Decoherence in the Josephson Charge Qubits

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Outline

- Introduction
- Cooper-pair box and the Josephson charge qubit
- Noise and decohernce. The qubit as a spectrometer of the enviromental noise
- Measurements of the qubit relaxaton
- Low frequency 1/*f*-noise and high frequency *f*-quantum noise

Classical and Quantum Computers

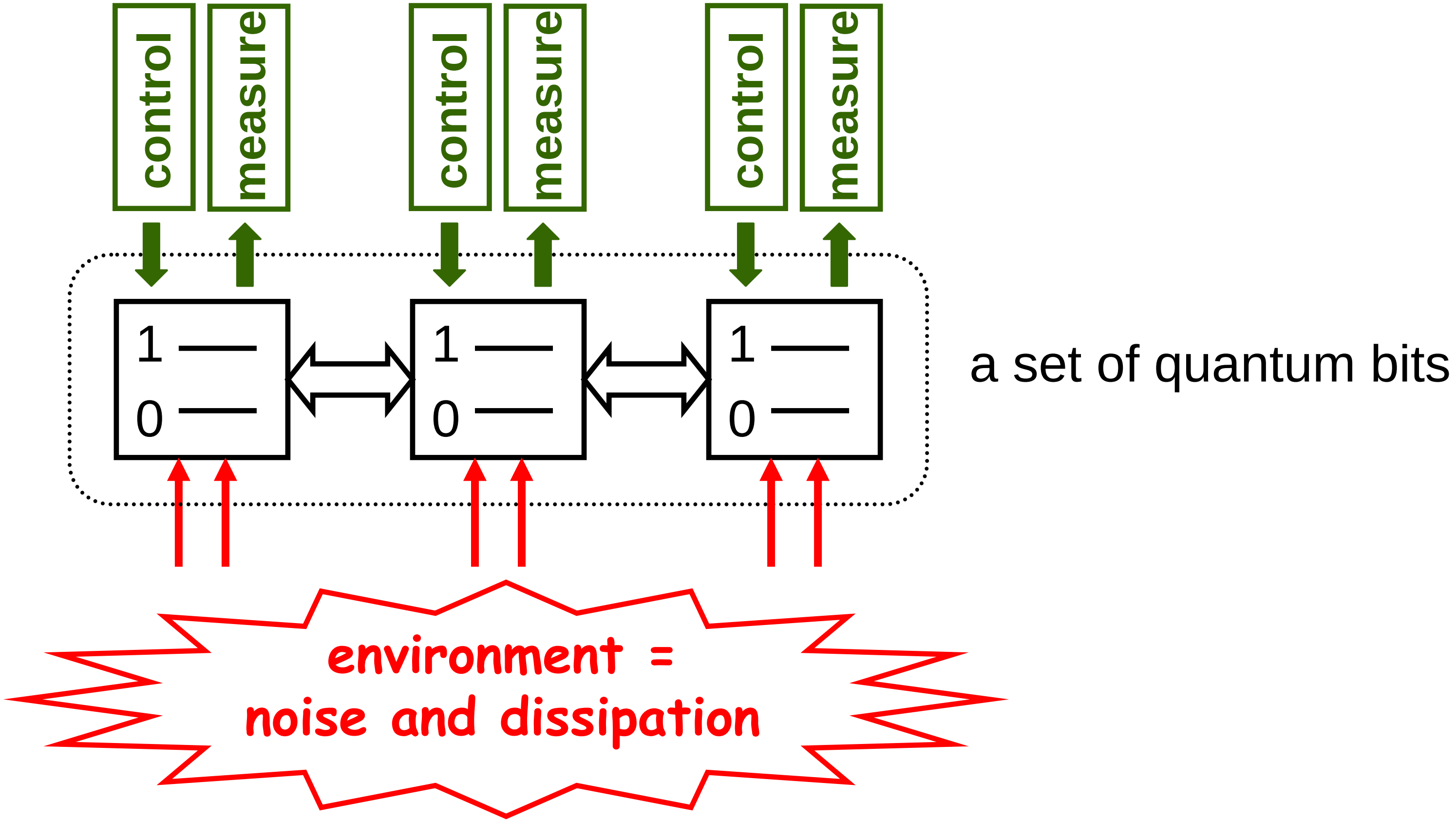
Classical bit: 0 or 1 110100…01

Quantum bit: $\alpha|0\rangle + \beta|1\rangle$

Quantum parallelism:

$c_0\ |110100\ldots01\rangle + c_1\ |010101\ldots10\rangle + \ldots$

Multi-qubit Integrated Circuit



Experimental challenge:
couple qubits to each other, control, & measure
not noise and dissipation

Requirements for quantum computing

DiVincenzo

- Scalable qubits
- Initialization
  - preparation of $|0\rangle$ state
- Universal quantum gate
  - one-qubit gate and two-qubit gate
- Readout : projective measurement
  - $\alpha|0\rangle + \beta|1\rangle$ $\Rightarrow$ 0 with probability $|\alpha|^2$
    1 with probability $|\beta|^2$
- Long decoherence time
  - much longer than gate operation time

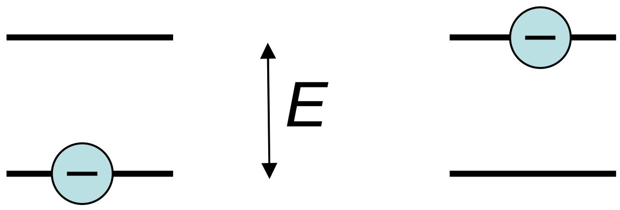
    $\sim 10^4 - 10^6$ times

    : threshold for applicability of quantum error correction

Qubit

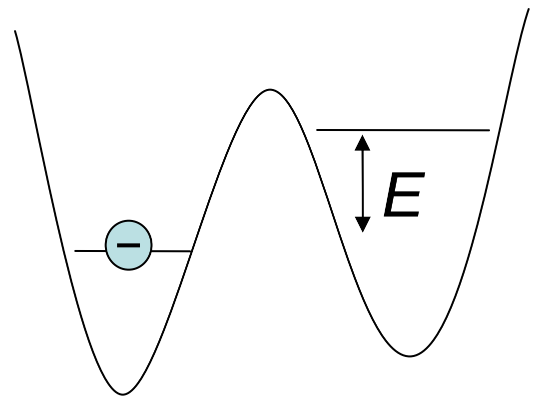
Two-level system (TLS)

Ground state | Excited state

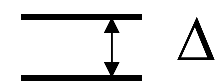


$|0\rangle$ $\quad$ $|1\rangle$

$$H|0\rangle = -\frac{E}{2}|0\rangle \qquad H|1\rangle = \frac{E}{2}|1\rangle$$



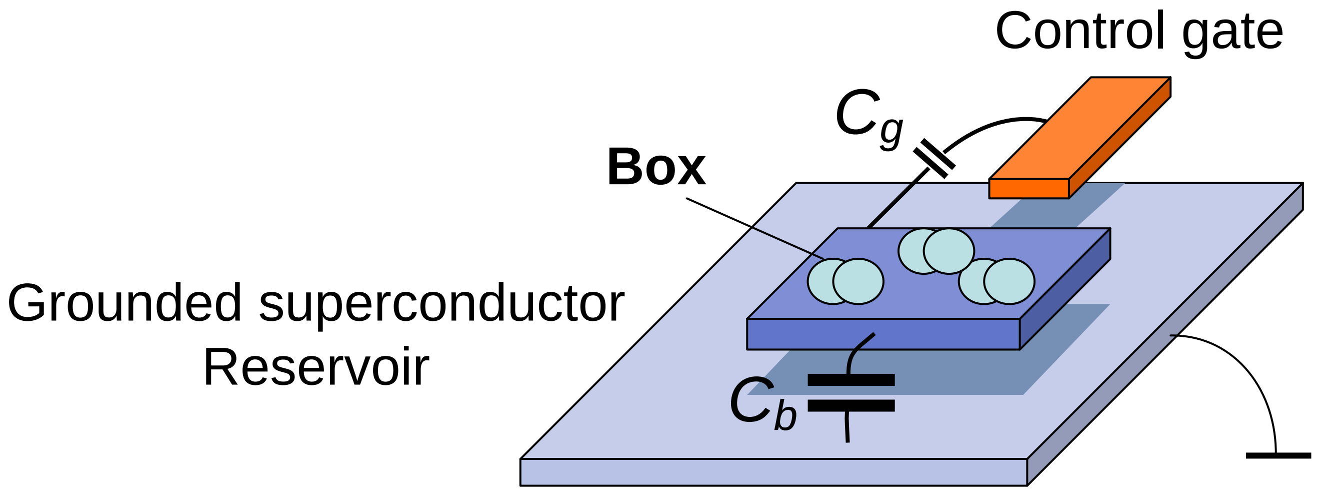
$$|+\rangle = |0\rangle + |1\rangle$$



$$|-\rangle = -|0\rangle + |1\rangle$$

$$H = -\frac{E}{2}\left(|0\rangle\langle 0| - |1\rangle\langle 1|\right) - \frac{\Delta}{2}\left(|0\rangle\langle 1| + |0\rangle\langle 1|\right)$$

Cooper pair box



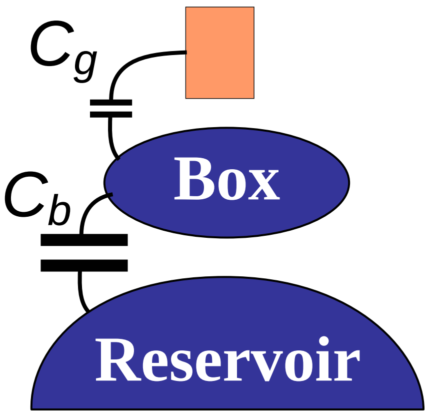
$C_b >> C_g$

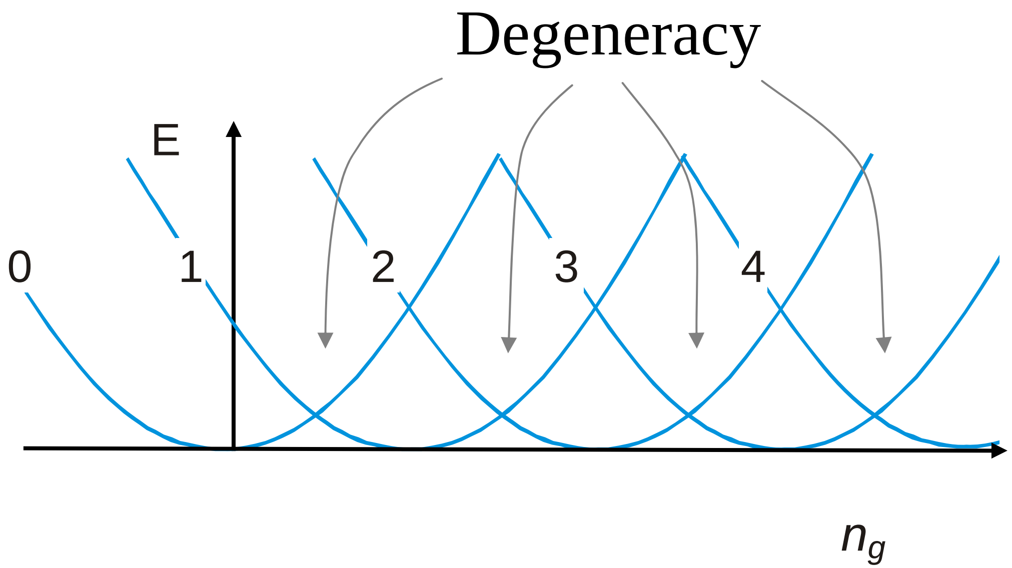
$$U = \frac{q^2}{2C_b} \qquad q = 2ne + 2n_g e \qquad n_g = \frac{V_g C_g}{2e}$$

$$U = E_c (n - n_g)^2$$

Charging energy: $E_c = \frac{(2e)^2}{2C_b}$

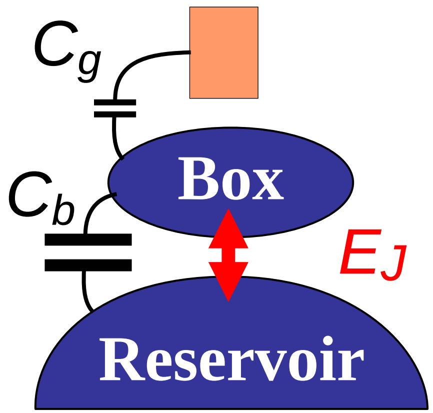
The Josephson Charge Qubit

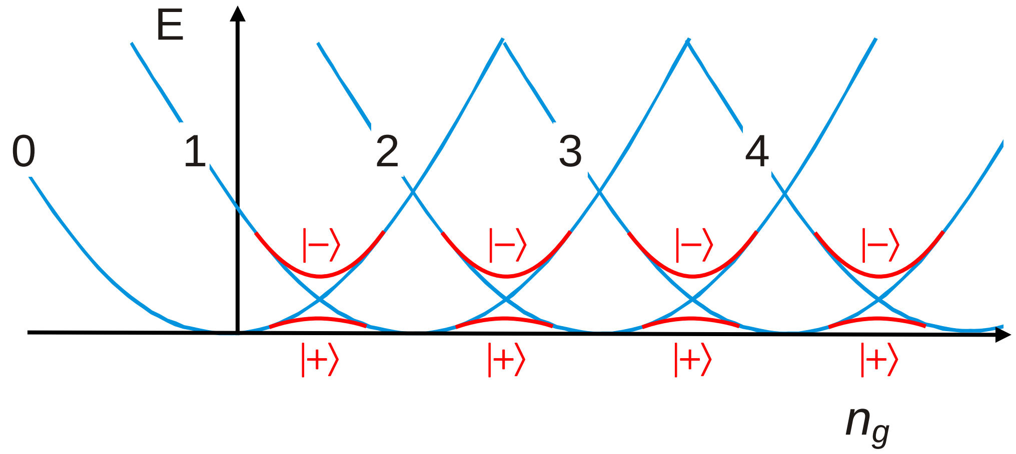




Charging energy (for Cooper pair): $E_c >> kT$

The Josephson Charge Qubit





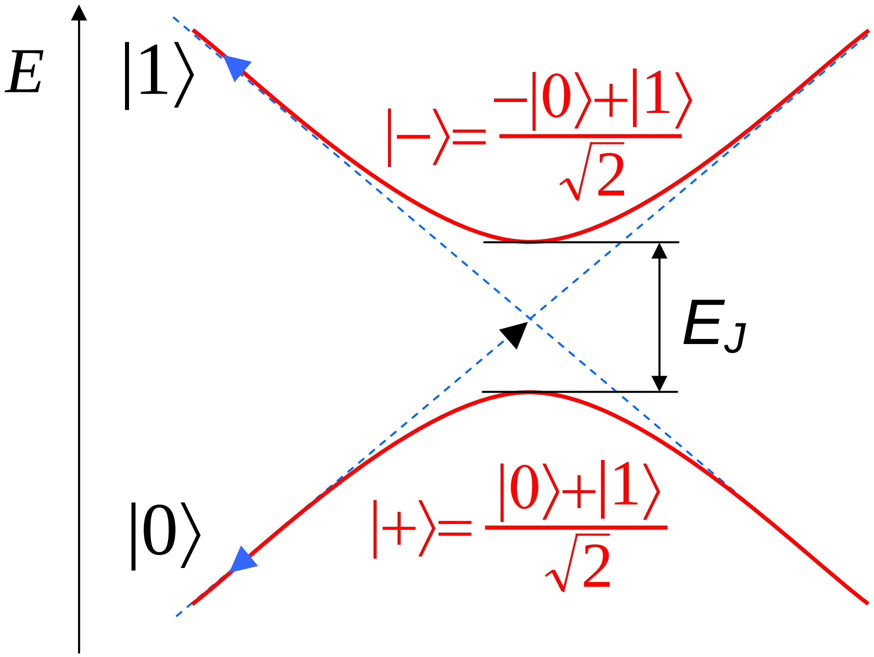
Charging energy (for Cooper pair): $E_c >> kT$

Josephson energy: E_J

Charge qubit: $E_c >> E_J$

The Hamiltonian

$$H = -\frac{U_{10}}{2}\left(|0\rangle\langle 0| - |1\rangle\langle 1|\right) - \frac{E_J}{2}\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right)$$



$$U_{10} = U(1) - U(0) = E_c n_g$$

$$\Delta E = \sqrt{U_{10}^2 + E_J{}^2}$$

Matrix Form of the Hamiltonian

$$\hat{H} = \frac{1}{2}\begin{pmatrix} -U_{10} & -E_J \\ -E_J & U_{10} \end{pmatrix}$$

Charge basis:

$$|0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$U_{10} = \frac{2e^2 n_g}{C}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

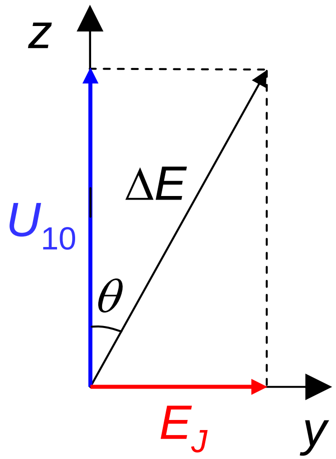
$$H = -\frac{\Delta E}{2}\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix} = -\frac{\Delta E}{2}\left(\sigma_z \cos\theta + \sigma_x \sin\theta\right)$$

$$\Delta E = \sqrt{U_{10}^2 + E_J{}^2} \qquad \tan\theta = \frac{E_J}{U_{10}}$$

Eigenstates, Eigenenergies

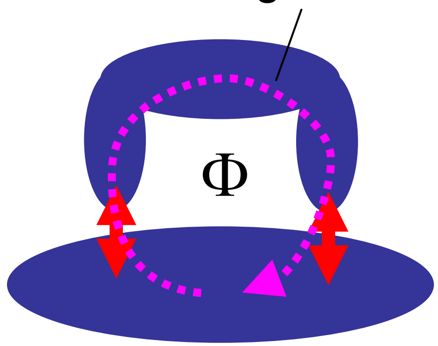
$$|+\rangle = \begin{pmatrix} \sin\theta/2 \\ \cos\theta/2 \end{pmatrix}, -\frac{\Delta E}{2} \qquad |-\rangle = \begin{pmatrix} \cos\theta/2 \\ -\sin\theta/2 \end{pmatrix}, \frac{\Delta E}{2}$$

Coupling between two charge states



$$\tan\theta = \frac{E_J}{U_{10}}$$

$$\Delta E = \sqrt{E_J^2 + U_{10}^2}$$



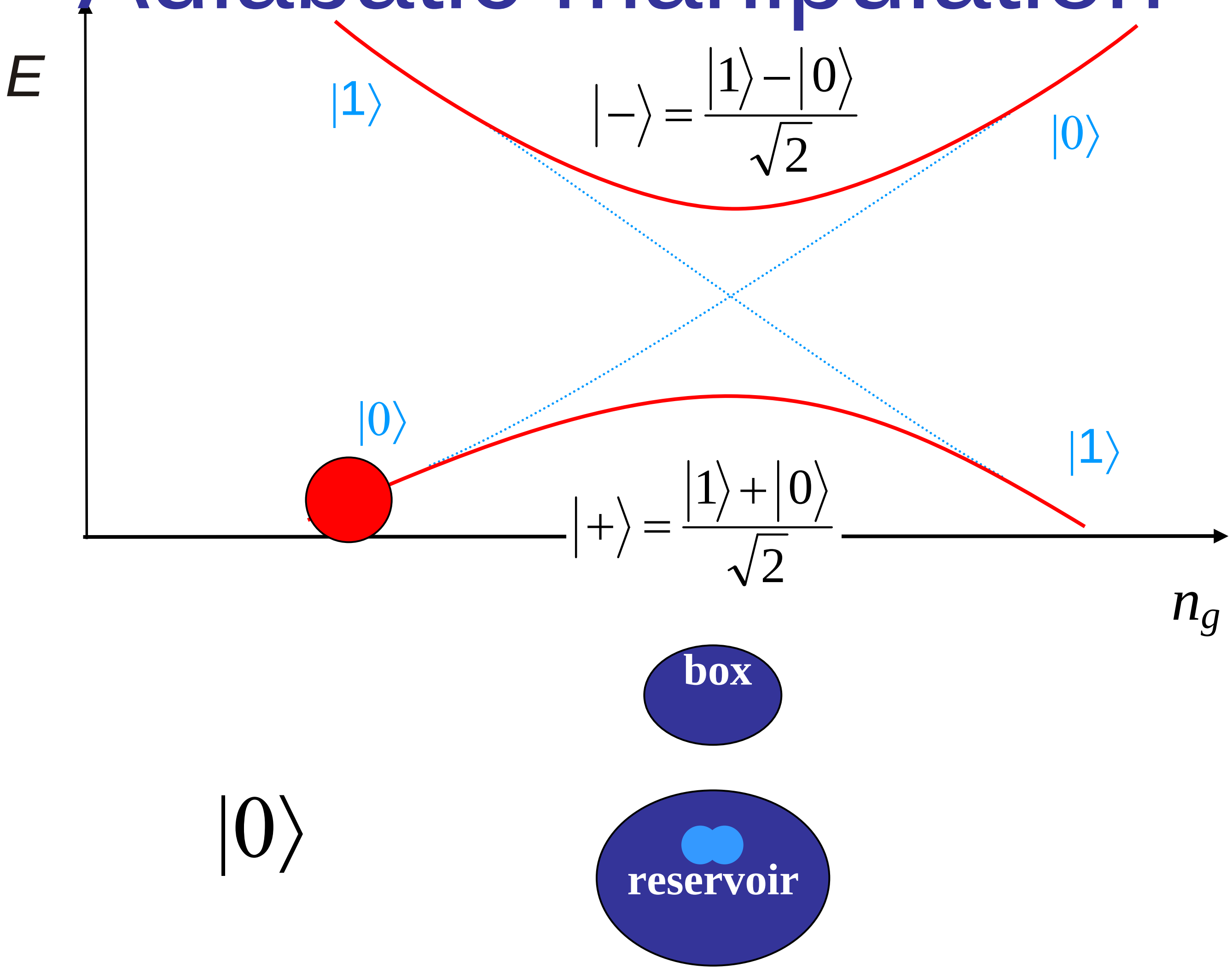
$$E_J = E_{J\max}\cos\left(\frac{\pi\,\Phi}{\Phi_0}\right)$$

Qubit energies are controlled by external fields

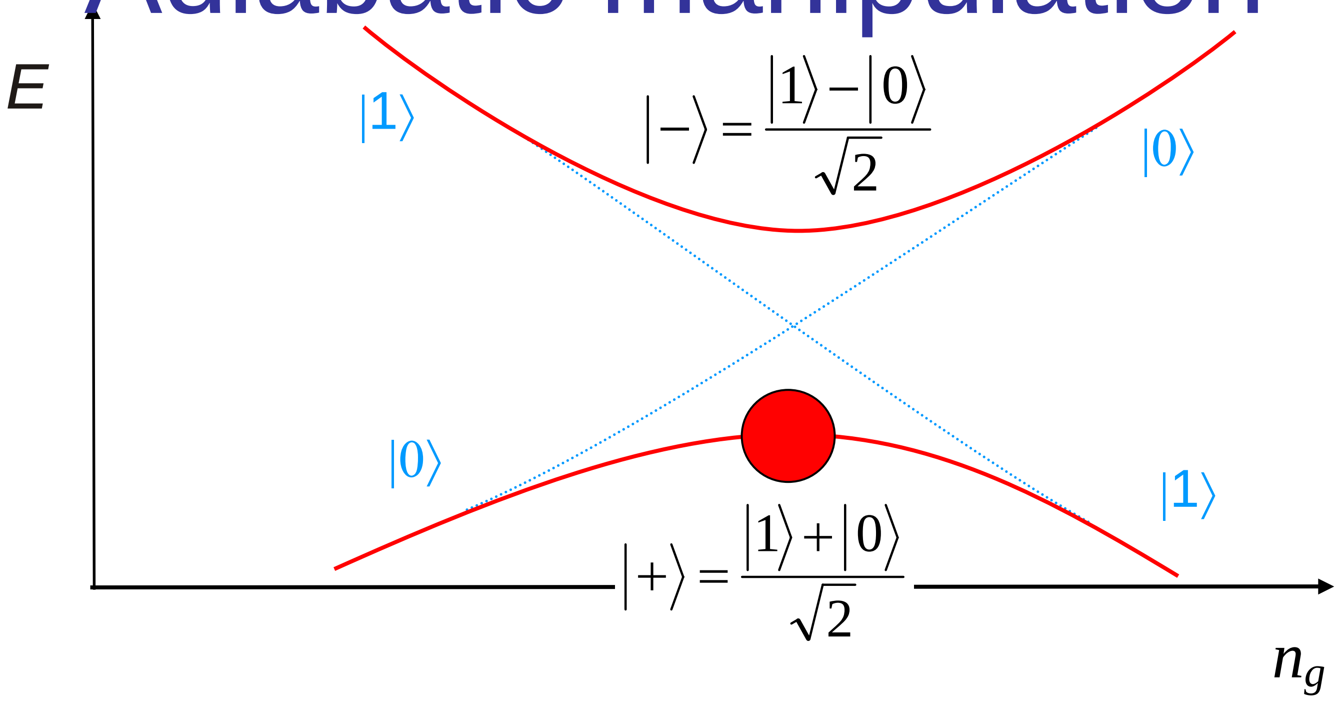
Charge basis: $$H = -\frac{\Delta E}{2}\left(\sigma_z\cos\theta + \sigma_x\sin\theta\right)$$

Eigenbasis: $$H = -\frac{\Delta E}{2}\sigma_z$$

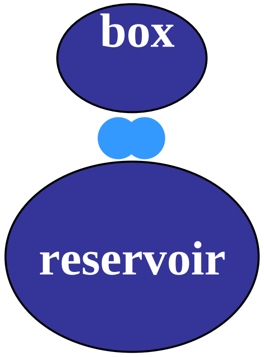
Adiabatic manipulation



Adiabatic manipulation



$$|+\rangle = \frac{|1\rangle + |0\rangle}{\sqrt{2}}$$



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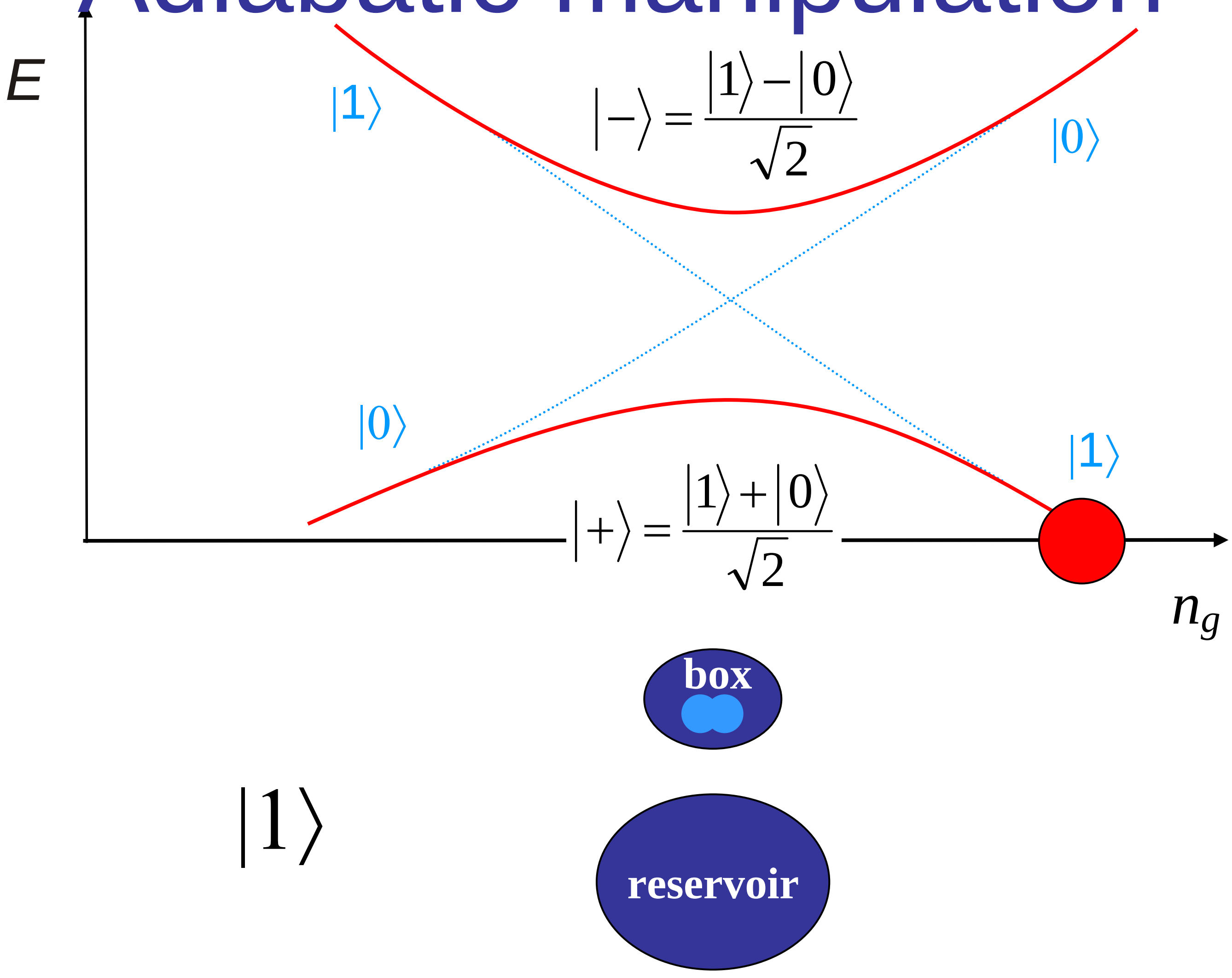


Time evolution

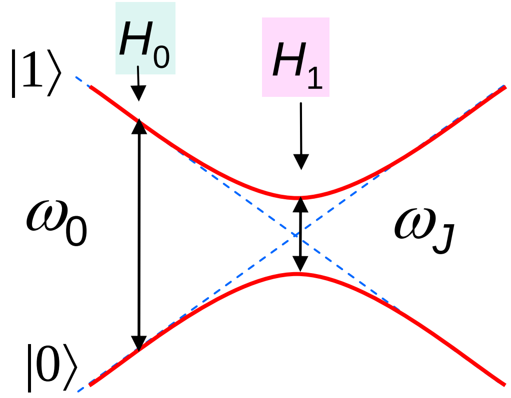
$$i\eta \frac{\partial \psi}{\partial t} = H\psi \qquad \psi(t) = \exp\left[-i\frac{H}{\eta}t\right]\psi(0)$$

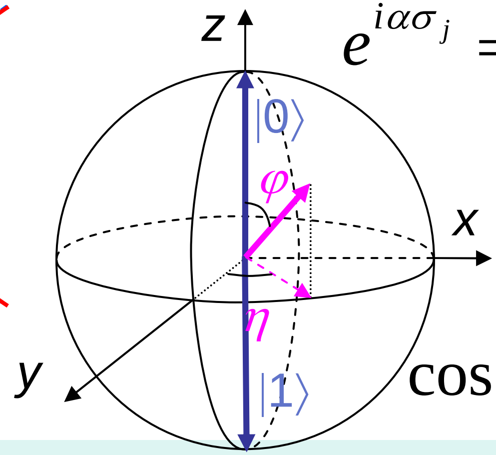
Eigenstates ($|0\rangle$, $|1\rangle$): $\qquad \exp\left[-i\frac{E_n}{\eta}t\right]|n\rangle$

Arbitrary initial state: $\qquad \psi(0) = \alpha|0\rangle + \beta|1\rangle$

$$\psi(t) = \alpha|0\rangle \exp\left[-i\frac{E_0}{\eta}t\right] + \beta|1\rangle \exp\left[-i\frac{E_1}{\eta}t\right]$$

Evolution in the charge qubit





$$e^{i\alpha\sigma_j} = \sum_{k=0}^{\infty} \frac{(i\alpha)^{2k+1}}{(2k+1)!}\sigma_j^{2k+1} + \frac{(i\alpha)^{2k}}{(2k)!}\sigma_j^{2k}$$

$$i\sin\alpha\,\sigma_j + \cos\alpha\, I$$

$$\cos\frac{\varphi}{2}|0\rangle + e^{i\eta}\sin\frac{\varphi}{2}|1\rangle$$

Initial point:

$$H_0 \approx -\frac{\eta\omega_0}{2}\sigma_z$$

$$\psi(t) = e^{\frac{i}{2}\eta\sigma_z}\psi(0) = \begin{pmatrix} e^{\frac{i}{2}\eta} & 0 \\ 0 & e^{-\frac{i}{2}\eta} \end{pmatrix}\psi(0)$$

$$\eta = \omega_0 t$$

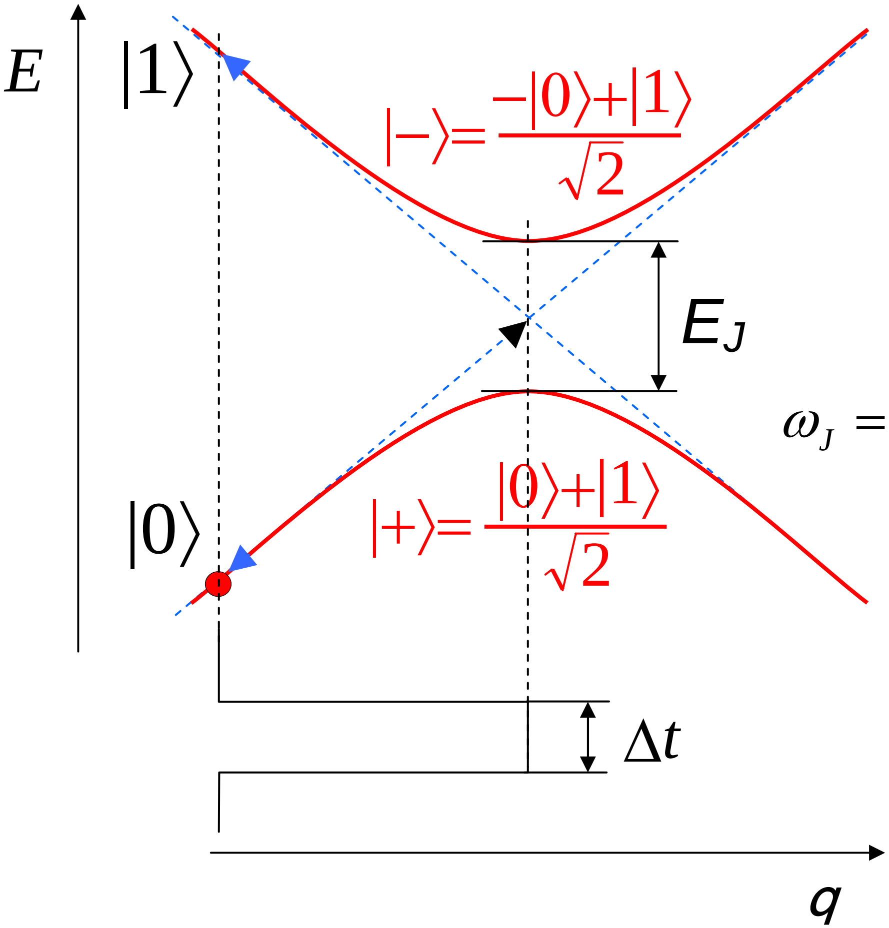
Degeneracy point:

$$H_1 = -\frac{\eta\omega_J}{2}\sigma_x$$

$$\psi(t) = e^{\frac{i}{2}\varphi\sigma_x}\psi(0) = \begin{pmatrix} \cos\varphi/2 & i\sin\varphi/2 \\ i\sin\varphi/2 & \cos\varphi/2 \end{pmatrix}\psi(0)$$

$$\varphi = \omega_J t$$

Coherent Oscillations



$$\omega_J = \frac{E_J}{\eta}$$

t = 0: $|0\rangle = \frac{|+\rangle - |-\rangle}{\sqrt{2}}$

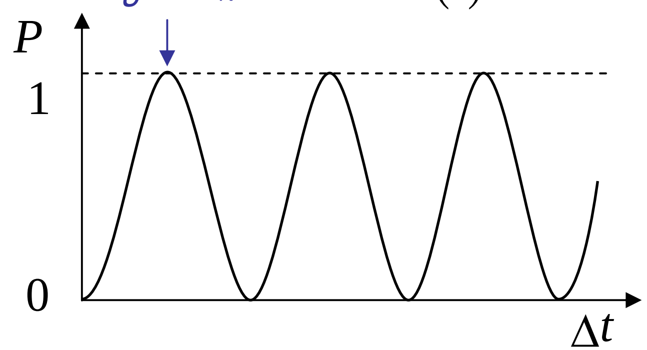
$$e^{\frac{i}{2}\omega_J t \sigma_x}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\cos\omega_J t/2\\ i\sin\omega_J t/2\end{pmatrix}$$

$$|0\rangle\cos\omega_J t/2 + i|1\rangle\sin\omega_J t/2$$

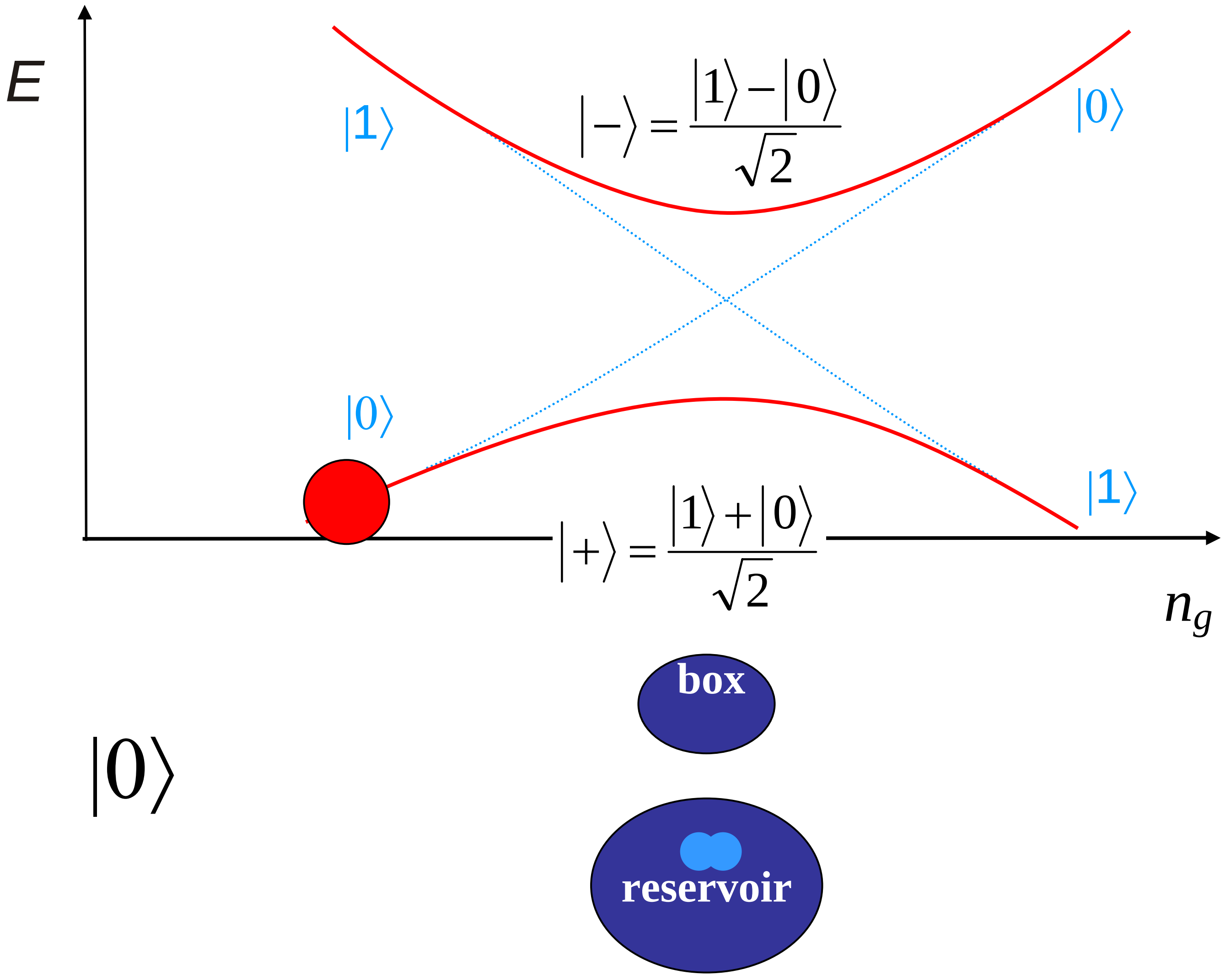
Amplitude of state |1⟩

π−pulse:
$\omega_J \Delta t_\pi = \pi$

$$P(1) = \frac{1-\cos(\omega_J t)}{2}$$



Non-adiabatic manipulation: $\omega \Delta \tau = \pi + 2\pi k$



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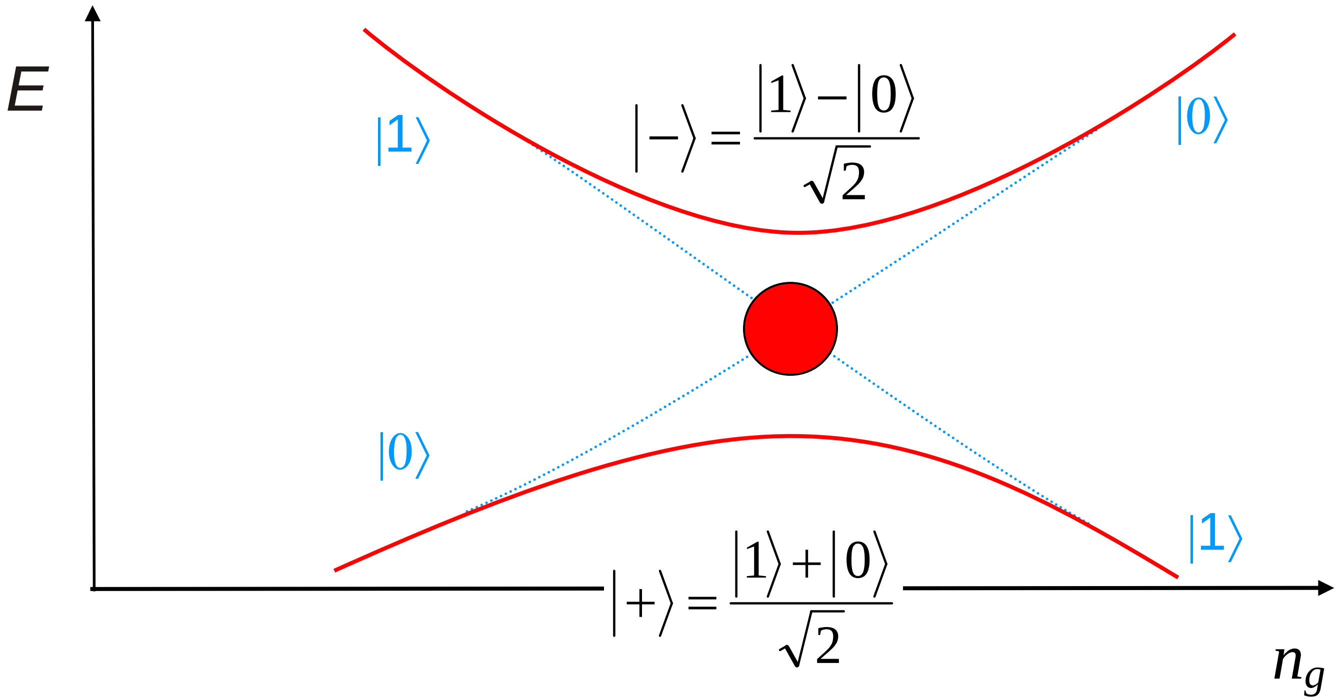
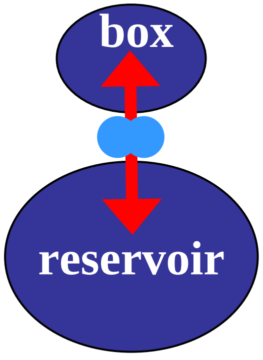


$$\cos(\frac{\omega t}{2})|0\rangle + i\sin(\frac{\omega t}{2})\,|1\rangle$$



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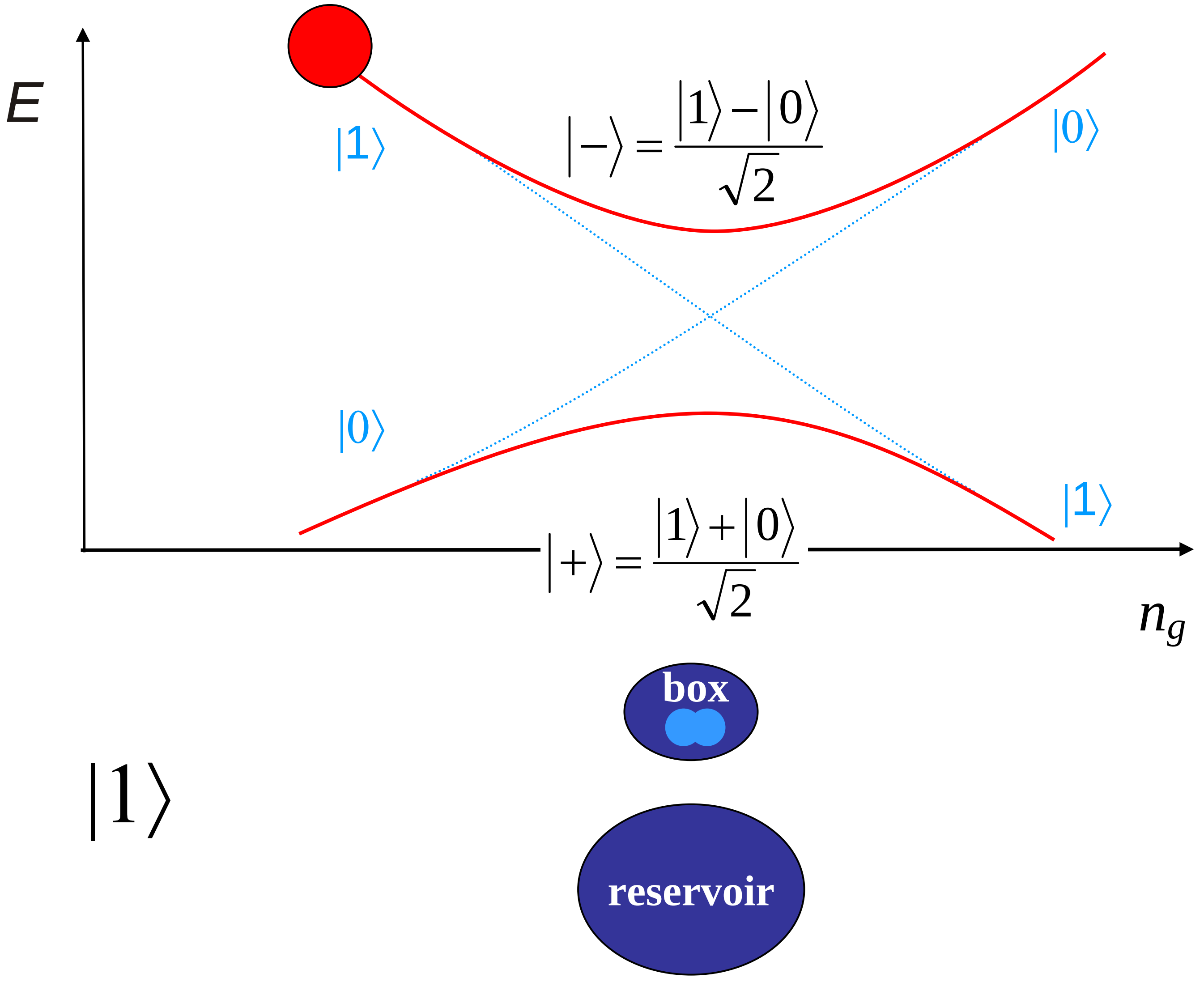
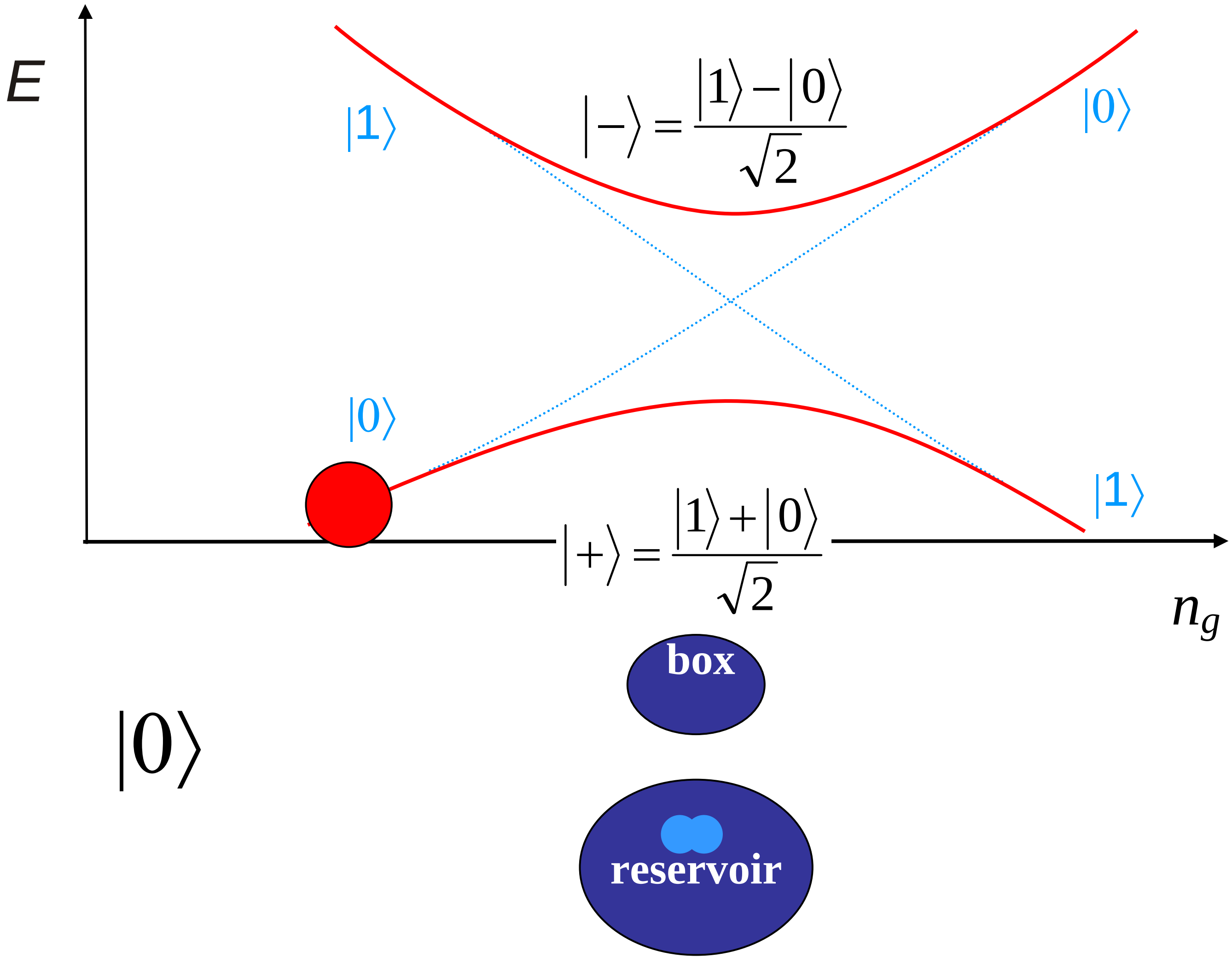
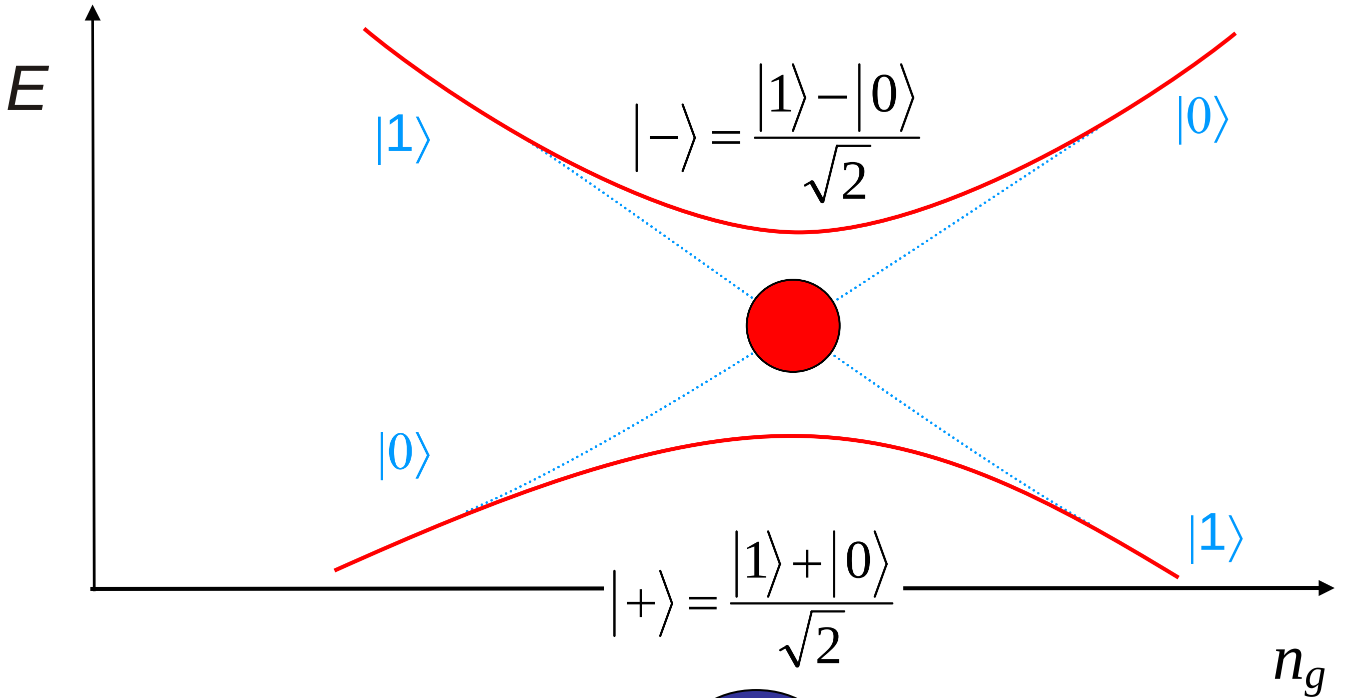


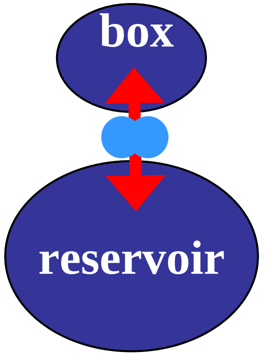
Non-adiabatic manipulation $\omega\Delta t = 2\pi k$



Non-adiabatic manipulation $\omega \Delta t = 2\pi k$



$$\cos(\frac{\omega t}{2})|0\rangle + i\sin(\frac{\omega t}{2})\,|1\rangle$$



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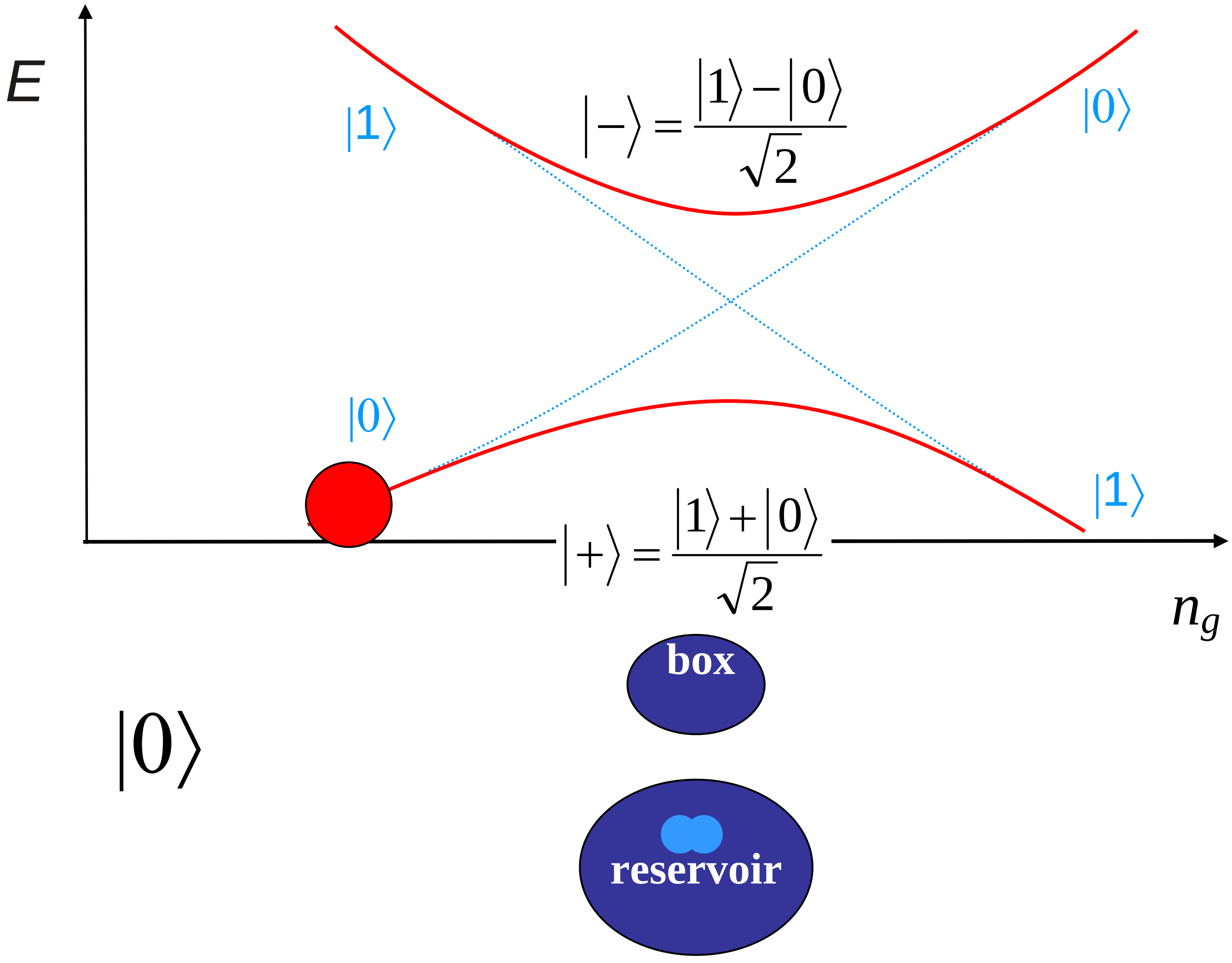
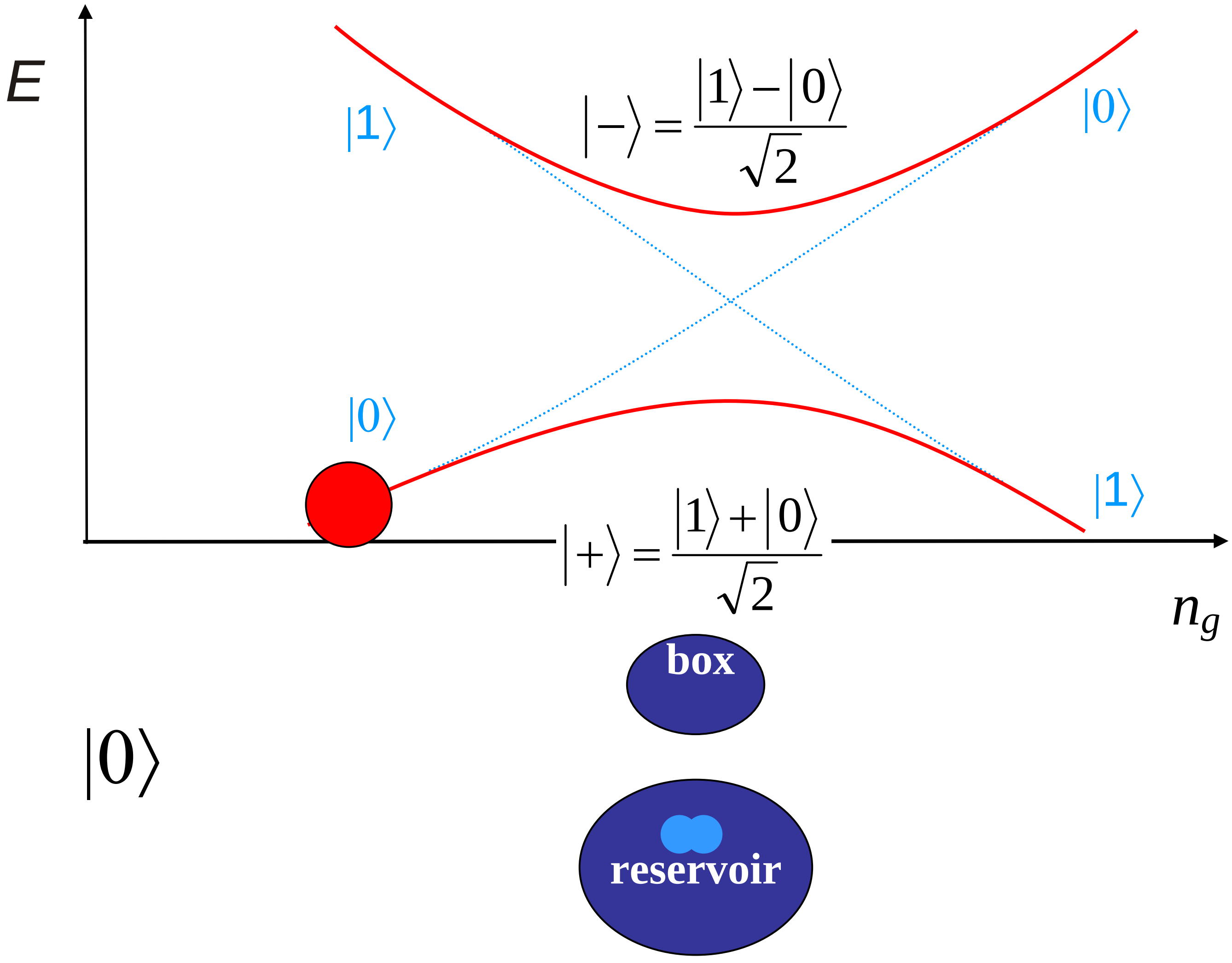


Non-adiabatic manipulation $\omega\Delta t \neq \pi k$



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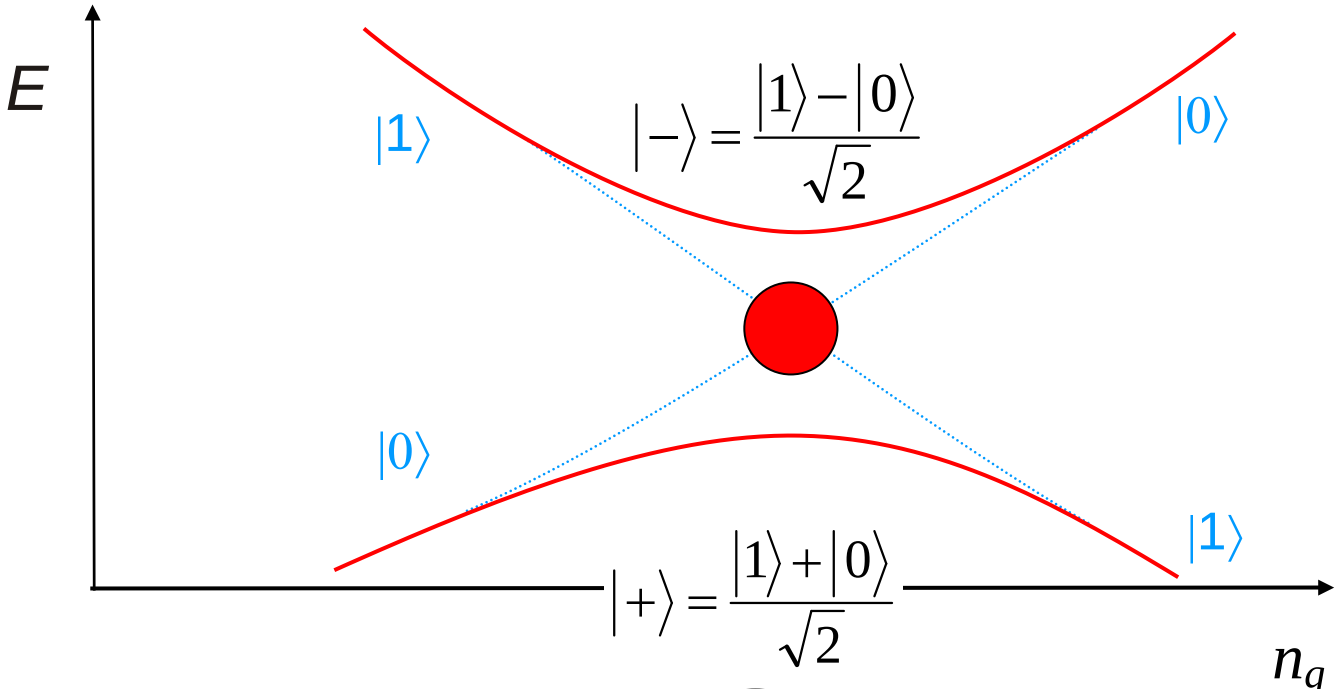
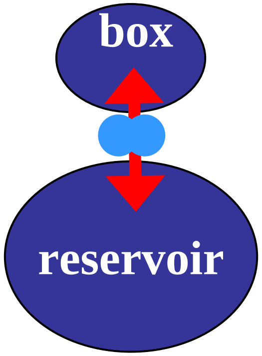


$$\cos(\frac{\omega t}{2})|0\rangle + i\sin(\frac{\omega t}{2})\,|1\rangle$$



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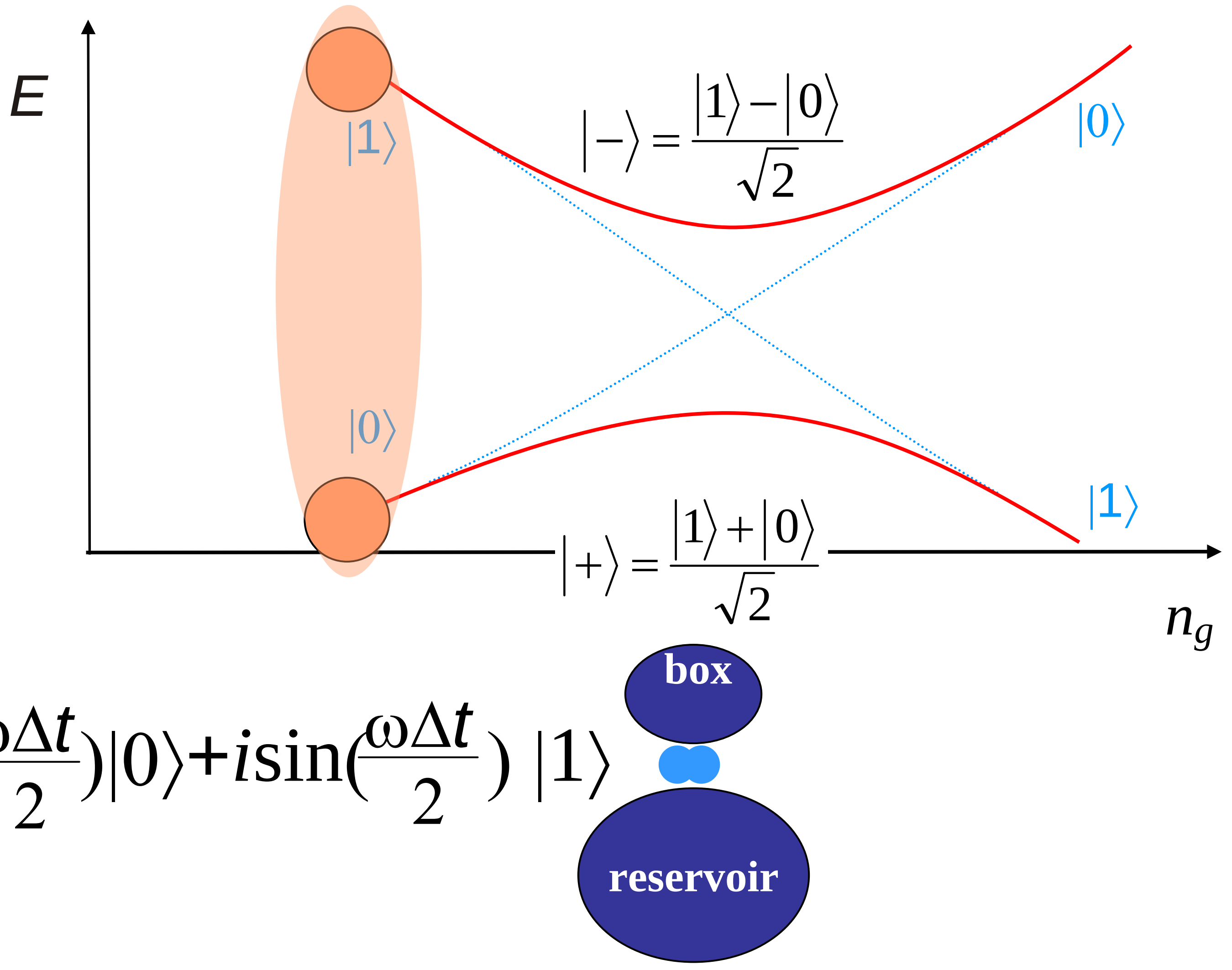
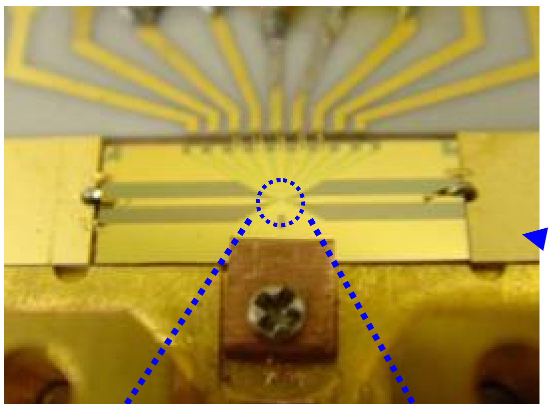
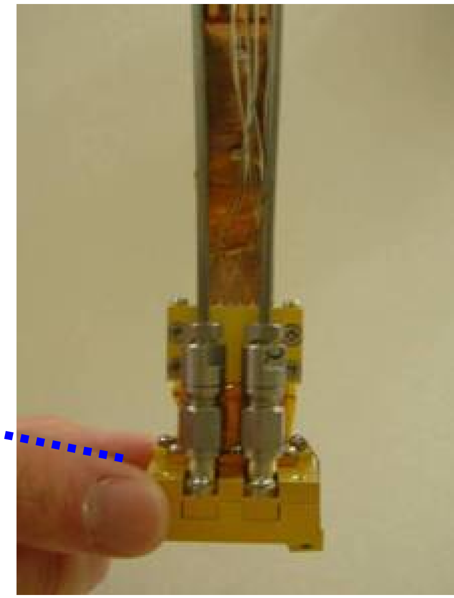
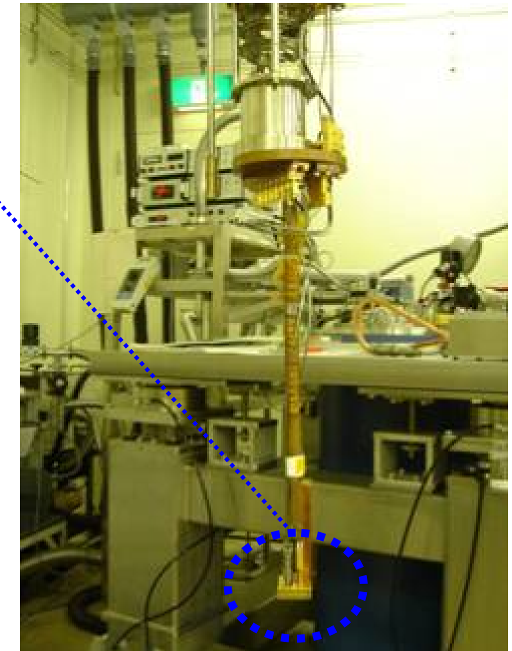


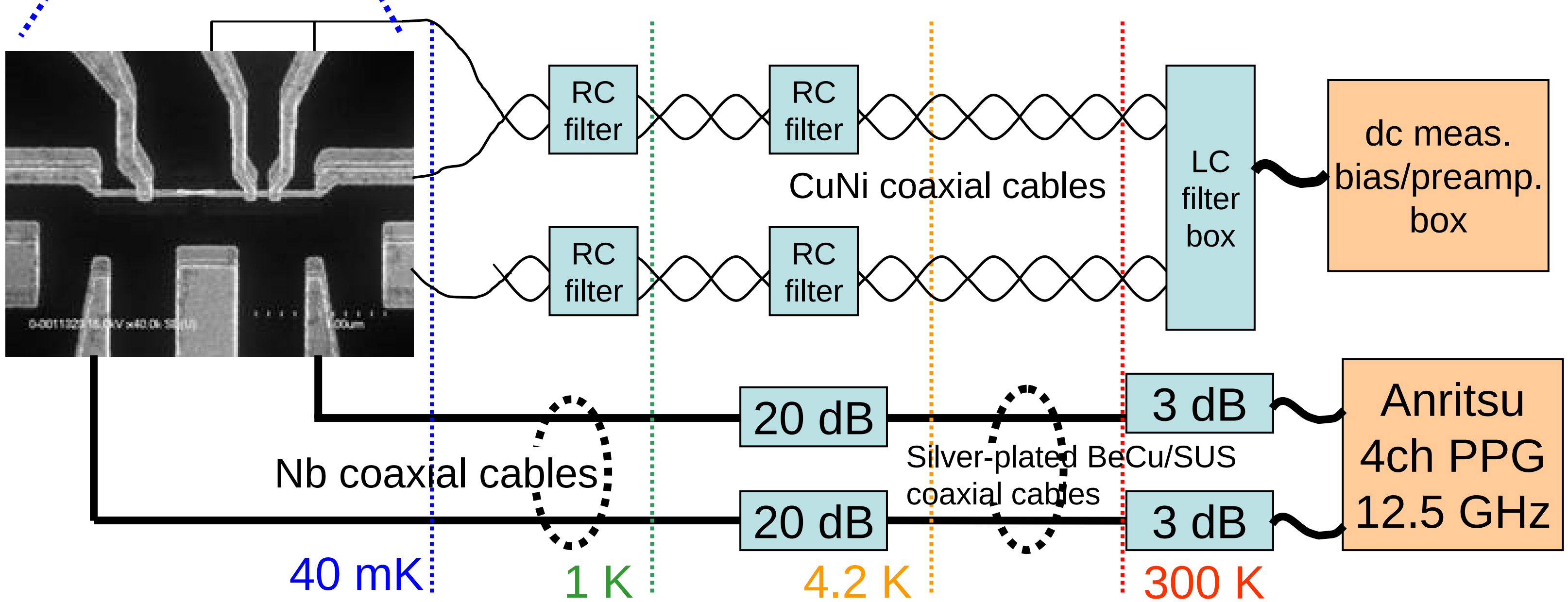
$$\cos\left(\frac{\omega \Delta t}{2}\right)|0\rangle + i\sin\left(\frac{\omega \Delta t}{2}\right)|1\rangle$$

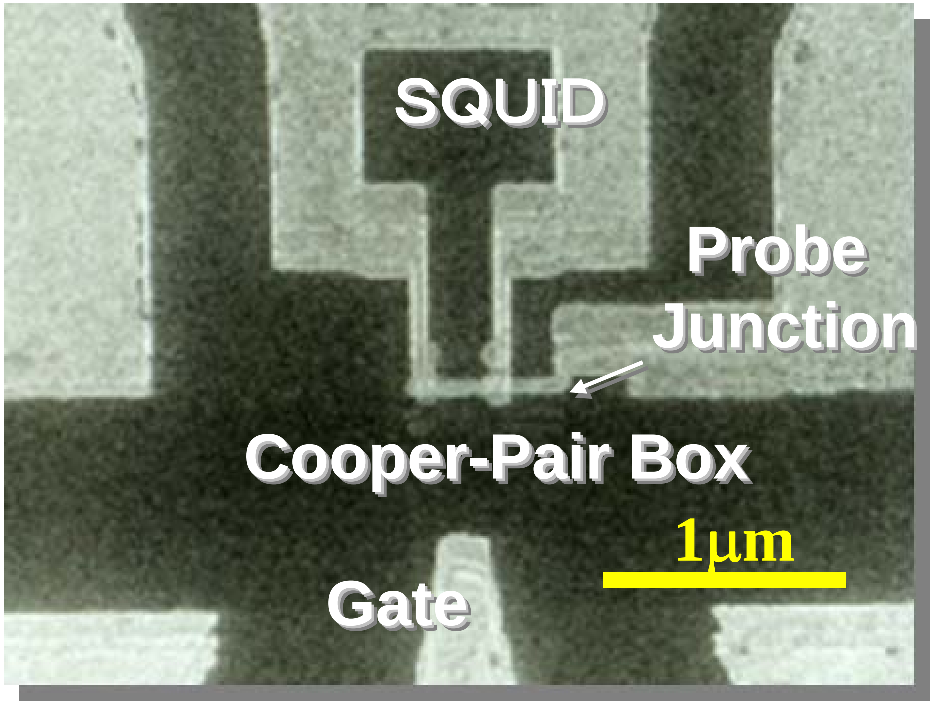
Experimental setup



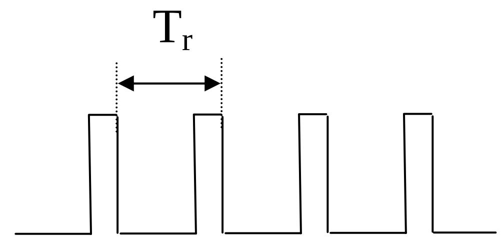




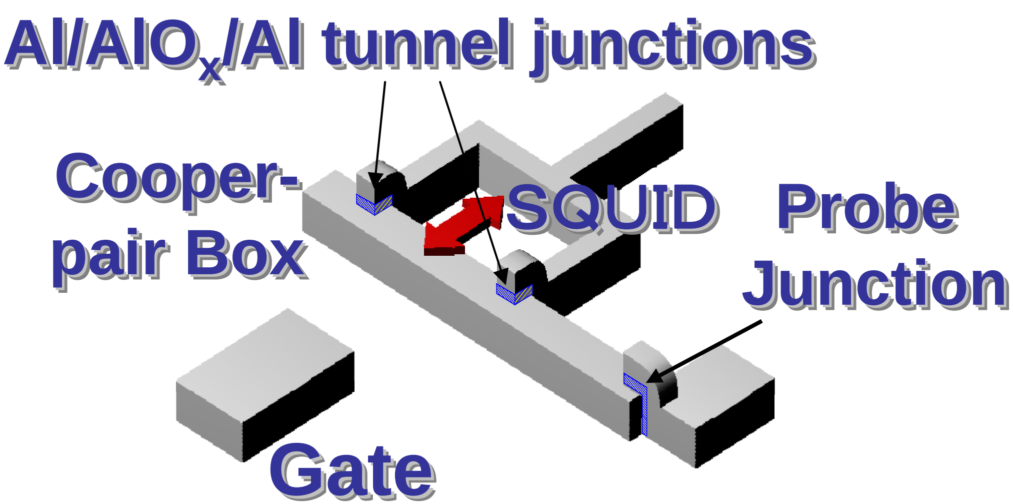




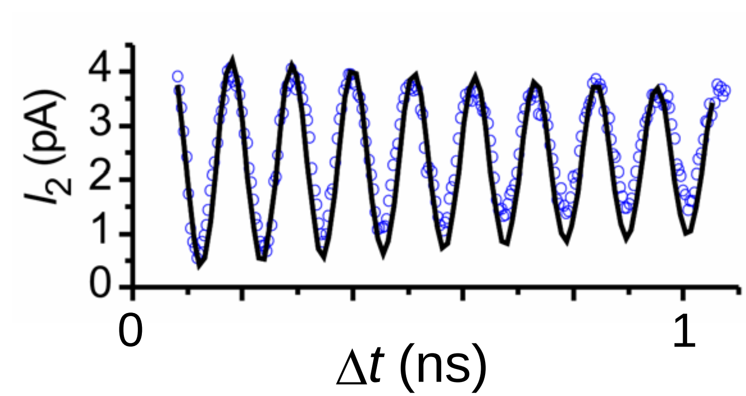
Control pulse sequence



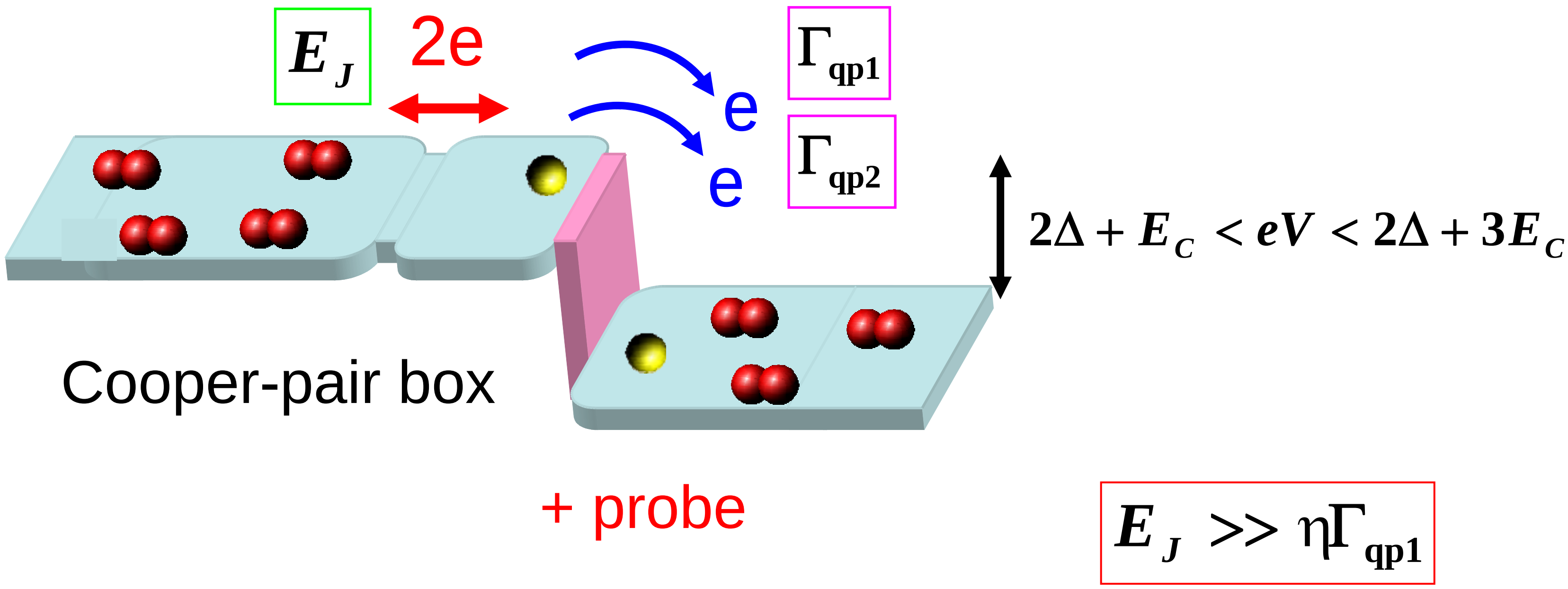
Pulse induced current in SQUID – box – probe junction circuit is measured



$\alpha|0\rangle+\beta|1\rangle \quad I = 2e\,|\alpha|^2/T_r$

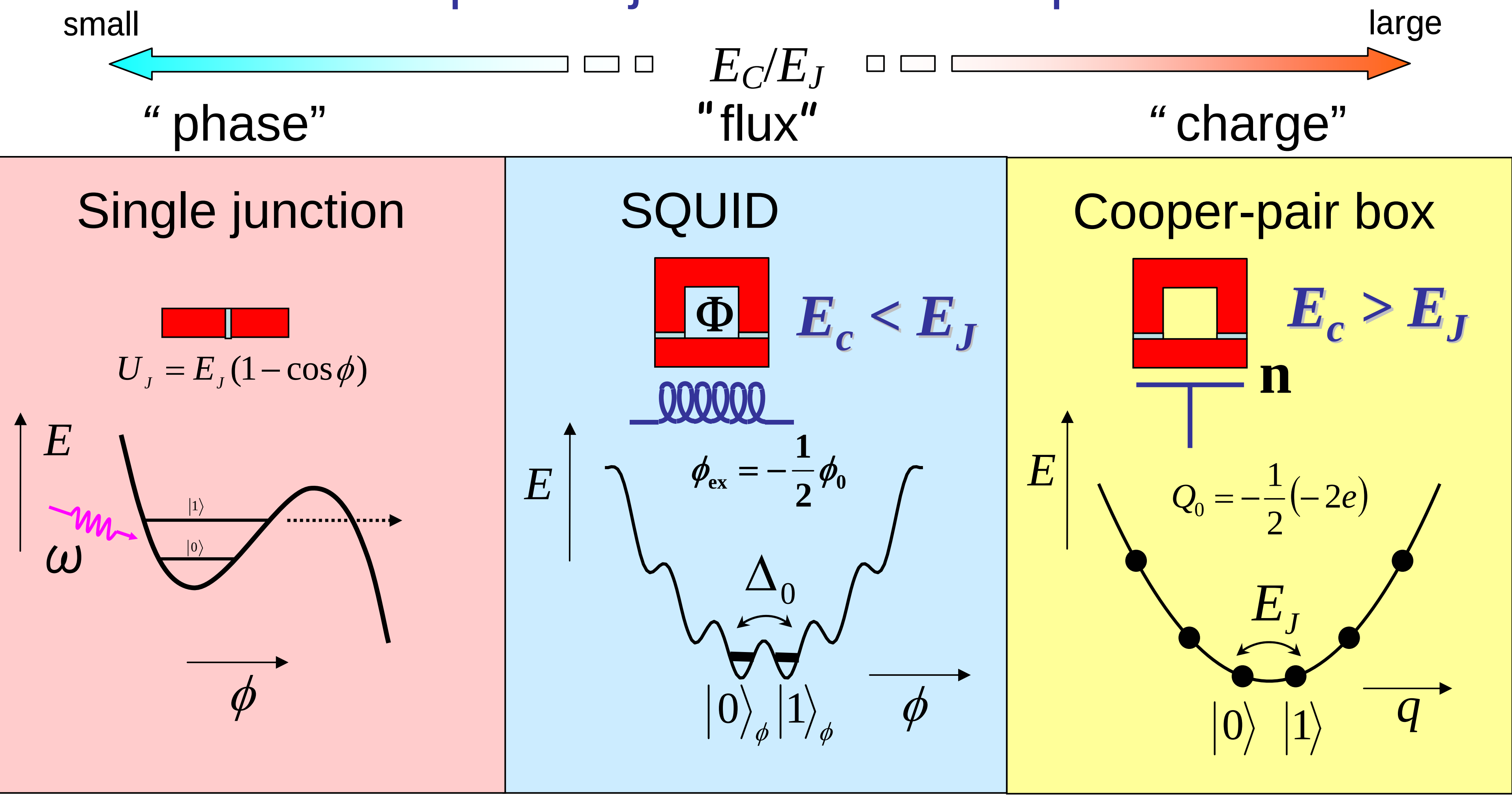


Final state read-out



To break a Cooper pair into two quasiparticles we bias the probe junction to $V_b \approx 2\Delta/e$

Josephson-junction-based qubits



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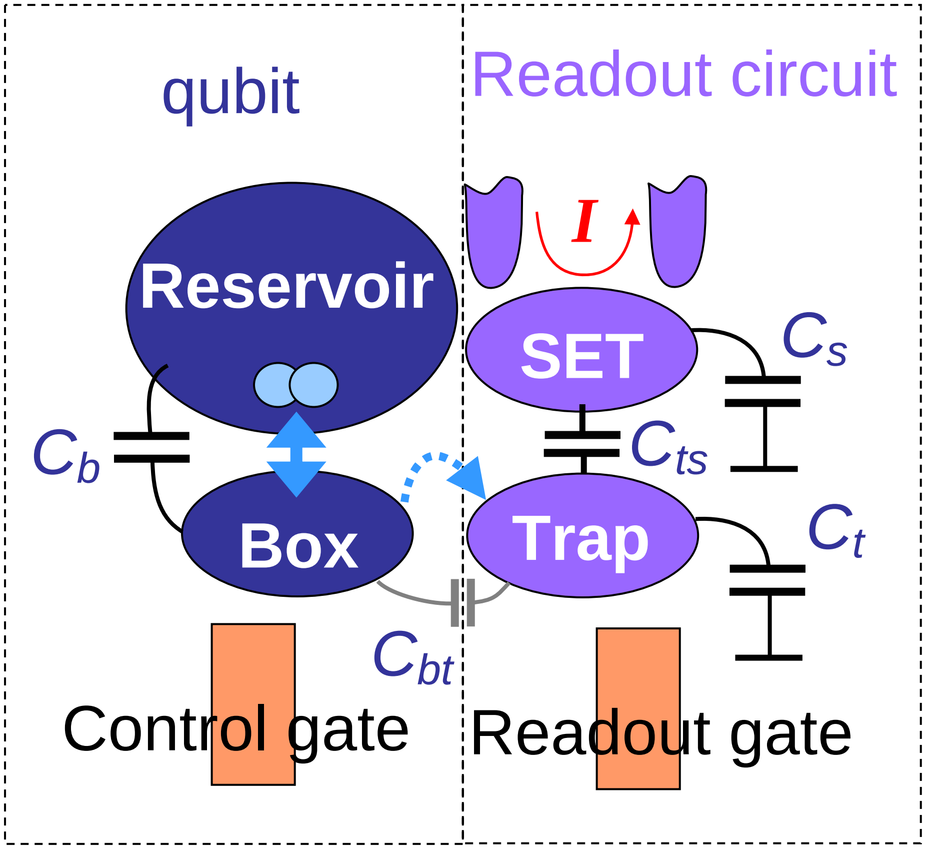
Qubit manipulation $\Rightarrow$ Ensemble averaging

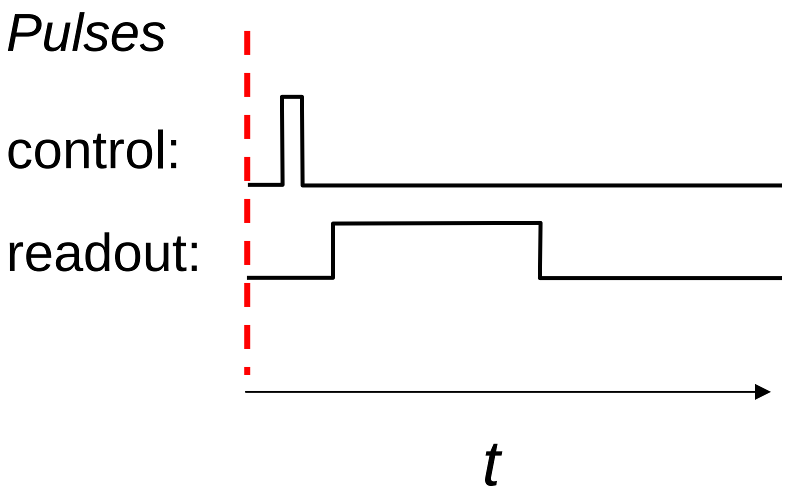
Measurement of results of each quantum state manipulation

Single-shot measurements

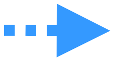
Quantum bit $\Rightarrow$ Classical bit $\Rightarrow$ Readout

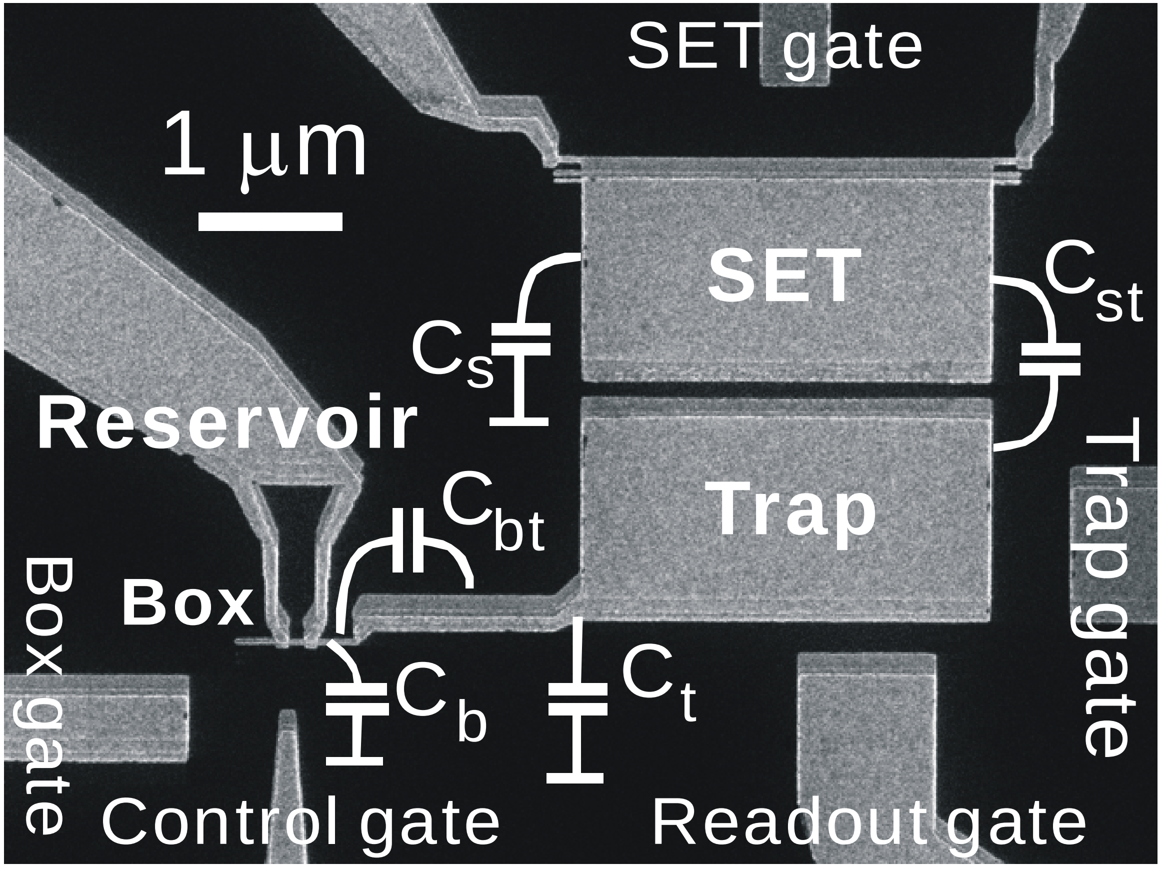
Single-shot Readout



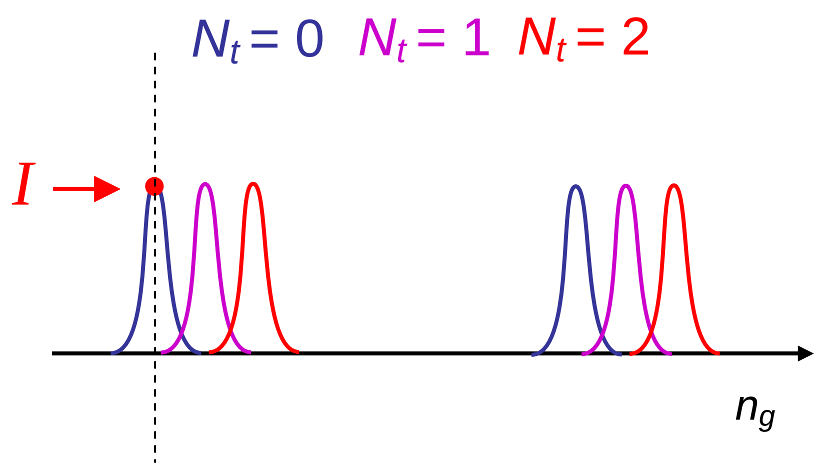


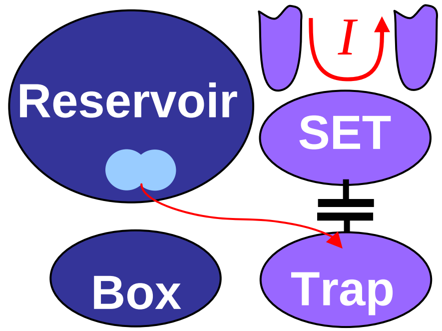
 Cooper pair oscillations

 Quasiparticle tanneling (when the trap is biased by $2\Delta/e$)

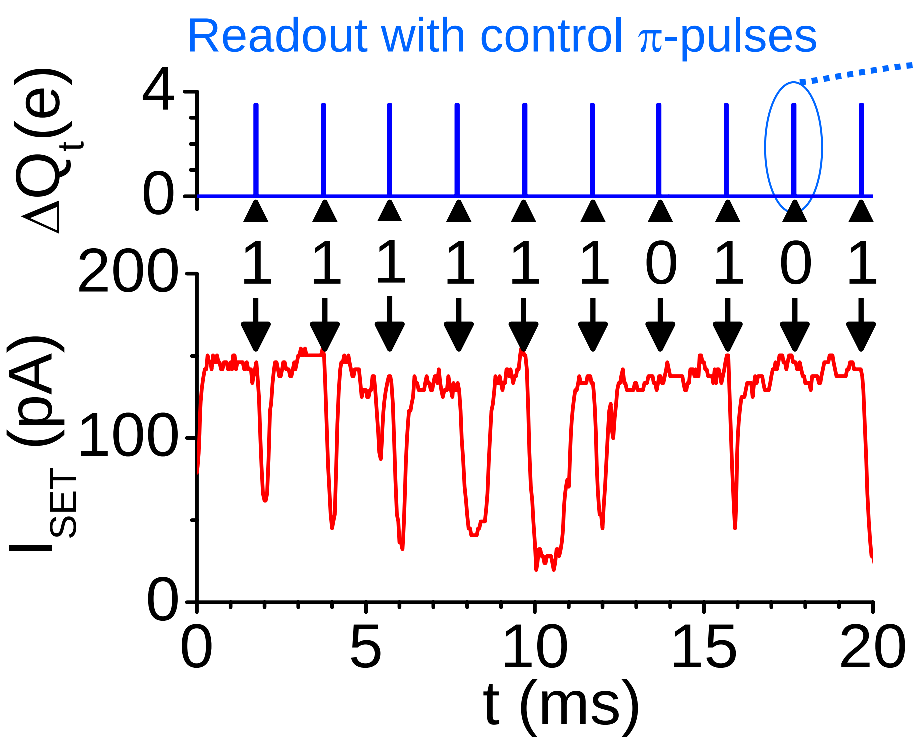


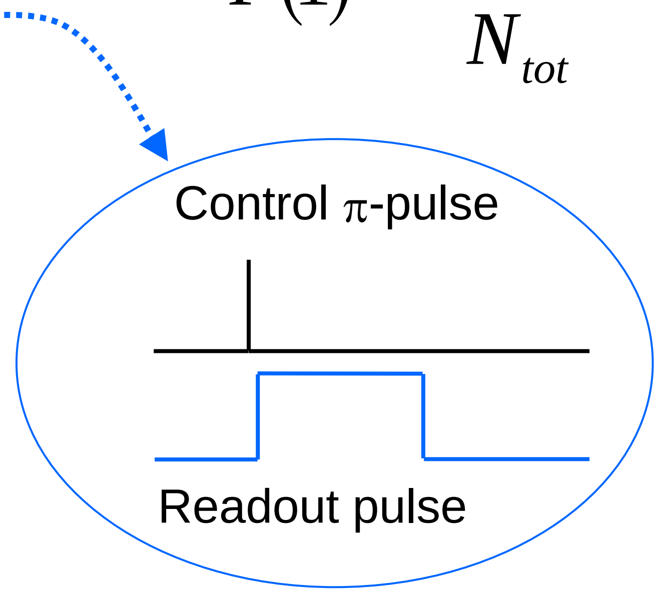
- *Measurement circuit is electrostatically decoupled from the qubit*
- *After reading out the final states, coherent state manipulation is completed*



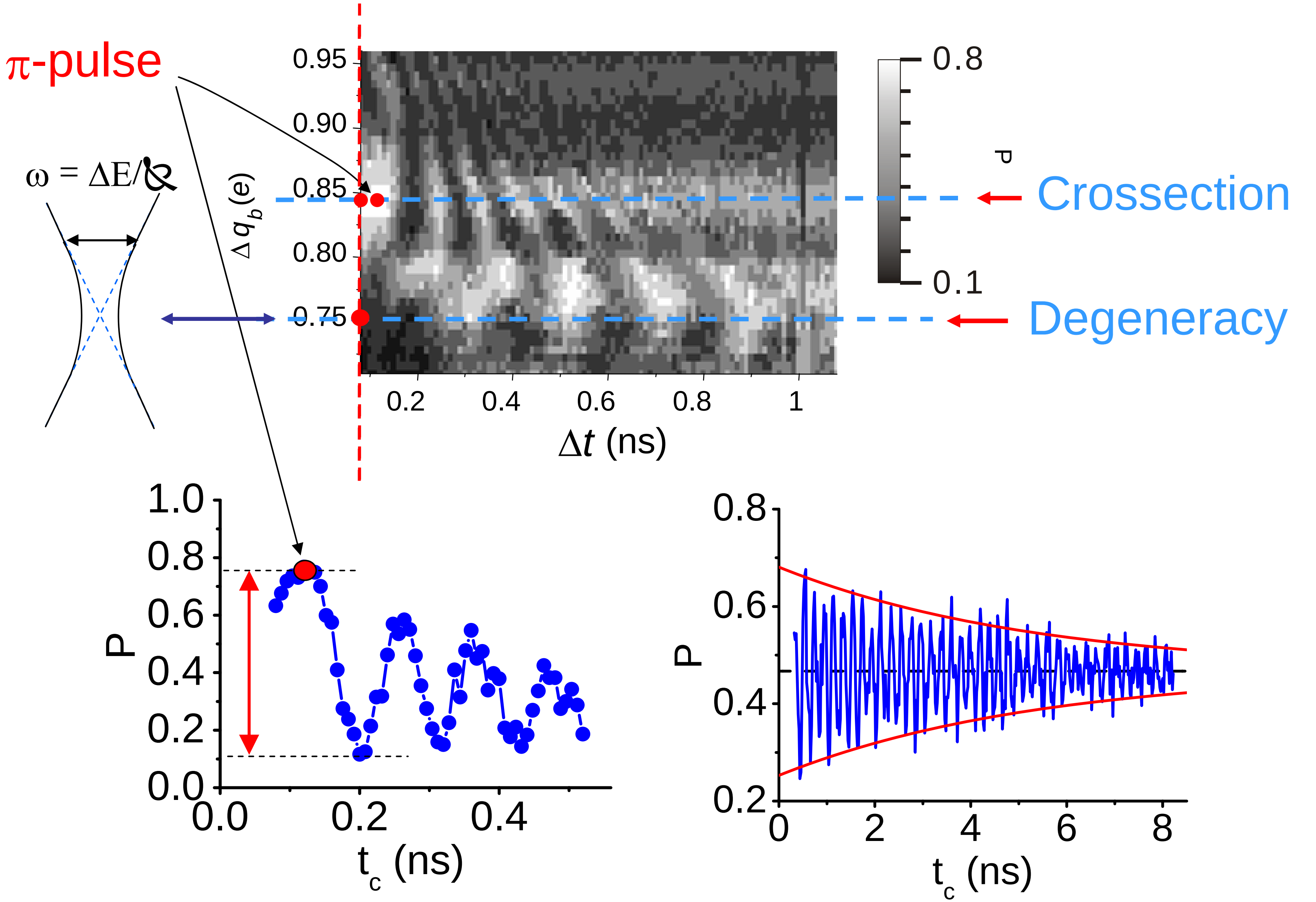


$$P(1) = \frac{N_{switch}}{N_{tot}}$$





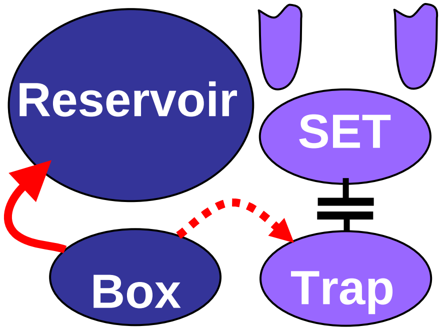
Quantum Oscillations

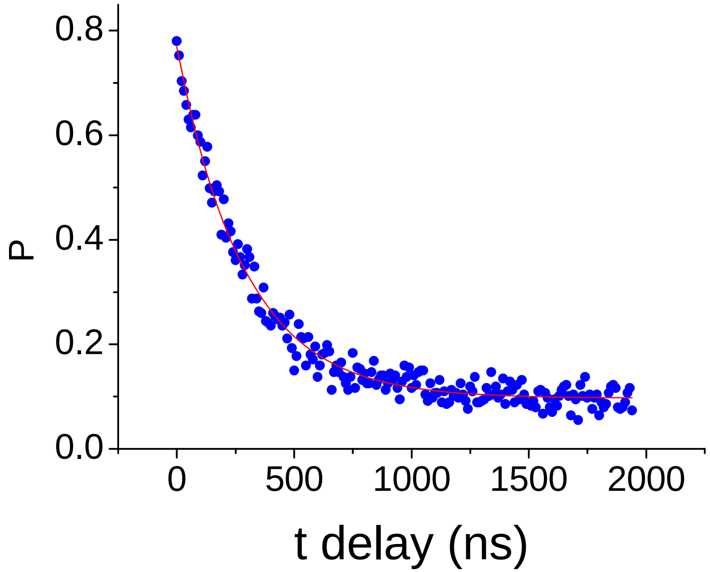


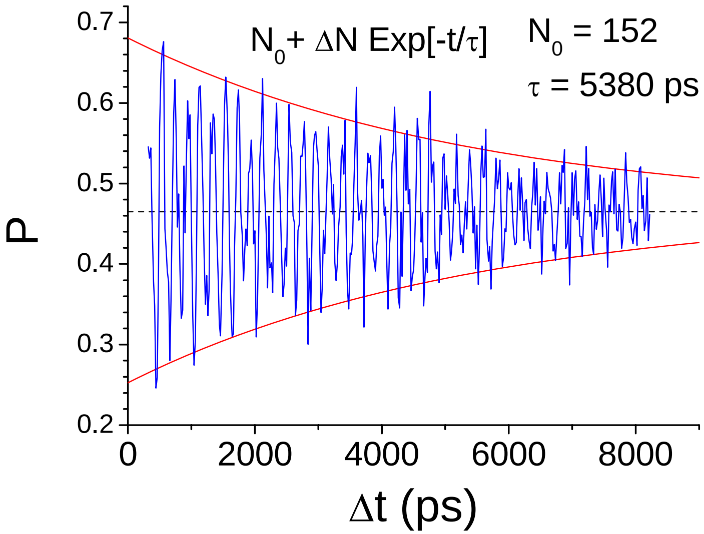
Readout efficiency

$$P_1(|1\rangle) = \frac{T_1^{res}}{T_1^{trap} + T_1^{res}} = \frac{288ns}{288ns + 31ns} \approx 0.90$$

$$P_0(|0\rangle) \approx 0.90$$



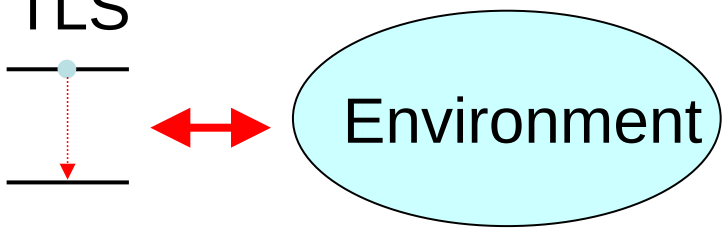


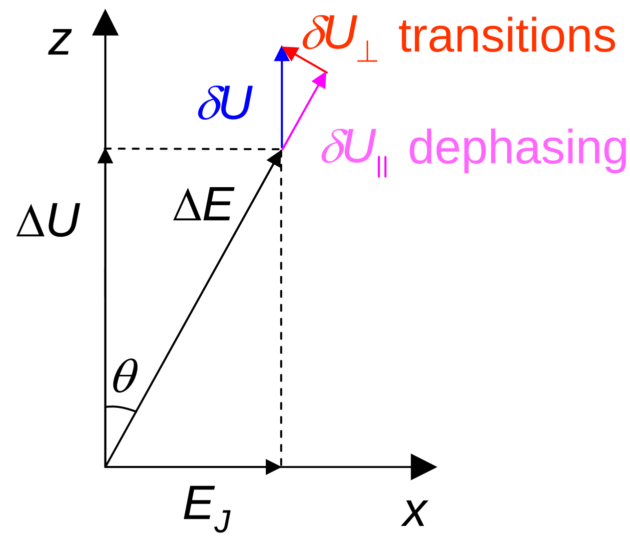


Two-level System as a Quantum Noise Spectrometer

Two-level system

TLS





$$\tan\theta = \frac{E_J}{\Delta U} \qquad \Delta E = \sqrt{E_J^2 + \Delta U^2}$$

Charge basis:

$$H = -\frac{\Delta E}{2}\left(\sigma_z \cos\theta + \sigma_x \sin\theta\right) - \frac{\delta U(t)}{2}\sigma_z$$

Electrostatic energy fluctuations

Eigenbasis:

$$H = -\frac{\hbar\omega_0}{2}\sigma_z - \frac{\delta U(t)}{2}\left(\sigma_z \cos\theta + \sigma_x \sin\theta\right)$$

Dephasing

Transitions

Charge noise

Charge noise spectral density:

$$S_q(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle q(0)q(\tau) \rangle e^{-i\tau\omega} d\tau \qquad \langle q(0)q(\tau) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_0^T q(t)q(t+\tau)dt$$

Relationship between electrostatic energy and charge noises:

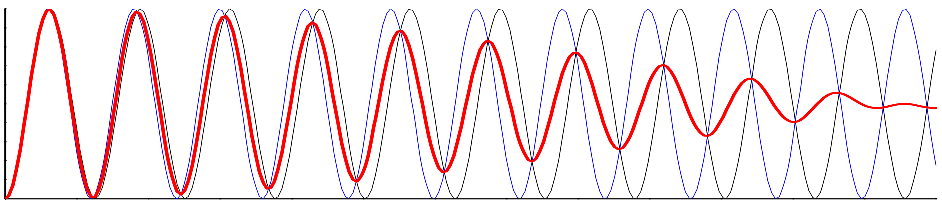
$$\delta U = \delta q \frac{\partial U_{10}}{\partial q} = \delta q \frac{E_C}{e} \qquad S_U = \left(\frac{E_C}{e} \right)^2 S_q$$

Electrostatic energy fluctuation: $\delta U^2 = \int_{\omega_0}^{\omega_1} S_U(\omega) d\omega$

Pure Dephasing

Energy fluctuations: δE

Probability to detect state $|1\rangle$: $$P(1) = \frac{1}{2}\left[1 - \cos\left(\frac{E_0 + \delta E}{\eta} t\right)\right]$$

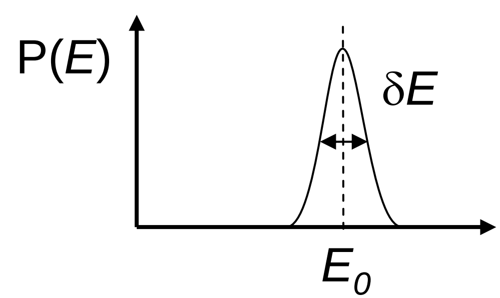


$$\int \cos\left(\frac{E}{\eta} t\right) P(E)\, dE$$

Gaussian distribution: $$P(E) = \frac{1}{\sqrt{2\pi\langle \delta E^2 \rangle}} \exp\left[-\frac{1}{2}\frac{(E - E_0)^2}{\langle \delta E^2 \rangle}\right]$$

$$\cos\left(\frac{E_0}{\eta}\right) \exp\left[-\frac{t^2}{2T_2^2}\right]$$

$$\frac{1}{T_2} = \frac{\sqrt{\langle \delta E^2 \rangle}}{\eta}$$



Free induction decay

$$H = -\frac{\hbar\omega_0}{2}\sigma_z - \frac{\delta U(t)}{2}(\sigma_z \cos\theta + \sigma_x \sin\theta)$$

$$\frac{1}{T_2} = \frac{\sqrt{\langle \delta U^2 \rangle}\cos\theta}{\hbar}$$

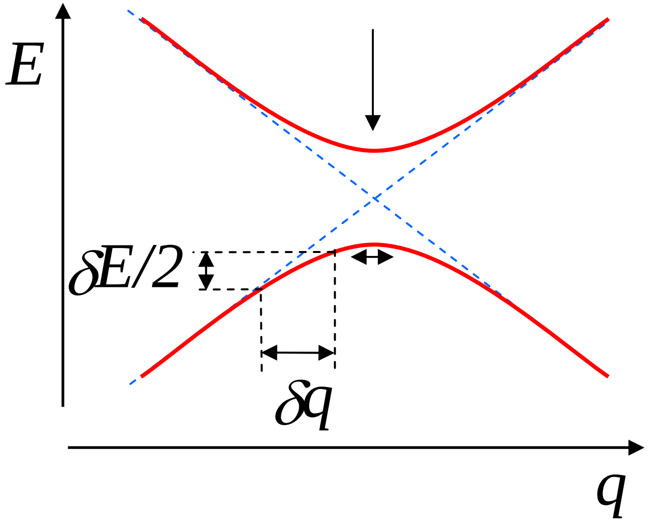
Far away from the magic point:

$$\hbar\omega >> E_J \text{, cos } \theta \approx 1$$

$$T_2 = \frac{\hbar}{\sqrt{\langle \delta U^2 \rangle}}$$

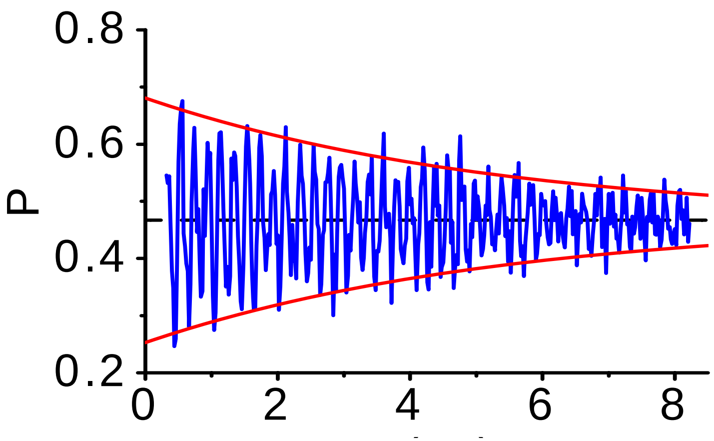
Magic point (δE=0):

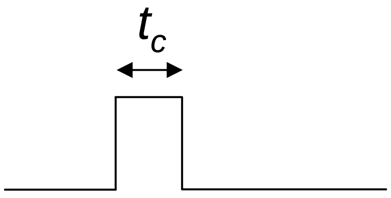
$\theta = \pi/2 \quad T_2 \to \infty$

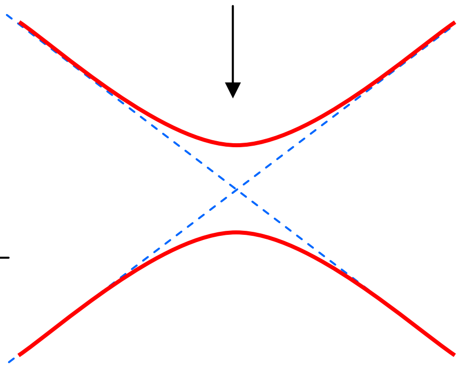


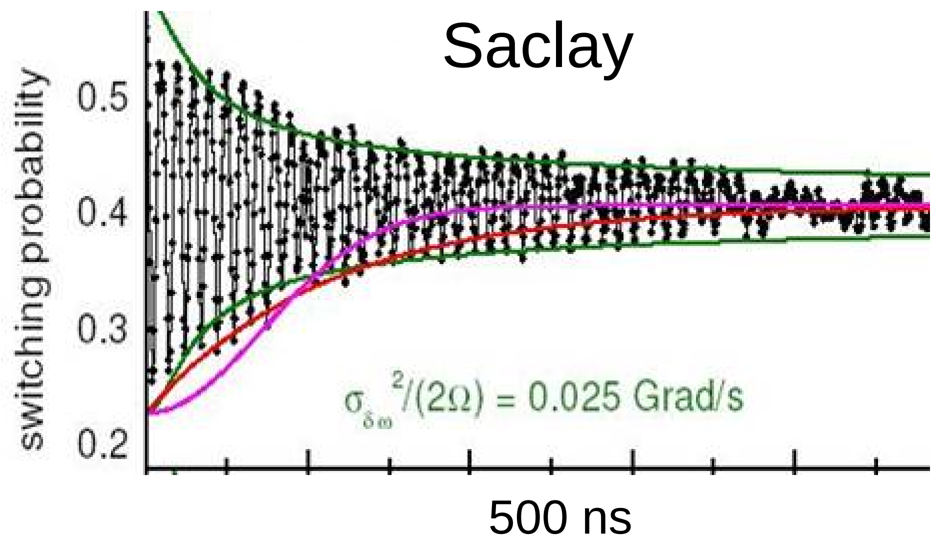
Magic point: no energy fluctuations in the first order!

Dephasing at magic (degeneracy) point

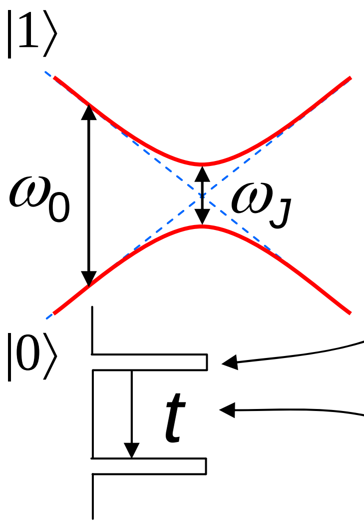








Ramsei interference



$$e^{\frac{i}{2}\varphi\sigma_x} = \begin{pmatrix}\cos\varphi/2 & i\sin\varphi/2 \\ i\sin\varphi/2 & \cos\varphi/2\end{pmatrix} \qquad \varphi = \omega_J t$$

$$\varphi = \frac{\pi}{2} \qquad \frac{1}{\sqrt{2}}\begin{pmatrix}1 & i \\ i & 1\end{pmatrix}\begin{pmatrix}1 \\ 0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 \\ i\end{pmatrix}$$

$$e^{\frac{i}{2}\eta\sigma_z} = \begin{pmatrix}e^{\frac{i}{2}\eta} & 0 \\ 0 & e^{-\frac{i}{2}\eta}\end{pmatrix}$$

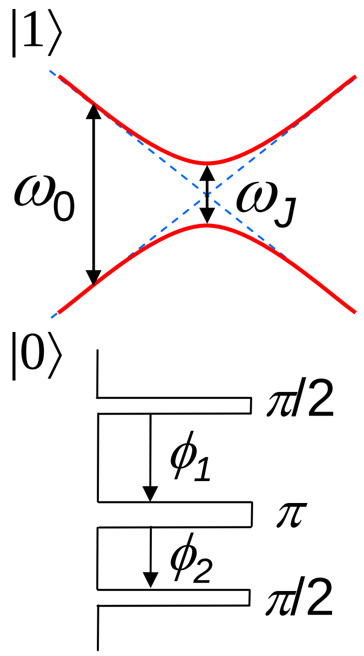
$$\phi = \omega_0 t$$

π/2 rotations at the degeneracy point

$$\psi(t) = e^{\frac{i}{2}\frac{\pi}{2}\sigma_x} e^{\frac{i}{2}\eta\sigma_z} e^{\frac{i}{2}\frac{\pi}{2}\sigma_x}|0\rangle = \frac{1}{2}\begin{pmatrix}1 & i \\ i & 1\end{pmatrix}\begin{pmatrix}e^{\frac{i}{2}\eta} & 0 \\ 0 & e^{-\frac{i}{2}\eta}\end{pmatrix}\begin{pmatrix}1 & i \\ i & 1\end{pmatrix}\begin{pmatrix}1 \\ 0\end{pmatrix} = \begin{pmatrix}\cos\eta/2 \\ i\sin\eta/2\end{pmatrix}$$

Evolution far away from the degeneracy point

Echo



π/2 pulse

$$e^{\frac{i}{2}\frac{\pi}{2}\sigma_x} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

π pulse

$$e^{\frac{i}{2}\pi\sigma_x} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} = i\sigma_x$$

$$\psi(t) = e^{\frac{i}{2}\frac{\pi}{2}\sigma_x} e^{\frac{i}{2}\eta_2\sigma_z} e^{\frac{i}{2}\pi\sigma_x} e^{\frac{i}{2}\eta_1\sigma_z} e^{\frac{i}{2}\frac{\pi}{2}\sigma_x} \left|0\right\rangle$$

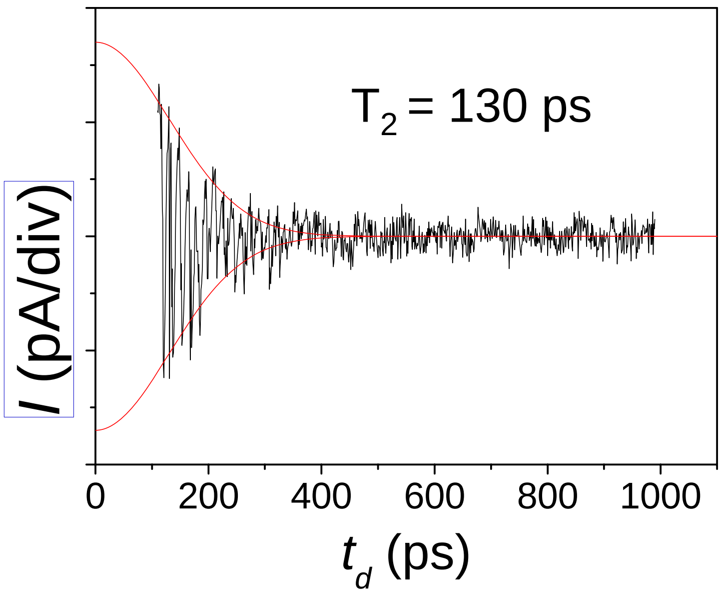
$$i\sigma_x e^{\frac{i}{2}(\eta_1-\eta_2)\sigma_z}$$

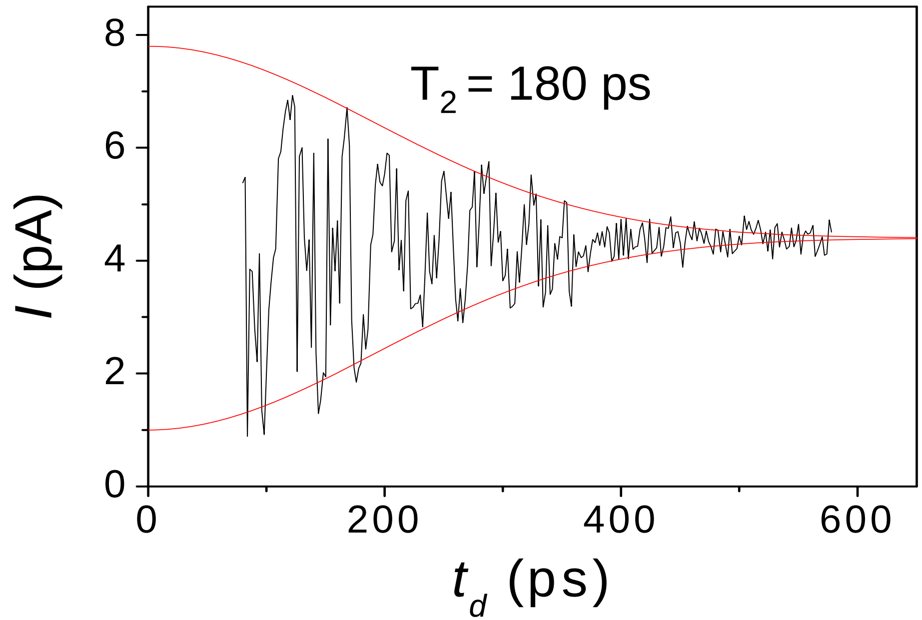
π-pulse reverses phase

$$\psi(t) = e^{-\frac{i}{2}\frac{\pi}{2}\sigma_x} e^{\frac{i}{2}(\eta_1-\eta_2)\sigma_z} e^{i\frac{i}{2}\frac{\pi}{2}\sigma_x} \left|0\right\rangle$$

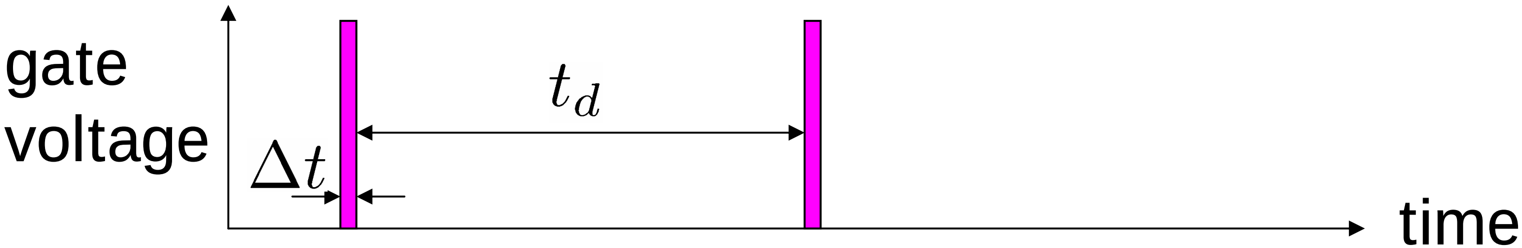
Ramsei interfernce: $e^{\frac{i}{2}\frac{\pi}{2}\sigma_x} e^{\frac{i}{2}\eta\sigma_z} e^{\frac{i}{2}\frac{\pi}{2}\sigma_x}$

Free-induction decay



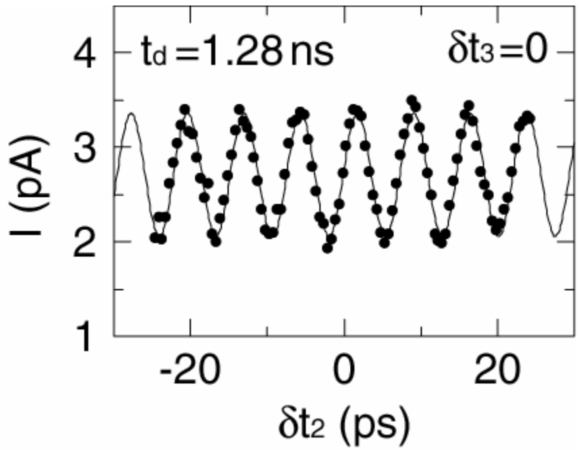


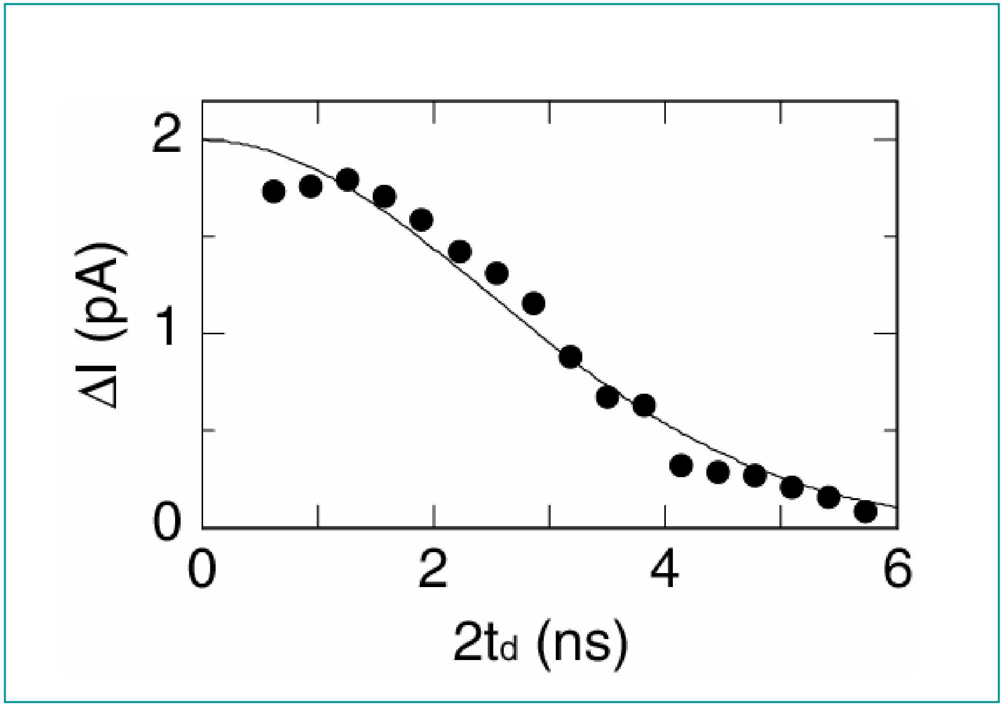
$\left(\frac{\pi}{2}\right)_x$ $\varphi = \frac{\Delta E}{\hbar} t_d$ $\left(\frac{\pi}{2}\right)_x$

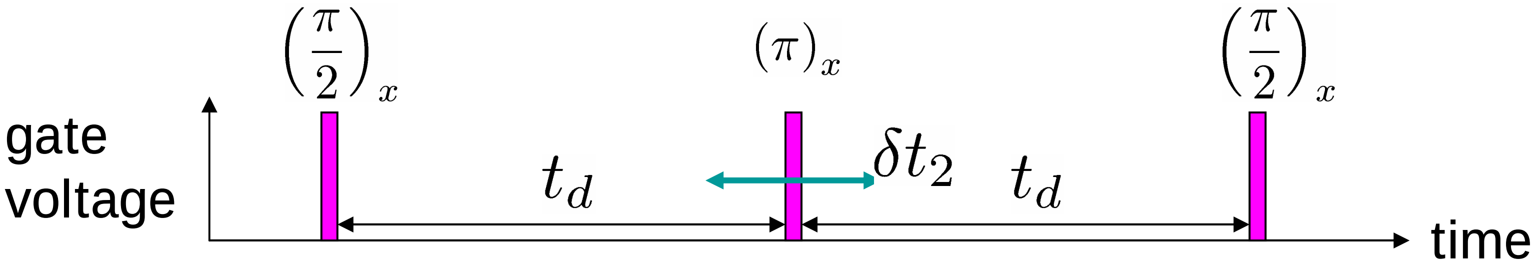


Decay of echo signal

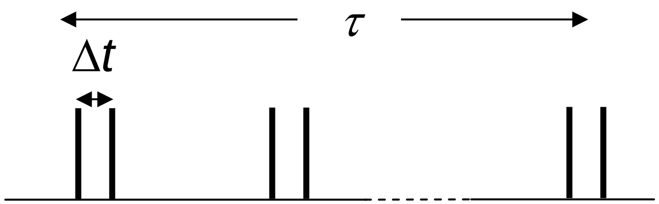
- Current vs.
  Position of 2nd pulse







Noise in Ramsei interference



$$\frac{1}{T_2} = \frac{\sqrt{\left\langle \delta U^2 \right\rangle} \cos\theta}{\eta}$$

$$\delta U^2 = \int_{\omega_0}^{\omega_1} S_U(\omega)d\omega$$

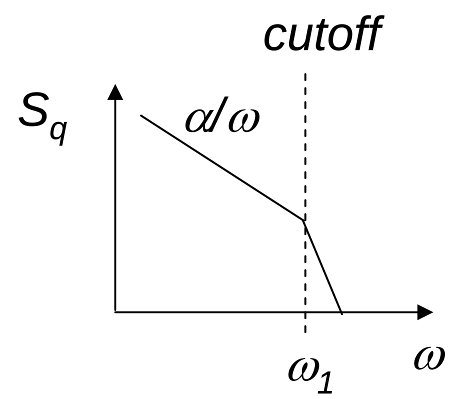
$\omega_1 \approx \pi/\Delta t$ $\Delta t \sim 300$ ps

$\omega_0 \approx \pi/\tau$ $\tau \sim 0.1$ s (averaging time)

1/f noise (Gaussian case):

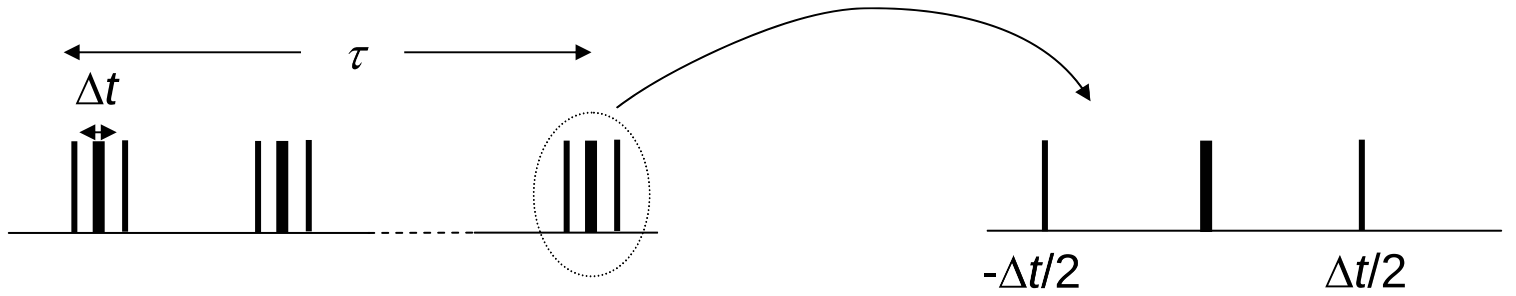
$$S_q(\omega) = \frac{\alpha}{\omega} \quad \Rightarrow \quad S_U(\omega) = \frac{E_C^2}{e^2}\frac{\alpha}{\omega}$$

$$\delta U^2 = \frac{E_C^2}{e^2}\alpha \ln(\omega_1 \tau) \qquad T_2 = \frac{e\eta}{E_C\sqrt{\alpha \ln(\omega_1 \tau)}}$$



Low frequency 1/f noise mainly contributes to the dephasing

Noise in Echo experiments



$$\langle \delta U^2 \rangle = \left\langle \left| U(t-\Delta t/2) - U(t+\Delta t/2) \right|^2 \right\rangle = 2\langle U^2 \rangle - 2\langle U(t-\Delta t/2)U(t+\Delta t/2) \rangle$$

Frequencies of order of $\pi/\Delta t$ contribute into dephasing

$$\delta U^2 = \int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} S_U(\omega)d\omega$$

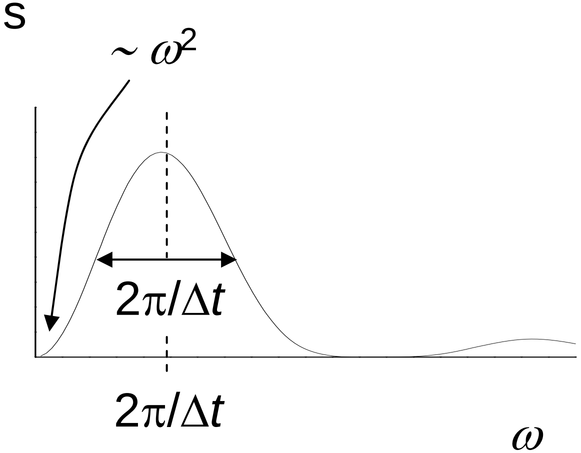
$\omega_1 \approx 2\pi/\Delta t$

$\Delta\omega \approx \pi/\Delta t$

$\Delta t \sim 1$ ns

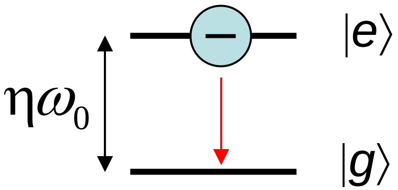
$$\delta U^2 = \int_{\pi/\tau}^{\infty} \left(\frac{\sin^2(\omega\Delta t/4)}{\Delta t\omega/4} \right)^2 S_U(\omega)d\omega$$

$\pi/\tau \to 0$



Transitions (relaxation, excitation)

$$H = -\frac{\eta\omega_0}{2}\sigma_z - \frac{\delta U(t)}{2}(\sigma_z \cos\theta + \sigma_x \sin\theta)$$



Transition amplitude over time *t*:

$$a_{eg} = -\frac{1}{2}\int_0^t \delta U(t_1)\sin\theta e^{i\omega_0 t_1} dt_1$$

Transition occurs under driving field of the noise at the resonant frequency:

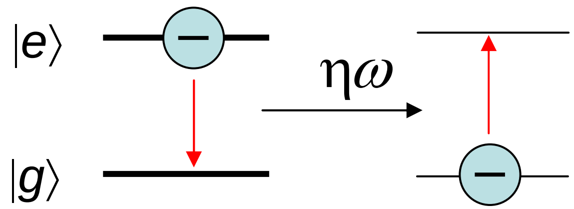
Transition probability:

$$|a|^2 = \frac{\sin^2\theta}{4\eta^2}\int_0^t\int_0^t \delta U(t_1)\delta U(t_2)e^{i\omega_0(t_1-t_2)}dt_1 dt_2$$

$$\int_0^t\int_0^t \delta U(\tau)\delta U(\tau + t_{21})e^{-i\omega_0 t_{21}} d\tau dt_{21} = 2\pi S_U(\omega_0)t$$

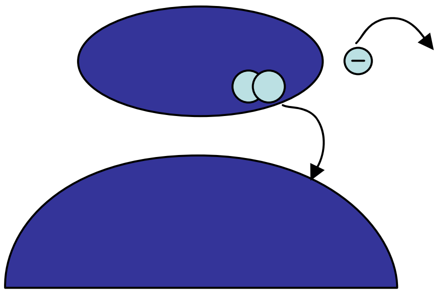
$$\Gamma_\downarrow = \frac{\pi S_U(\omega_0)\sin^2\theta}{2\eta^2} \qquad \Gamma_\uparrow = \frac{\pi S_U(-\omega_0)\sin^2\theta}{2\eta^2}$$

Quantum noise



$T = 0$

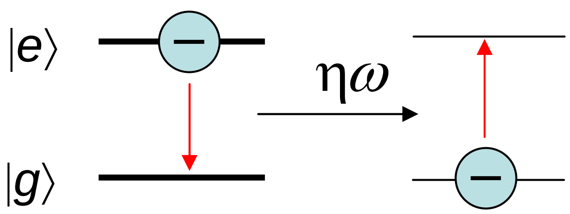
δU_q – induced electrostatic energy



Relaxation rate (Fermi Golden Rule):

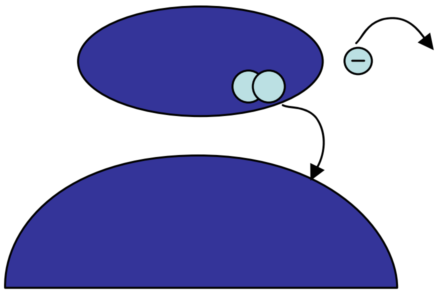
$$\Gamma_{\downarrow} = \frac{2\pi \left| \delta U_q \sin\theta \right|^2}{\eta} \delta(E_q - \eta\omega)$$

Quantum noise



$T = 0$

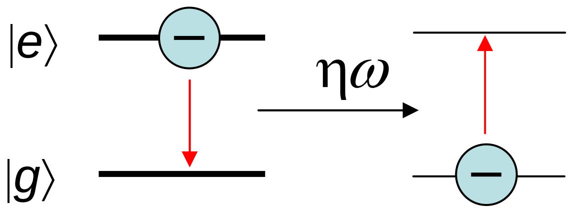
δU_q – induced electrostatic energy



Relaxation rate (Fermi Golden Rule):

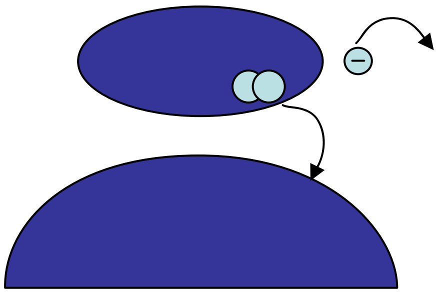
$$\Gamma_{\downarrow} = \frac{2\pi \left| \delta U_q \sin\theta \right|^2}{\eta} \sum_k \delta(\eta\omega_0 - \eta\omega_k)$$

Quantum noise



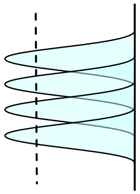
$T = 0$

δU_q – induced electrostatic energy

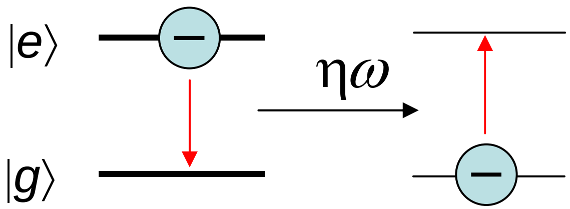


Relaxation rate (Fermi Golden Rule):

$$\Gamma_{\downarrow} = \frac{2\pi \left| \delta U_q \sin\theta \right|^2}{\eta} \sum_k g(\eta\omega_0 - \eta\omega_k)$$

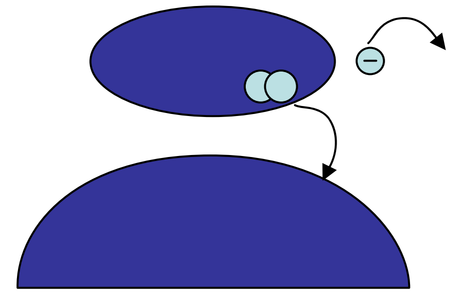


Quantum noise



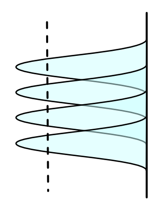
$T = 0$

δU_q – induced electrostatic energy



Relaxation rate (Fermi Golden Rule):

$$\Gamma_{\downarrow} = \frac{2\pi \left|\delta U_q \sin\theta\right|^2}{\eta} D(\eta\omega)$$



$$\sum \delta(E_q - \eta\omega_k) \to \sum g(E_q - \eta\omega_k) = D(\eta\omega)$$

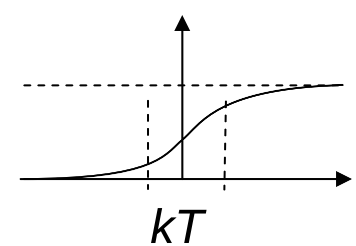
No excitations! $\Gamma_{\uparrow} = 0$

$$\Gamma \propto S_U(\omega)$$

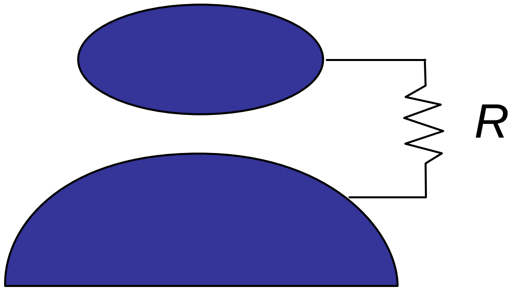
$S(-\omega) = 0 \qquad S(\omega) > 0$

$T \neq 0$

$$\frac{\Gamma_{\uparrow}}{\Gamma_{\downarrow}} = \frac{1}{2}\left[1 + \coth\left(\frac{\eta\omega}{kT}\right)\right]$$



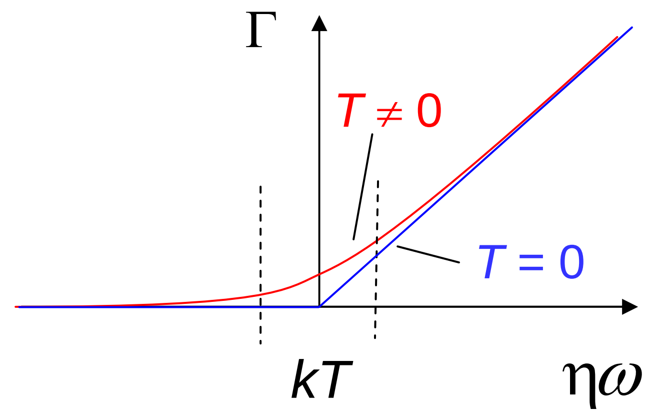
Ohmic environment at finite temperature



$$S_V(\omega) = 2R\eta\omega \frac{1}{2}\left[1 + \coth\left(\frac{\eta\omega}{kT}\right)\right]$$

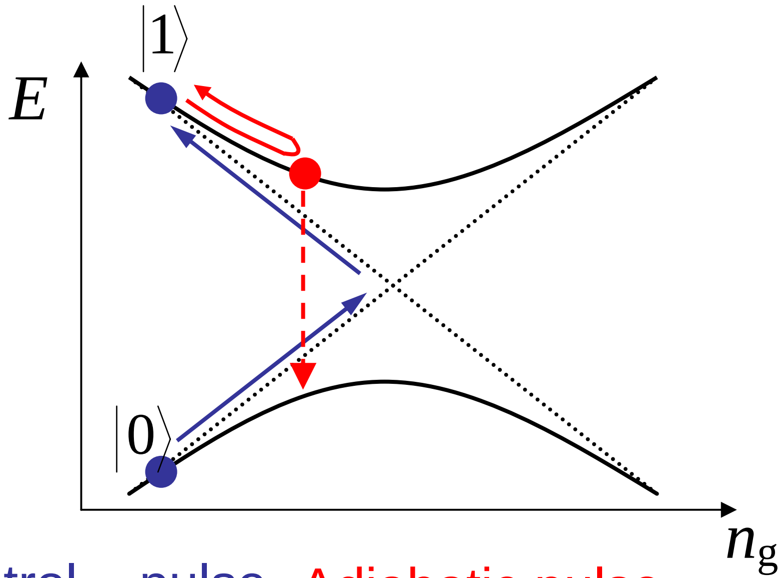
$$S_U(\omega) = (2e)^2 S_V(\omega)$$

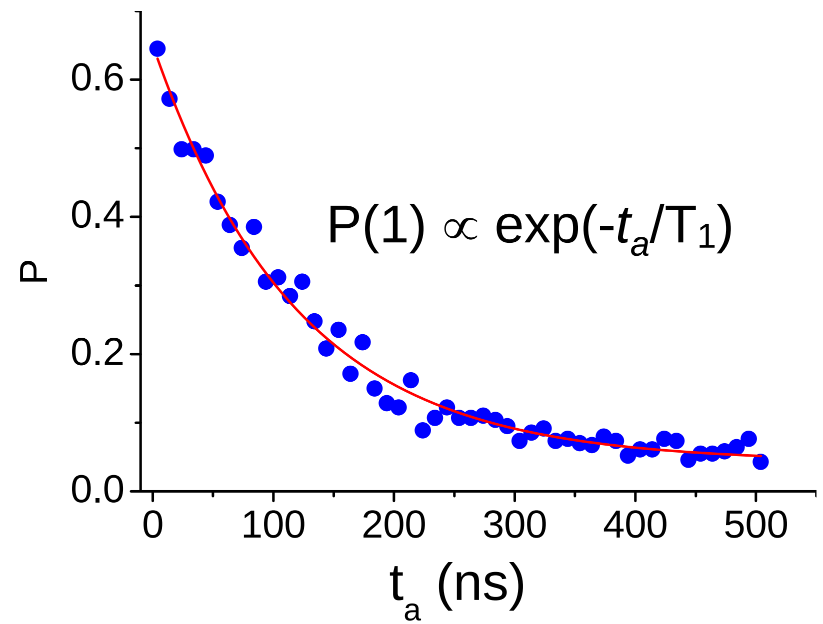
$$\Gamma_{\downarrow} = \frac{\pi 4e^2 R\eta\omega \sin^2\theta}{2\eta^2}\frac{1}{2}\left[1 + \coth\left(\frac{\eta\omega}{2kT}\right)\right]$$

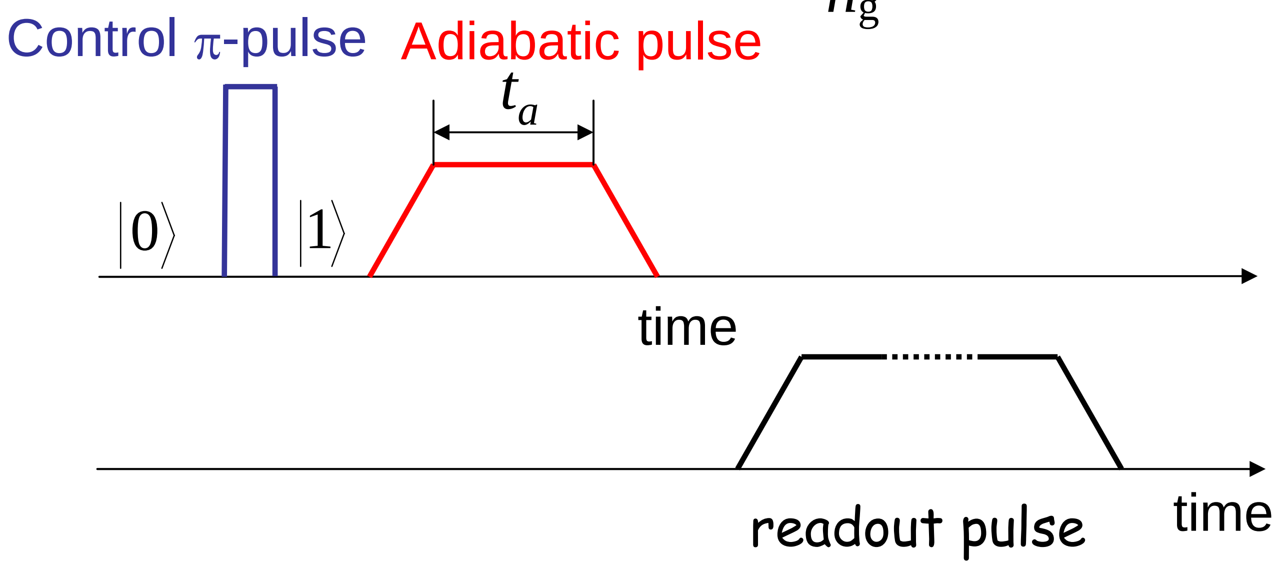


Environment with constant density of states and frequency Independent coupling behaves like ohmic

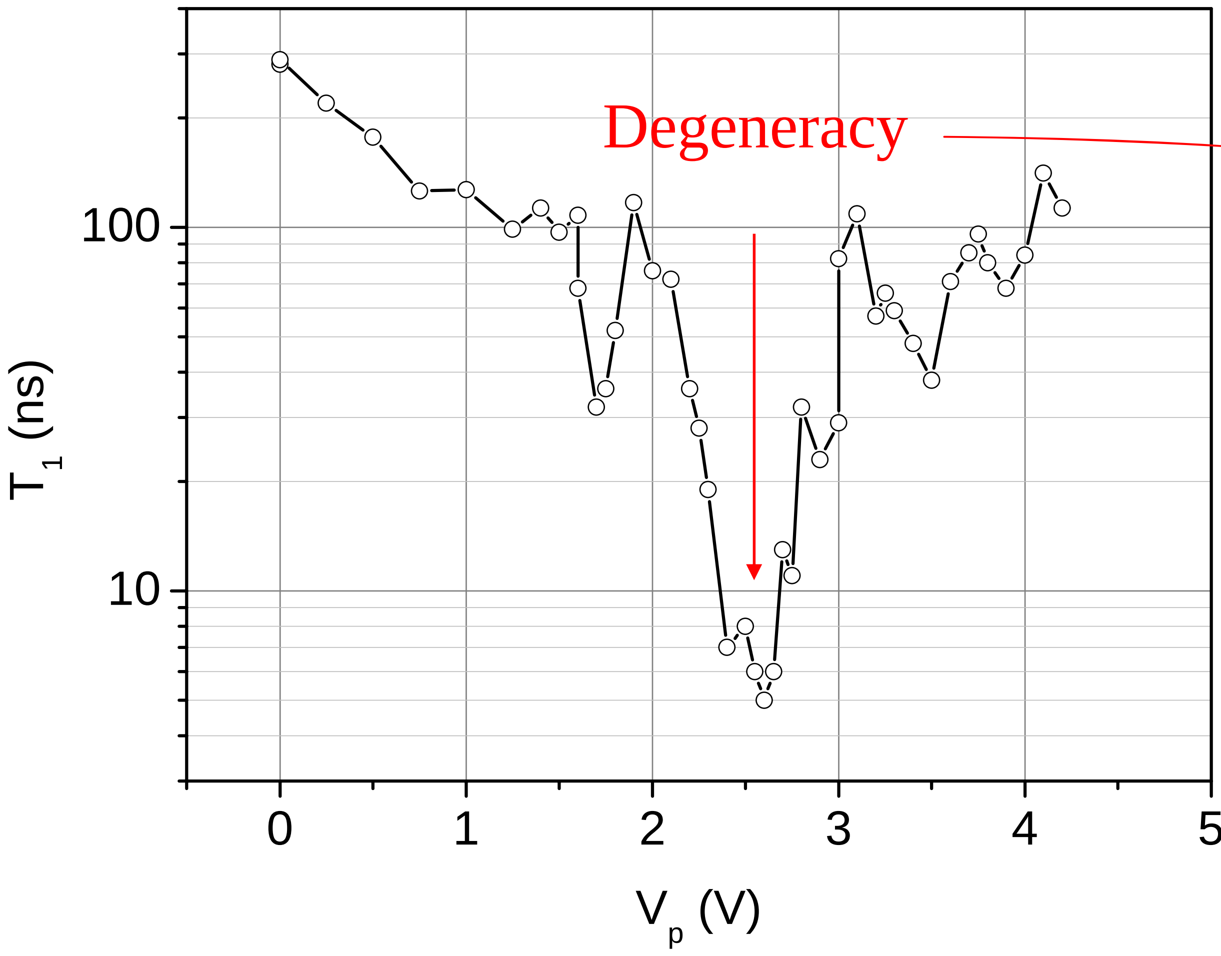
T_1 time measurements

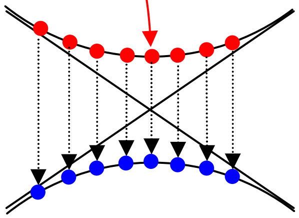


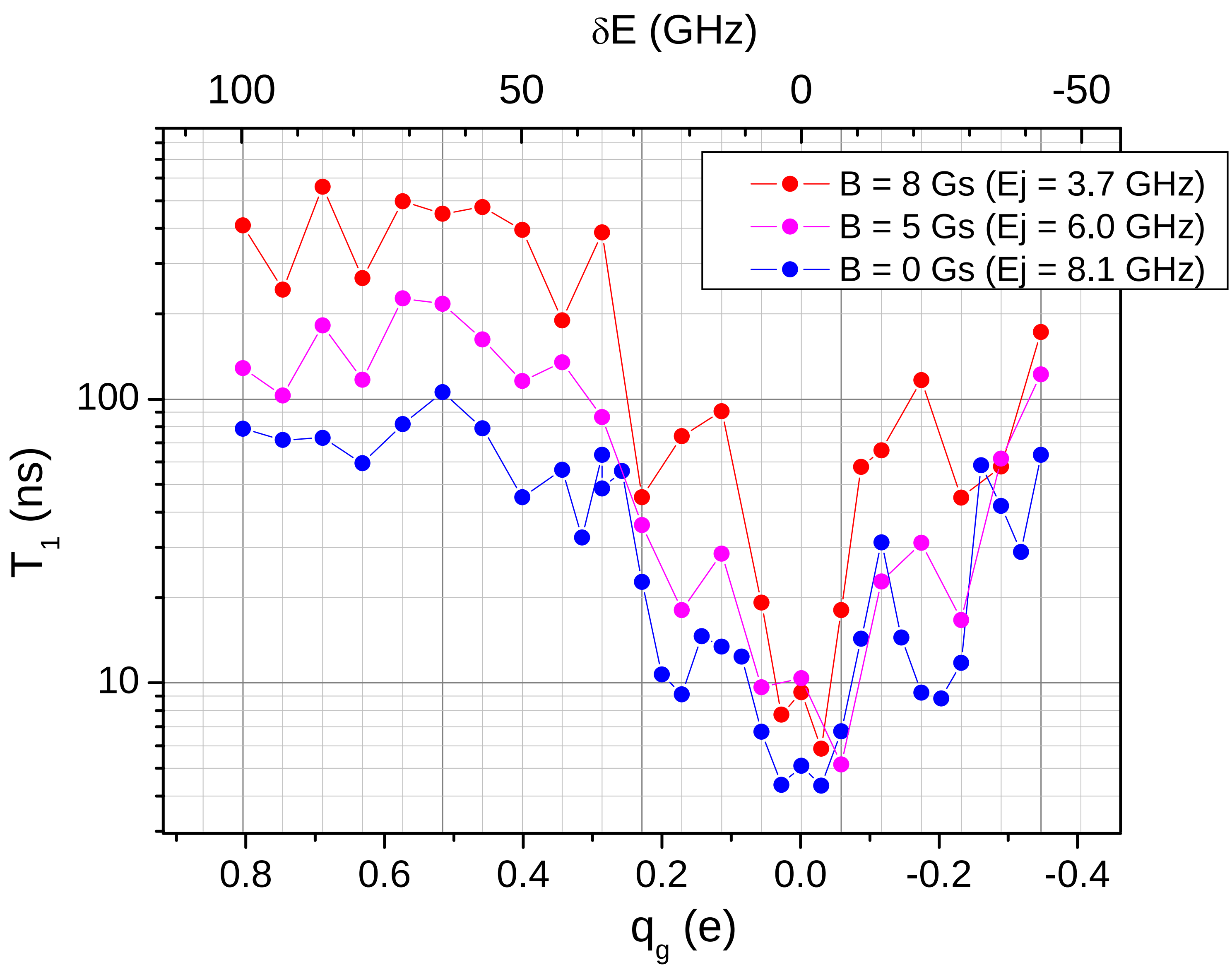




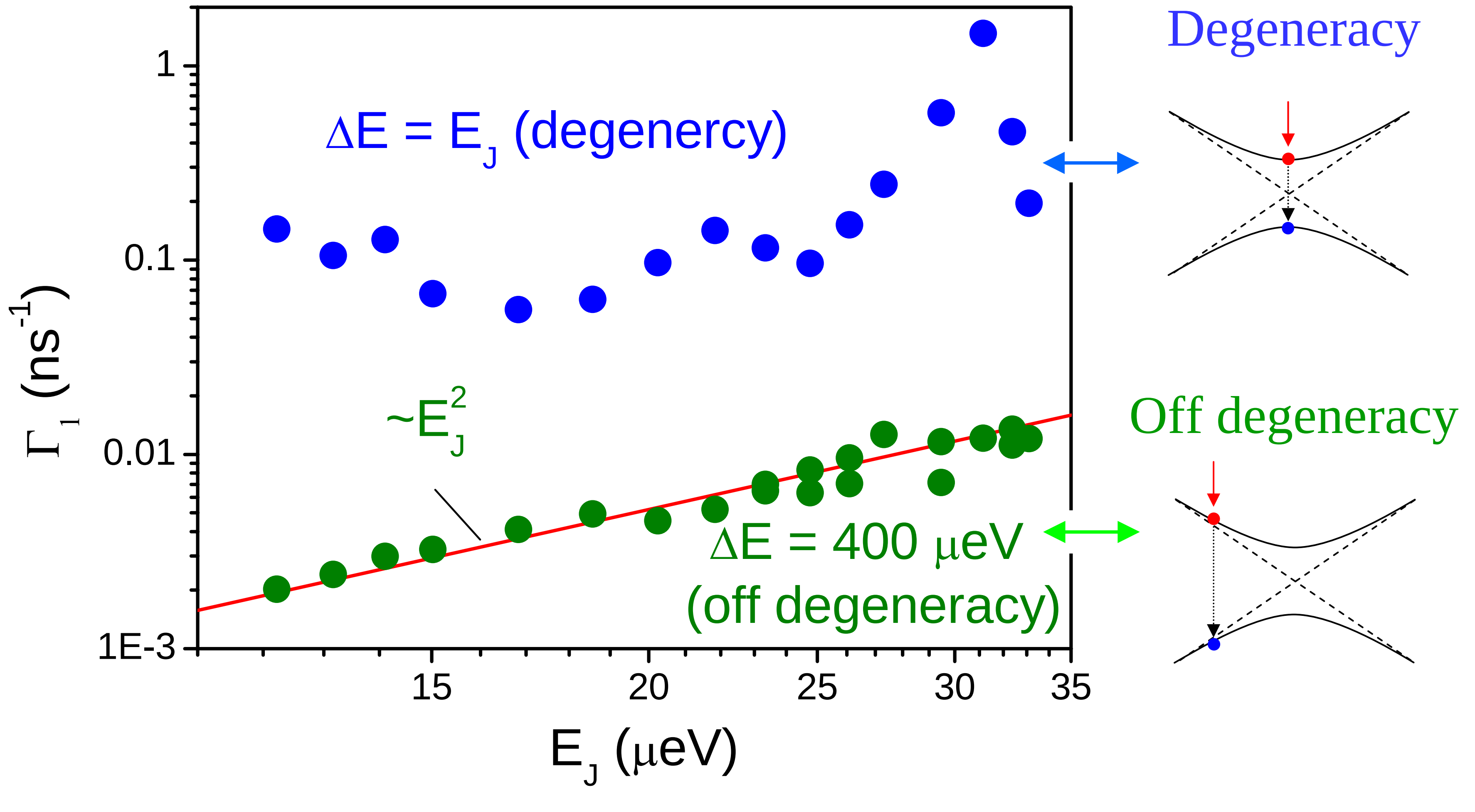
T_1 time vs Gate Voltage



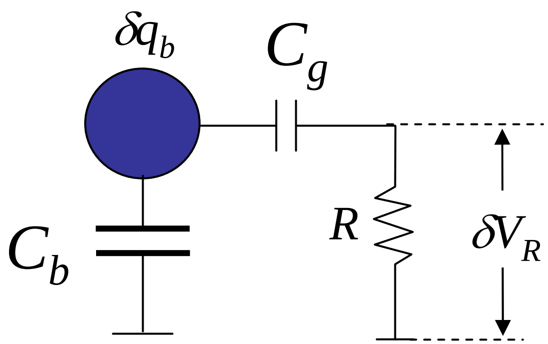



δE (GHz)
100
50
0
-50
B = 8 Gs (Ej = 3.7 GHz)
B = 5 Gs (Ej = 6.0 GHz)
B = 0 Gs (Ej = 8.1 GHz)
100
10
T1 (ns)
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
qg (e)

E_J-dependences



Qubit relaxation due to wiring and readout



Possible sources of decoherence:
- Absorption by resistor
- Noise of readout current

$$\delta q_{Cb} = C_g \delta V_R \qquad \delta U_{Cb} = \frac{E_C}{e} C_g \delta q_R$$

$$S_U(\omega) = \left(\frac{E_C}{e}\right)^2 C_g^2 S_V = 4e^2\kappa^2 S_V(\omega)$$

Ohmic environment

$$S_V^R = 2\,\mathrm{Re}(R)\eta\omega$$

Re(Z_b) = 1 Ω

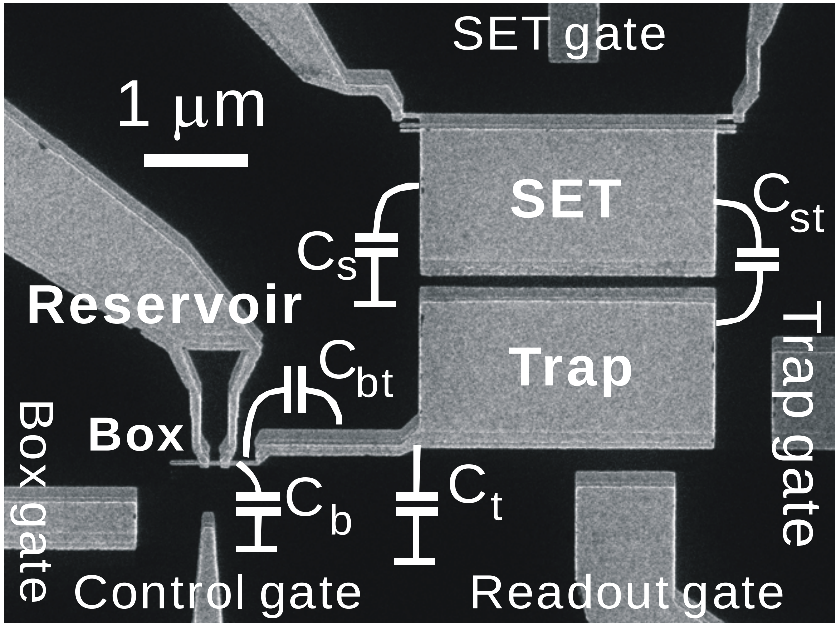
Γ_1 = (10 ns)$^{-1}$

f = 10 GHz

Shot noise of the readout SET

$$S_V^R = \frac{2Ie}{(\omega C_{SET})^2}$$

Coupling to Environment through Electrical Leads



Gates:

$$\kappa = \frac{C_g}{C_b} = \frac{1aF}{600aF} \approx 1.7\times10^{-3}$$

$$\mathrm{Re}(Z_b) \approx 3\times10^{-4}\,\Omega \qquad \Gamma \approx 30\,\mu s \quad f = 10GHz$$

SET:

$$\kappa = \frac{C_{bt}C_{ts}}{C_b C_t} = \frac{30aF\times100aF}{600aF\times1000aF} \approx 5\times10^{-3}$$

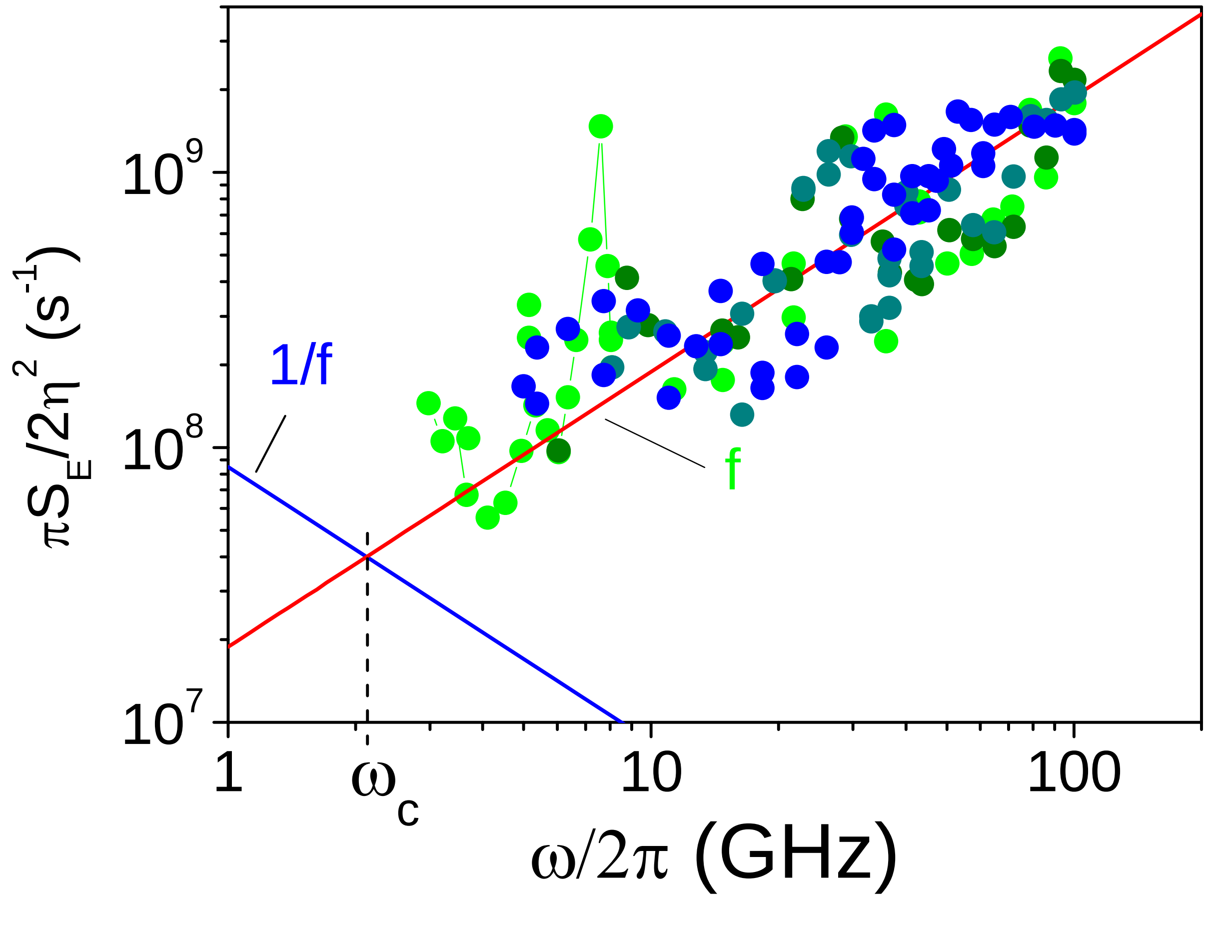
$$\mathrm{Re}(Z) < 10^{-1}\,\Omega \qquad \Gamma \approx 1\mu s$$

Measurement current of the SET: $I_{SET} = 100$ pA $\quad \Gamma \approx 1\mu s$

Measured relaxation time can not be explained by coupling to the external environment through electrical leads

The noise derived from T_1 time

$$\Gamma_1 = \frac{\pi S_U(\omega_0)}{2\hbar^2}\sin^2\theta$$



$$T_c = \frac{\eta\omega_c}{k} \approx 100mK$$

Classical ⇔ Quantum Noise

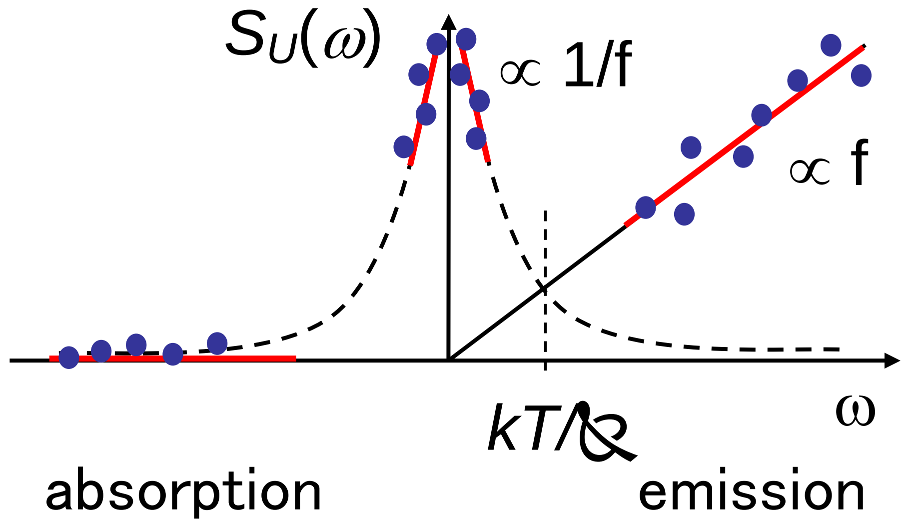
Do low frequency 1/f and high frequency f noises have common origin?

Quantum f-noise:

$$\beta\hbar\Theta(\omega)$$

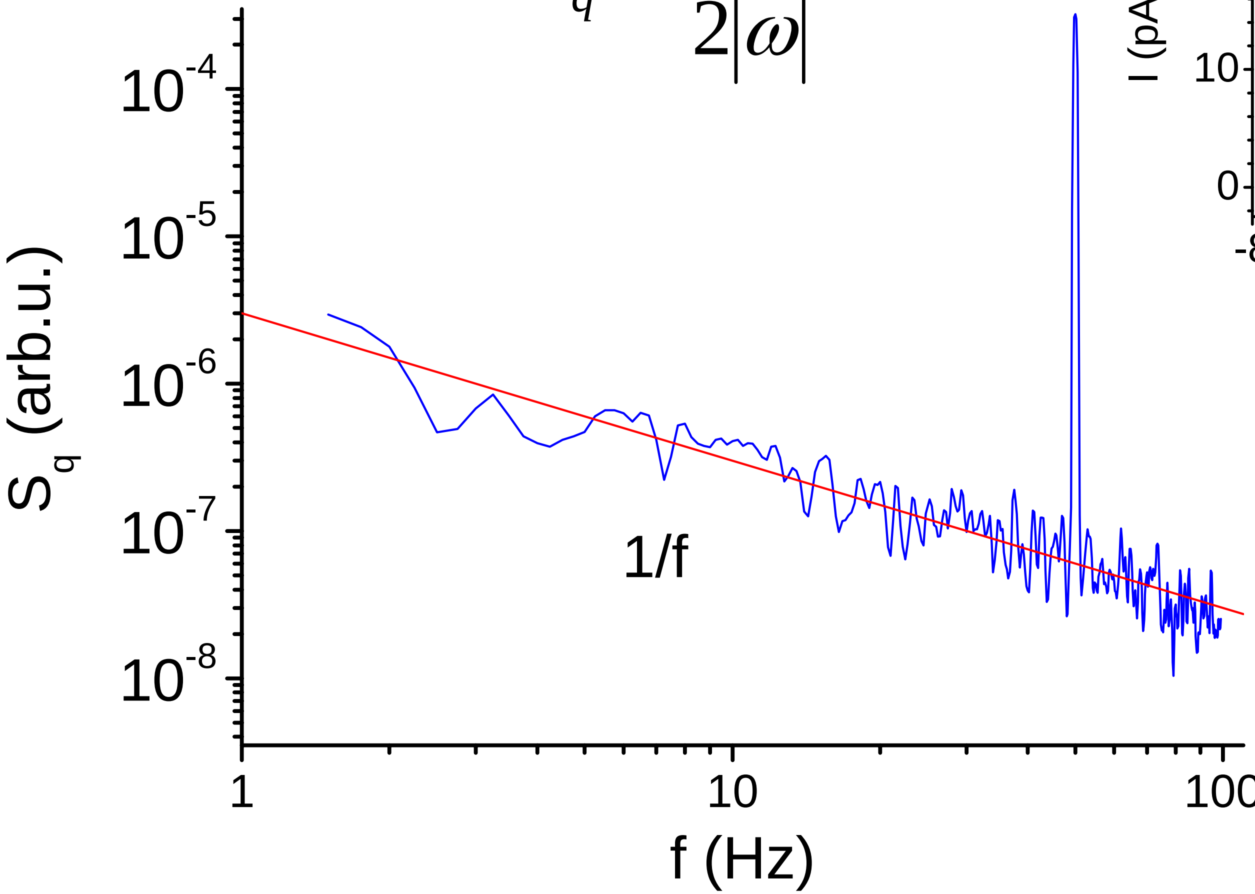
Classical 1/f-noise:

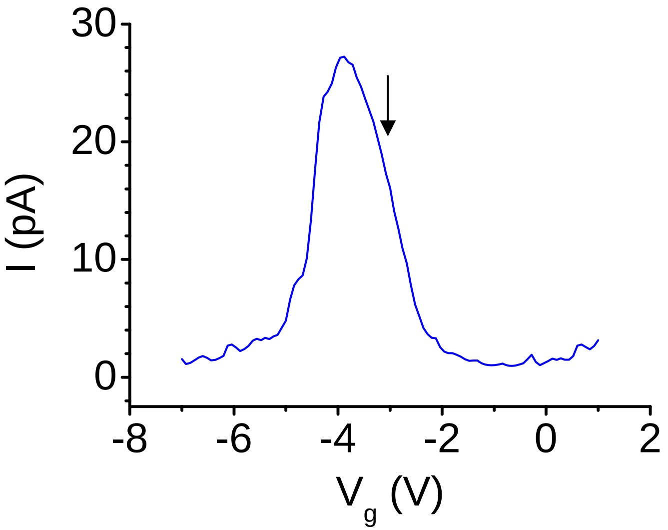
$$\frac{\beta (kT_{eff})^2}{\hbar|\omega|}$$



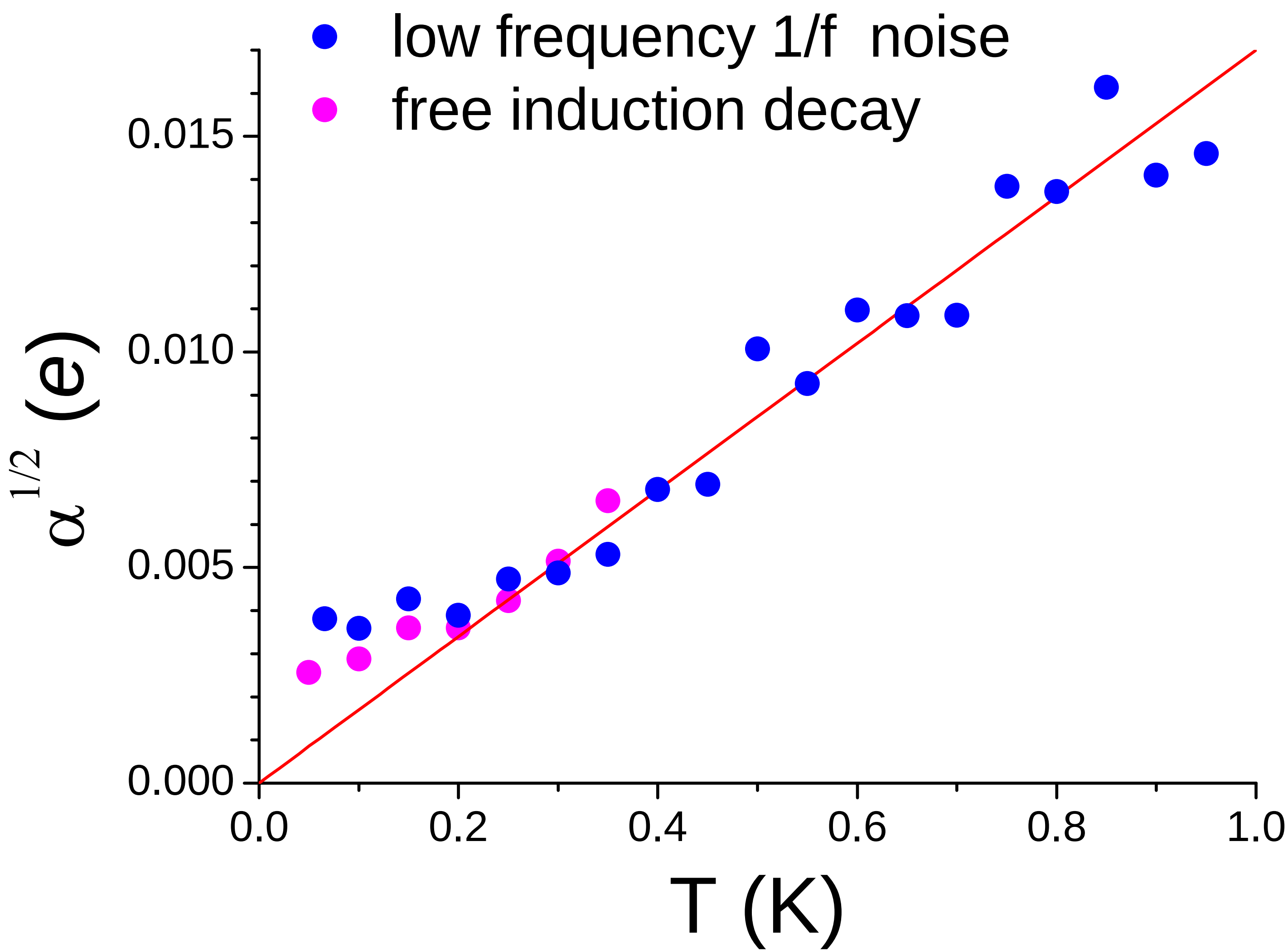
1/f charge noise measured by SET

$$S_q = \frac{\alpha}{2|\omega|}$$

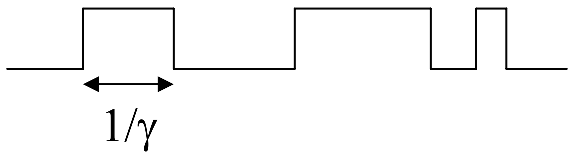




Temperature dependence of 1/f charge noise

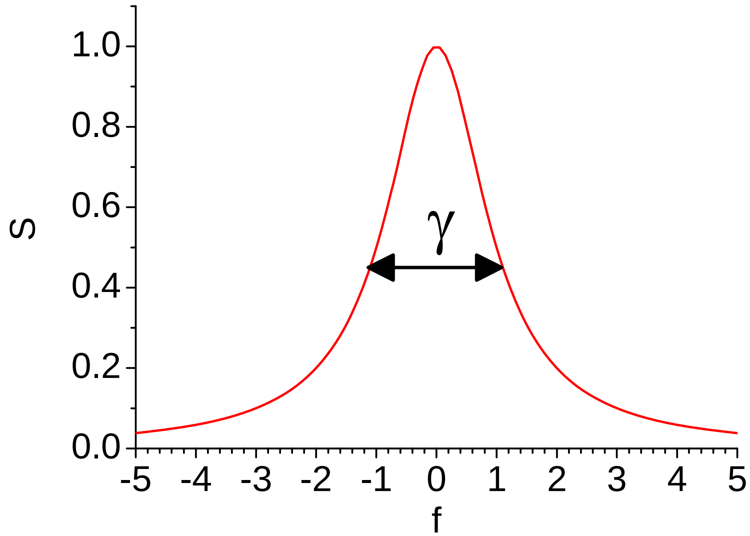


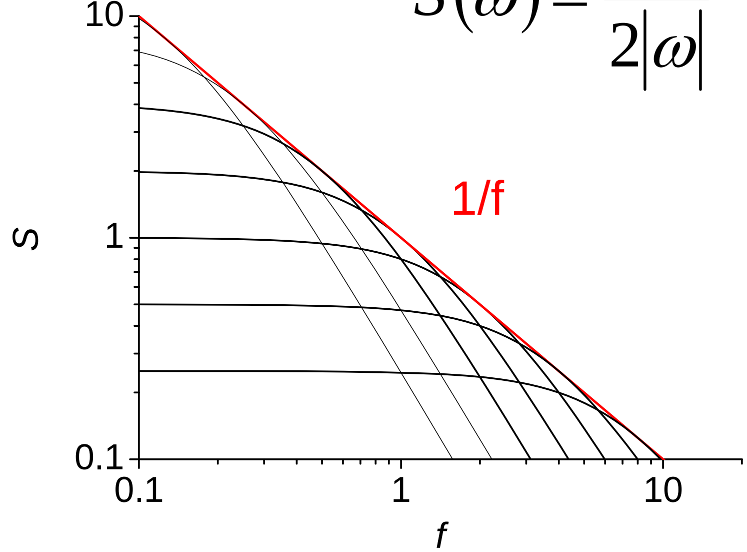
Basic properties of the 1/f noise



$$s(\omega,\gamma)=\frac{\gamma}{\gamma^2+\omega^2} \qquad S(\omega)=\int_{-\infty}^{\infty}P(\gamma)s(\omega,\gamma)d\gamma$$

$$S(\omega)=\frac{\alpha}{2|\omega|}$$





$$\gamma = \gamma_0 \exp\left[-\frac{\varepsilon}{kT}\right] \qquad \Rightarrow \qquad d\varepsilon = -kT\,\frac{d\gamma}{\gamma}$$

Noise temperature dependences

$$P(\varepsilon) = P_0 \qquad \Rightarrow \qquad P_0 d\varepsilon = kTP_0\,\frac{d\gamma}{\gamma}$$

$$S(\omega) = \int_0^\infty P_0 s(\omega, \gamma(\varepsilon))d\varepsilon = kTP_0 \int_0^\infty \frac{d\gamma}{\gamma^2 + \omega^2} = \frac{2\pi}{\omega} kTP_0$$

T^2 dependence

$$P(\varepsilon) \propto \varepsilon \qquad Pd\varepsilon \propto (kT)^2 \ln\left(\frac{\gamma}{\gamma_0}\right)\frac{d\gamma}{\gamma}$$

$$S(\omega) \propto (kT)^2 P_0$$

Conclusion

- The qubit is a spectrometer of environmental noise
- Dephasing of the charge qubit is caused by charge fluctautors
- Energy relaxation caused by ohmic-like environment with *f* quantum noise
- Crossover of low frequency 1/*f* and high frequency *f* noises is *kT/h*
- T^2 dependence of the 1/*f* noise is observed