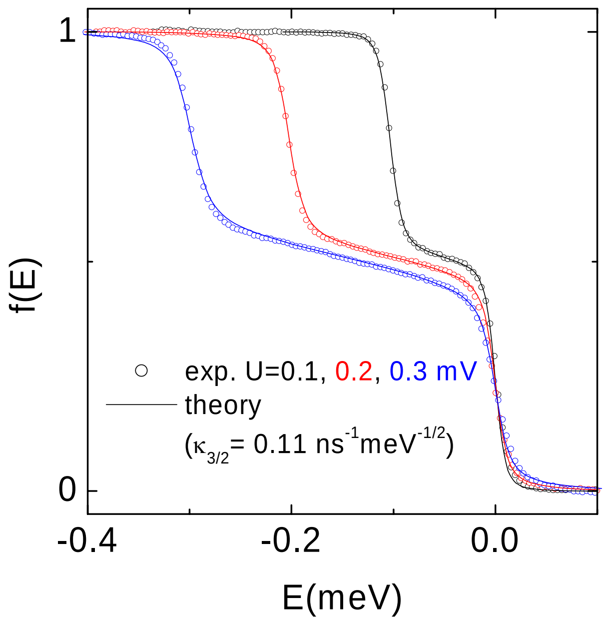
Conclusions – Lectures 1 & 2

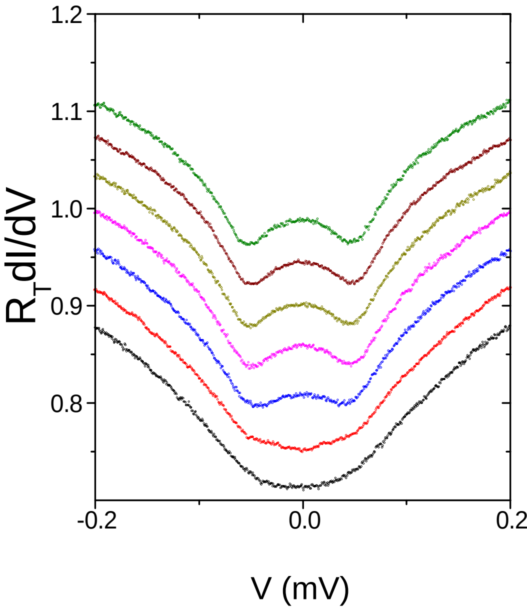
Two methods to investigate interactions in wires

$\tau_\phi(T)$



f(E)

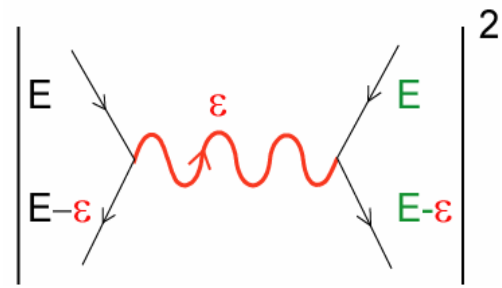




Moral of the story: even at concentrations as low as 1 ppm, magnetic impurities have a large influence on dephasing and energy exchange in metals at low-temperature.

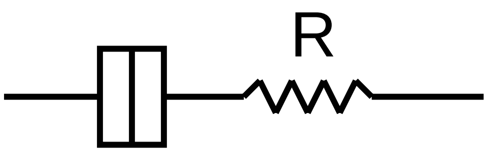
Summary: consequences of electron-electron interactions in quasi-1D diffusive wires (Altshuler & Aronov)

- loss of phase coherence: $\tau_\phi \sim T^{-2/3}$ (AA+Khmelnitskii)
  (subject of Lecture #1)
- energy exchange between quasiparticles (Lecture #2):



$\propto \varepsilon^{-3/2}$

- correction to resistance: $\delta R(T) \sim T^{-1/2}$ (Lecture #1)
- correction to tunneling DOS, or dynamic Coulomb blockade: $dI/dV \sim V^{2R/R_Q}$ (Lecture #2)



References

- Dephasing
  - Gougam, Pierre, Pothier, Esteve, Birge, J. Low Temp. Phys. **118**, 447 (2000).
  - Pierre & Birge, Phys. Rev. Lett. **89**, 206804 (2002).
  - Pierre, Gougam, Anthore, Pothier, Esteve, Birge, Phys. Rev. B 68, 085413 (2003).

- Energy Exchange
  - Pothier, Gueron, Birge, Esteve, Devoret, Phys. Rev. Lett. **79**, 3490 (1997).
  - Pothier, Gueron, Birge, Esteve, Devoret, Z. Physik. B **104**, 178 (1997).
  - Pierre, Pothier, Esteve, Devoret, J. Low Temp. Phys. **118**, 437 (2000).
  - Anthore, Pierre, Pothier, Esteve, Devoret, cond-mat/0109297 (2001).
  - Anthore, Pierre, Pothier, Esteve, Phys. Rev. Lett. **90**, 076806 (2003).

- Both
  - Pierre, Pothier, Esteve, Devoret, Gougam, Birge, cond-mat/0012038 (2000).
  - Pierre, Ann. Phys. Fr. **26**, N° 4 (2001).
  - Huard, Anthore, Birge, Pothier, Esteve, PRL **95**, 036802 (2005).