Decoherence in Metallic Conductors and Mesoscopic Devices (theory)

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Overview

- General concepts
- Bulk samples and devices with diffusive metallic transport
  - Quantum contributions to diffusive transport – weak localization
  - Mechanisms of decoherence
  - Spin effects
- Mesoscopic devices
  - Coherent properties of qubits

General concepts

Disorder versus interaction

e-e interaction

Electron gas → Fermi liquid → Wigner crystal

Superconductivity

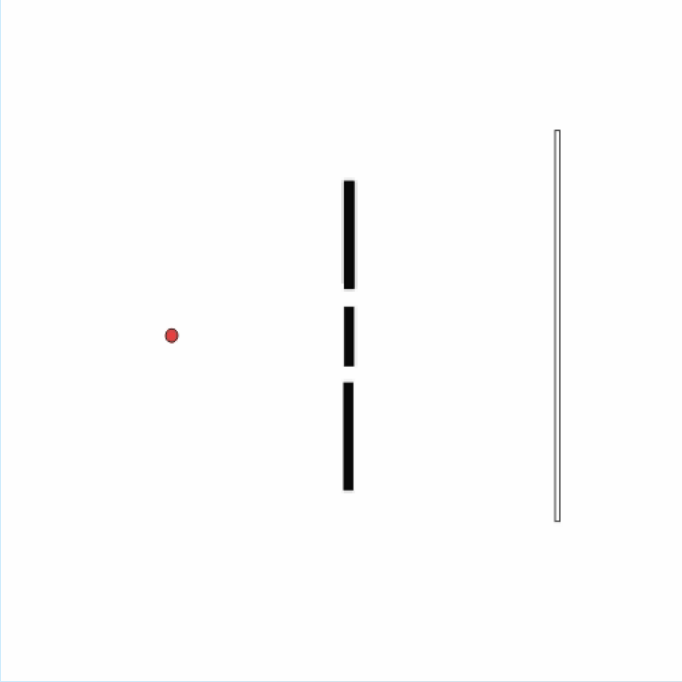
Weak localization → Disorder-enhanced e-e interaction

Strong (Anderson) localization

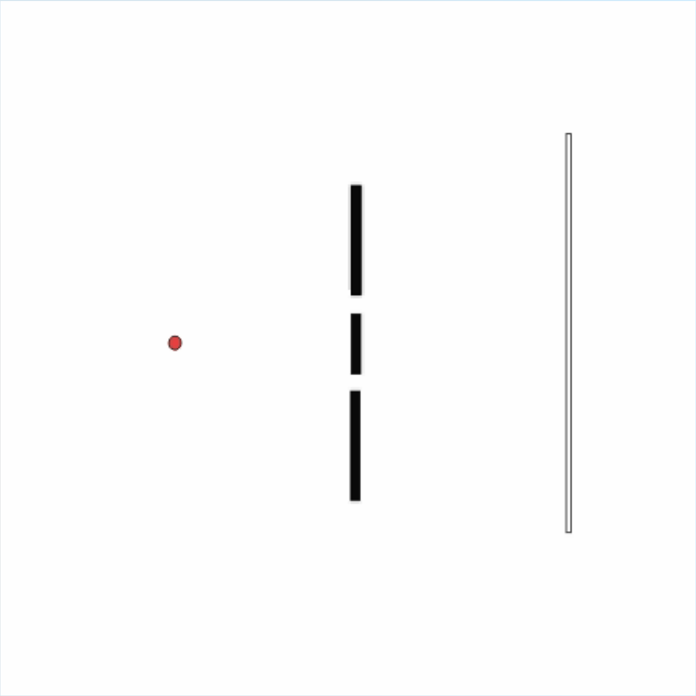
disorder

What is the phase coherence?

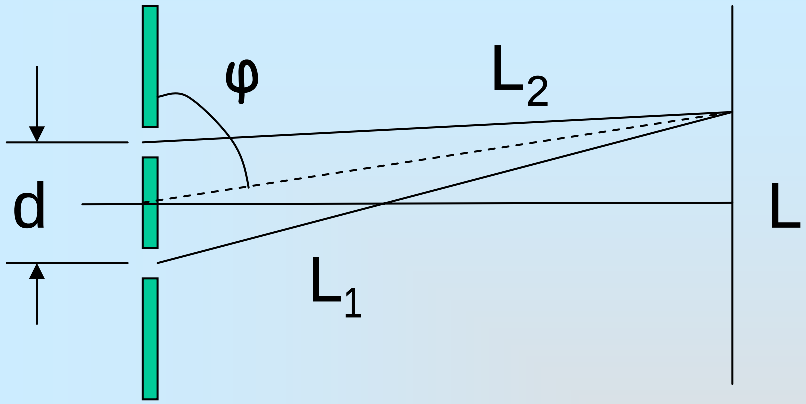
Two-slit experiment



Classical mechanics:
Particles



Quantum mechanics:
Waves



Sum of the signals:

$$\cos(kL_1+\chi_1)+\cos(kL_2+\chi_2)$$

$$=\boxed{2\cos\frac{k(L_1-L_2)+\chi_-}{2}}\cos\frac{k(L_1+L_2)+\chi_+}{2}$$

$$\chi_\mp=\chi_1\mp\chi_2$$

$$L_1-L_2\approx d\cos\varphi$$

$$2\cos\frac{kd\cos\varphi+\chi_-}{2}$$ Amplitude

What to we need to observe interference from two different sources?

Two signals are coherent if the phase difference, χ_- , is stable.

Bulk samples and devices with metallic conductance

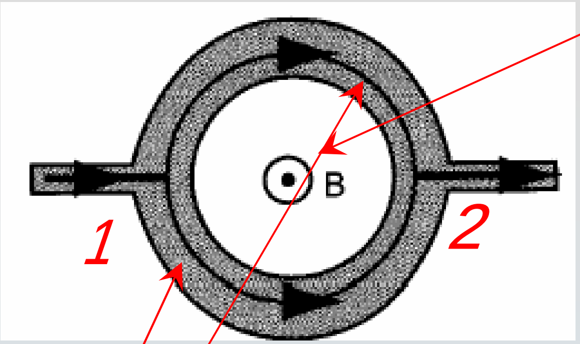
Phase coherence for electrons

Q: What is the difference between electromagnetic waves and charged quantum particles?

A: An important difference between electrons and electromagnetic waves is that electrons have a finite charge which interacts with magnetic field

To clarify this issue let us consider the Aharonov-Bohm effect, which would not exist in the absence of quantum interference

Aharonov-Bohm effect



Let us make a confined tube of magnetic field

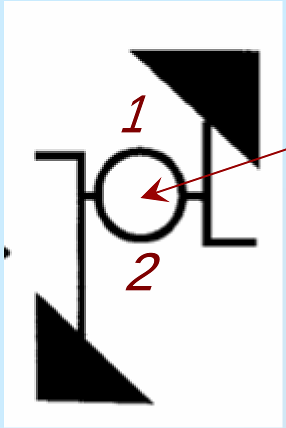
Will the interference pattern feel this magnetic field?

Wave function $\psi \propto e^{-i\vec{p}\vec{r}/\hbar}$

Phase gain between *1* and *2*: $\phi = \frac{1}{\hbar}\int_1^2 \vec{p}(\vec{r})\, d\vec{r}$

In magnetic field ⟶ $\vec{p} \to \vec{p} + e\vec{A}$

Additional phase difference between the paths: $\frac{e}{\hbar}\oint \vec{A}\, d\vec{r} = \frac{e}{\hbar}\Phi$



Φ - magnetic flux embedded in the ring.

Transition probability, *T*, is the squared modulus of the transition amplitude, *t*

$$T = |t_1 + t_2|^2, \qquad t_{1,2} = t_0 e^{i\varphi_{1,2}}$$

$$T = 2t_0^2 \left[1 + \cos(\varphi_1 - \varphi_2)\right]$$

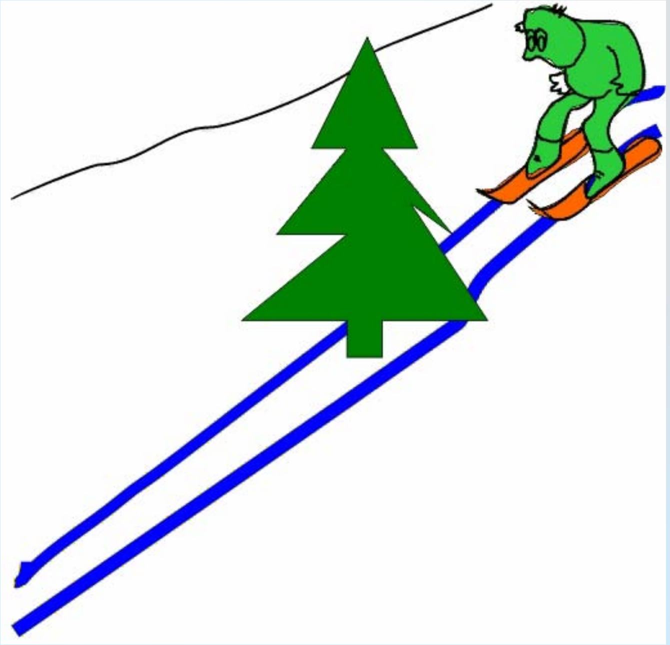
Due to the finite electron charge,

$$\varphi_1 - \varphi_2 = 2\pi \frac{\Phi}{\Phi_0} + \chi_-, \qquad \Phi_0 = \frac{e}{h}.$$

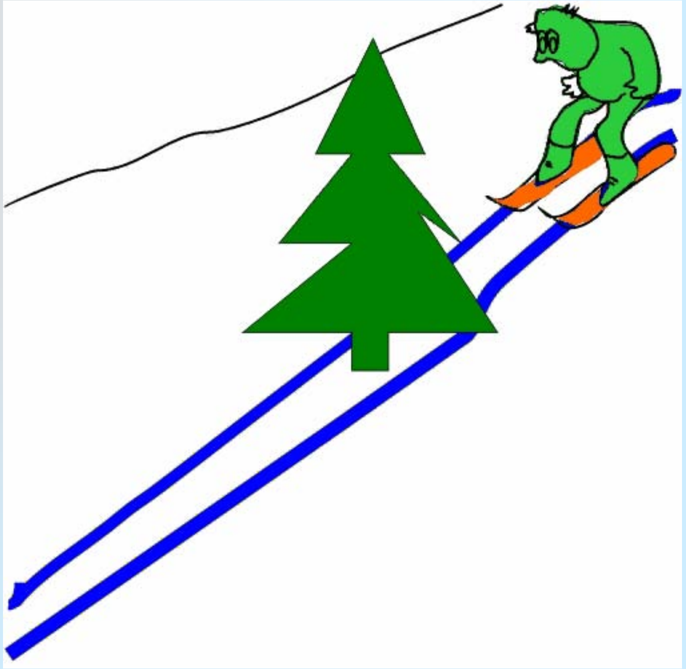
Thus, the conductance must periodically depend on magnetic field

$$G \propto \left[1 + \cos\left(2\pi \frac{\Phi}{\Phi_0} + \chi_-\right)\right]$$

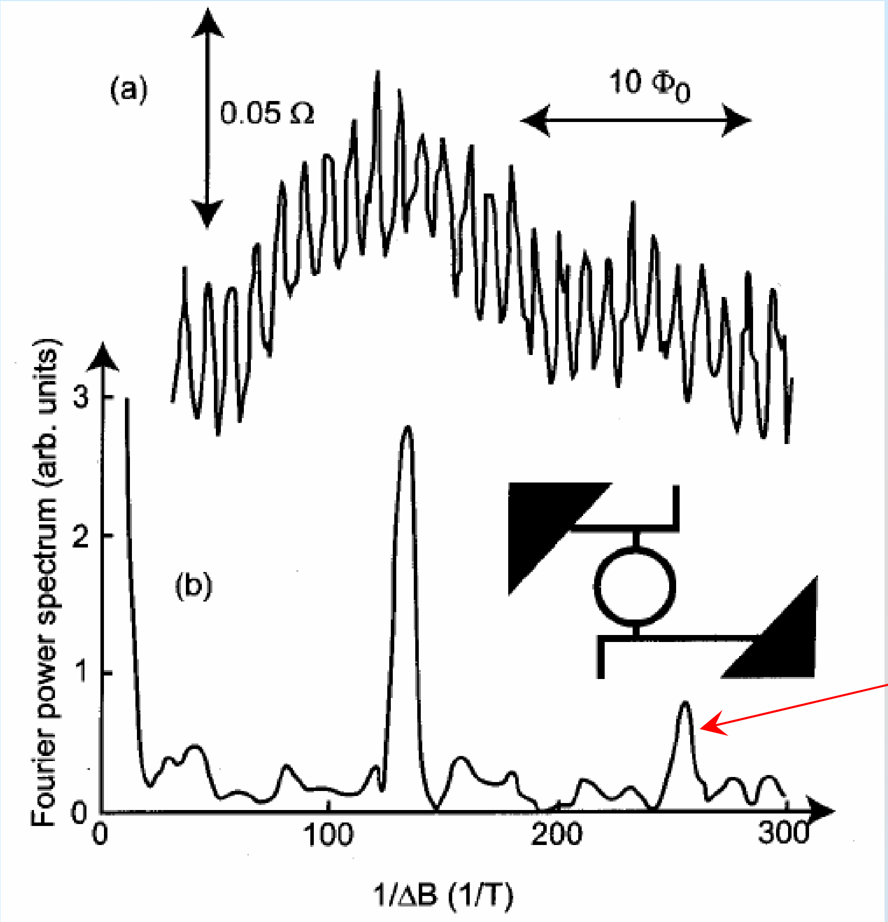
Aharonov-Bohm oscillations



Destructive and constructive interference



Experiment:



Aharonov-Bohm oscillations,

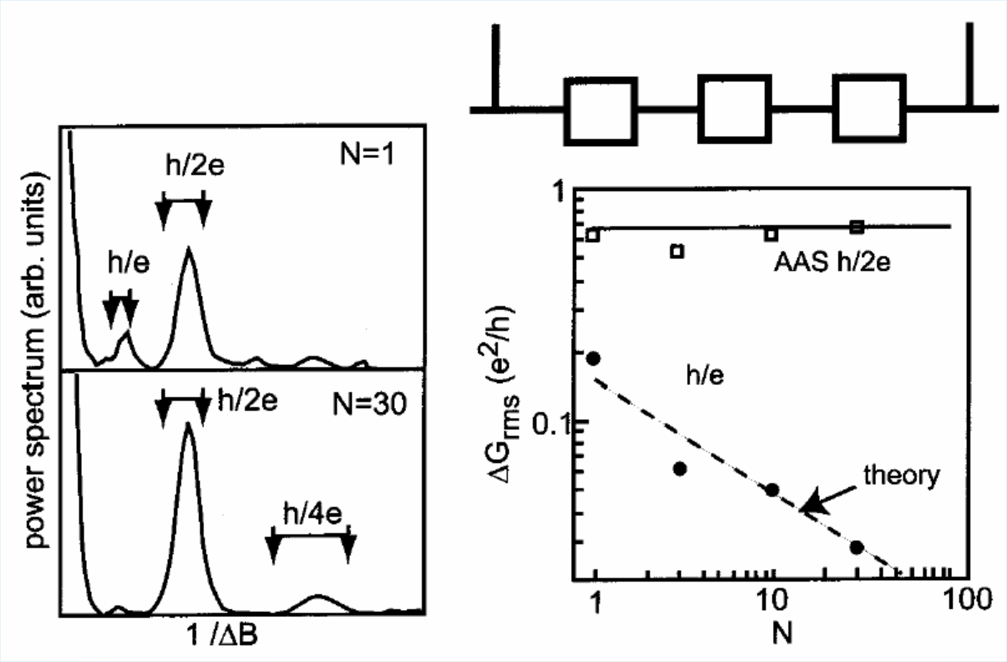
Webb 1985, Au

Fourier analysis shows that there are also weak oscillations with half period

Q: Will one observe the AB oscillations for a system of many rings connected in series? ?

Test of the ensemble averaging, Umbach 1986

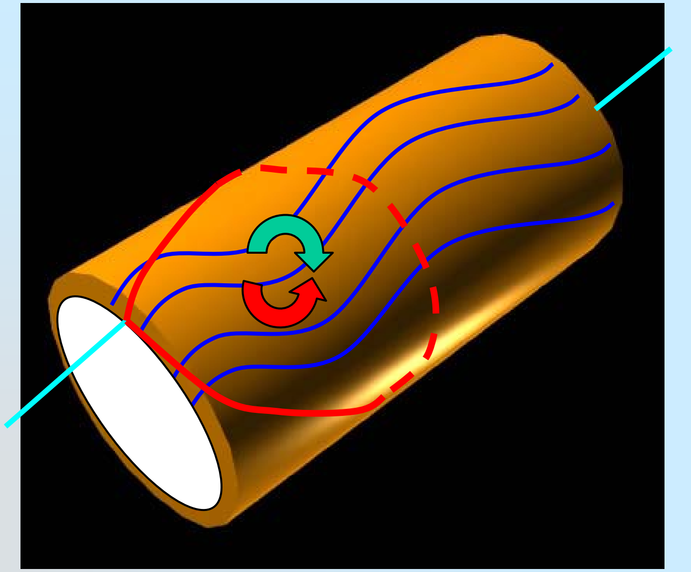
Ag loops, 940x940 nm^2, width of the wires 80 nm



Fourier series

N-dependence of the AB oscillations amplitude

Q: What happens in a long hollow cylinder?

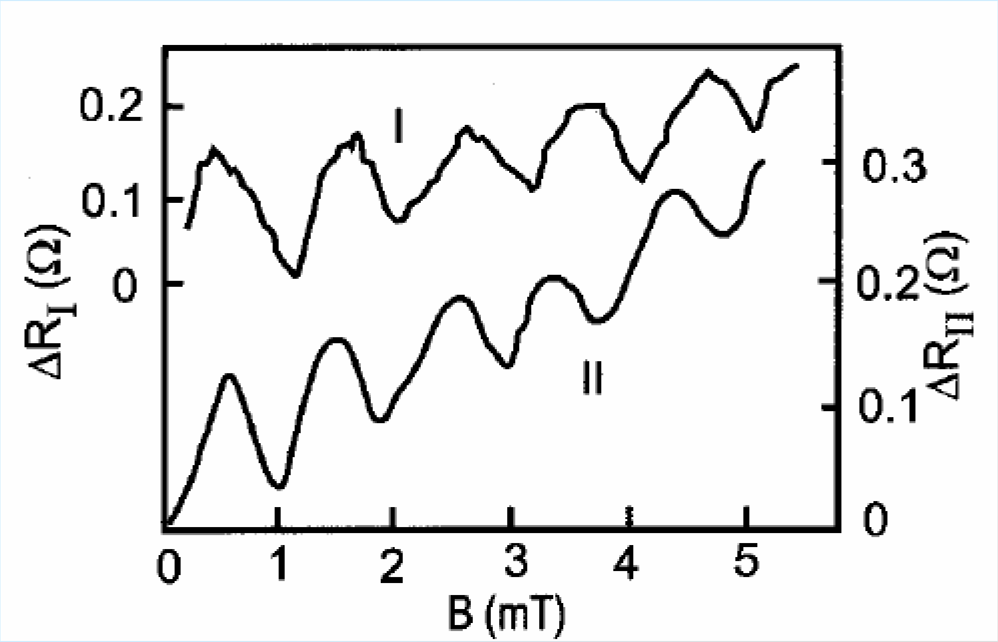


A: Amplitudes for different paths have different phases → interference disappears

But, there are closed loops, which can be propagated clockwise and counter-clockwise – they do interfere.

The clockwise and counter-clockwise paths are exactly the same → the backscattering increases.

Magnetic field destroys the interference, the period being a half of the period in the ring because the field-indices phase gains are opposite in sign.



Altshuler, Aronov, Spivak (AAS) oscillations

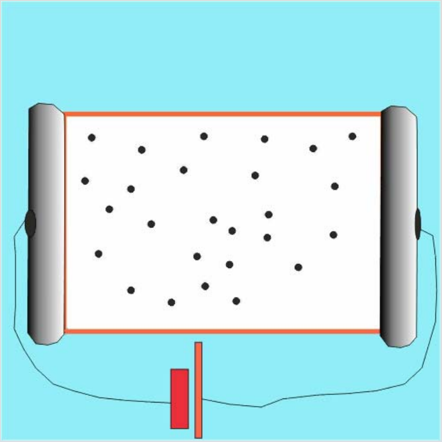
The period is $2\Phi_0$
Experiment by Sharvin, 1981, Mg-coated human hair

AB oscillations vanish in an ensemble of small rings since the phases χ_{-} are random.

In contrast, AAS oscillations survive ensemble averaging.

Weak localization in diffusive transport

The probability for an electron to move from point *1* to point *2* during time *t* in terms of the transition amplitude, A_i, along different paths.



$$P(t) = |\sum_i A_i|^2 = \sum_i |A_i|^2 + 2\Re \left(\sum_{i \neq j} A_i A_j^* \right)$$

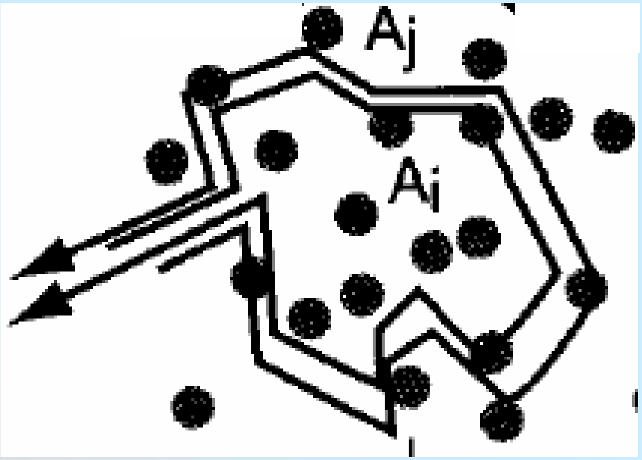
Classical probability

Interference contribution

Vanishes for most paths since phases are almost random

Consider now a close loop with 1=2.

Then the amplitude A_j is just a time reversal of A_i. Hence



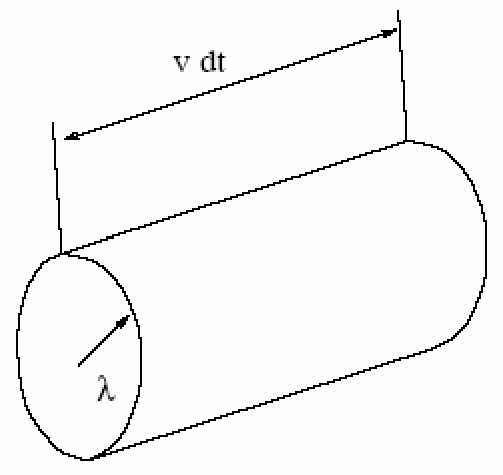
$$|A_i + A_j|^2 = |A_i + A_i^*|^2 = 4|A_i|^2$$

The backscattering probability is enhanced by factor 2!

This is a predecessor of localization.

This effect is called the weak localization since the relative number of closed loops is small.

However, the effect is very important since it is sensitive to very weak magnetic fields.



Let us calculate the probability for an electron to return to the "interference volume" during the time dt.

One obtains:

Interference volume

Volume of diffusive trajectory

$$\frac{\boxed{v\lambda^2\,dt}}{\boxed{(Dt)^{d/2}b^{3-d}}}$$

Thus, the relative quantum correction is

$$\frac{\Delta\sigma}{\sigma} \sim -\int_\tau \frac{v\lambda^2\,dt}{(Dt)^{d/2}b^{3-d}}$$

D – diffusion constant, b – thickness, d - dimensionality

decoherence time

Diverges for *d=2* !

In 2D case, introducing the sheet conductance $G = \sigma b$ we get

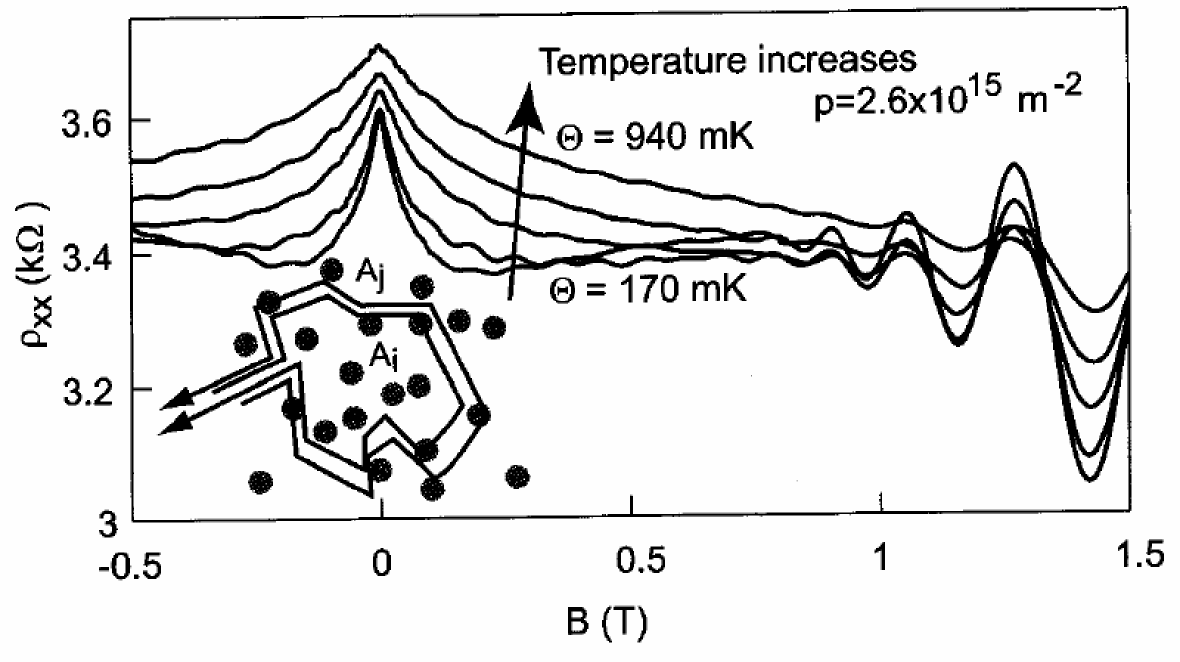
$$\delta G = -\frac{e^2}{\hbar} \ln \frac{L\varphi}{l_e}, \qquad L_\varphi = \sqrt{D\tau_\varphi}$$

This contribution is suppressed by very weak magnetic fields,

$$H \geq H_0 \sim \Phi_0/(D\tau_\varphi)$$

where bending of the trajectories by magnetic field is still not important

Anomalous magnetoresistance is a hall mark of the electron interference



Experiment:

Si/SiGe quantum well

Weak localization is a very important phenomenon – it allows find the decoherence time, spin-orbit interaction, etc.

Sources of decoherence

- Oscillations with the period $2\Phi_0$ are not affected by static disorder – this is why they survive averaging.
- Only the processes violating the time-reversal symmetry can contribute to decoherence.

  Among them are:
  - Magnetic field (in weak localization it switches off the destructive interference and the conductance increases);
  - Inelastic collisions;
  - Slowly varying non-stationary electrical or elastic fields (electrical fluctuations, low-frequency phonons, etc)
  - Spin degrees of freedom

Interference enhances electron-electron interaction

Let $|\xi|$ be the difference between the energies of 2 electrons.

Then they move coherently during the time $\hbar/|\xi|$ and the return probability is

$$\alpha_\xi = \int_\tau^{\hbar/|\xi|} \frac{v\lambda^2\, dt}{(Dt)^{d/2} b^{3-d}}$$

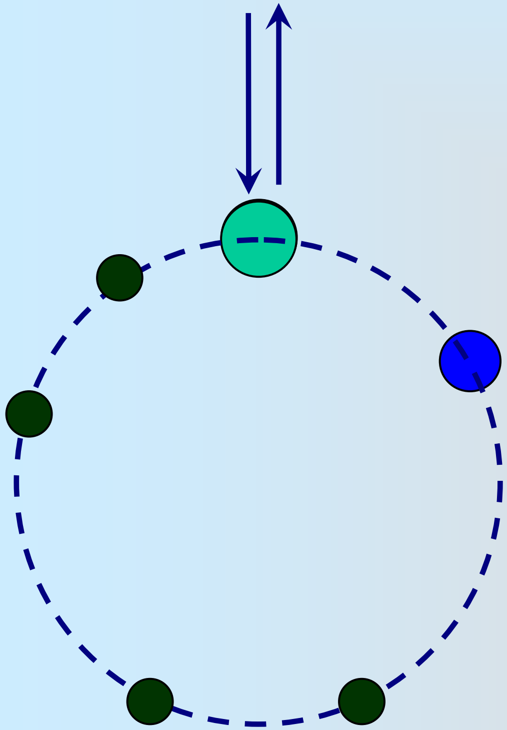
Result: $e_{eff}^2 = e^2(1+\alpha_\xi)$ → corrections to conductance dependent on temperature and dimensionality

(B. L. Altshuler & A. G. Aronov)

Comparison with weak localization:

- Interaction dominates in 3D case, has the same order in 2D case and less important in 1D case;
- Corrections differently depend on magnetic field

What is the difference between inelastic collisions and slow fields?



$$\varphi_\pm = \hbar^{-1}\int_{\mathcal{P}_\pm} \mathbf{p}\cdot d\mathbf{r}$$
$$= \hbar^{-1}\int_{\mathcal{P}_\pm} ds\,\sqrt{2m[\mathcal{E}-U(s,t)]}$$

Expanding in the potential energy of disorder, we get

$$\Delta\varphi_\pm = -\hbar^{-1}\int_{\mathcal{P}_\pm}\frac{ds}{v}U(s,t) = -\hbar^{-1}\int_{\mathcal{P}_\pm} dt\,U(s_t,t)$$

s_t is the electron's coordinate on the trajectory

Clockwise

$$(\Delta\varphi)_+ = -\frac{1}{\hbar}\int_0^{t_0} dt\, U(s_t, t)$$

Counter-clockwise

$$(\Delta\varphi)_- = -\frac{1}{\hbar}\int_0^{t_0} dt\, U(s_{t_0-t}, t)$$

t_0 is the total time for an electron to traverse the trajectory

Phase difference $\Delta\varphi \equiv (\Delta\varphi)_+ - (\Delta\varphi)_-$

Its mean squared fluctuation

$$\overline{(\Delta\varphi)^2} = \sum_{\pm} \left[\overline{(\Delta\varphi)_\pm^2} - \overline{(\Delta\varphi)_\pm(\Delta\varphi)_\mp}\right]$$

can be expressed through fluctuations of the potential,

$$\int_0^{t_0} dt \int_0^{t_0} dt'\, \overline{U(s_{t_i}, t)\, U(s_{t'_k}, t')} \qquad i,k = \pm,\; t_+ \equiv t,\; t_- \equiv t_0 - t$$

Assume: different scattering events are uncorrelated

$$\overline{U(s_{t_\pm},t)U(s_{t'_\pm},t')} \propto \overline{U^2(s,t)}\,\delta(t-t'),$$

$$\overline{U(s_{t_\pm},t)U(s_{t'_\mp},t')} \propto \boxed{\overline{U(s,t)U(s,t_0-t)}}\,\delta(t+t'-t_0)$$

Thus the phase fluctuation depends of the correlation of the random potential at different times!

Denote $\overline{U(s,t)U(s,t')} \equiv \overline{U^2}\boxed{f(t-t')}$

$\overline{U^2} \equiv \overline{U^2(s,t)}, f(0)=1$

Correlation function

$\overline{U^2}$ can be absorbed into partial relaxation rates, τ_s^{-1} due to different sources of decoherence.

$$\boxed{\overline{(\Delta\varphi)^2} \propto \sum_s \int_0^{t_0} \frac{dt}{\tau_s}[1 - f_s(2t - t_0)]}$$

Features of decoherence

Static potential , *f=1* $\longrightarrow$ no decoherence

Phase jumps, *f -> 0* $\longrightarrow$ $\overline{(\Delta\varphi)^2} \sim t_0/\tau_{\mathrm{in}}$

Phase wandering due to slow dynamics of environment

Sources of decoherence in bulk samples

- Thermal noise
- Electron-phonon interaction
- Dynamic defects (flicker noise)
- Spin-induced effects

Thermal (Nyquist) noise

Altshuler, Aronov, Khmelnitskii, 1981

Random non-stationary electric fields produced by thermal fluctuations in the electromagnetic environment

Interaction: $U(\boldsymbol{r}_t, t) = e\boldsymbol{\mathcal{E}}(t) \cdot \boldsymbol{r}_t$

Spectral density (Nyquist) -> $\langle \mathcal{E}^2 \rangle_\omega = \dfrac{kT}{G(\omega)}$

conductance

2D -> $G(\omega) = \sigma \mathcal{L}^2, \; \mathcal{L} = \sqrt{D t_0}$

Self-consistent estimate of τ_φ:

$$\frac{1}{\tau_\varphi} \approx \frac{1}{\tau_{ee}} \sim \frac{kT}{\hbar}\frac{e^2}{\hbar G}$$

Circuit parameters can influence the noise spectrum

Electron-phonon interaction

Altshuler, Aronov, Larkin, Khmelnitskii, 1981; Afonin, Gurevich, Y.G., 1985

$$\tau_\varphi \sim \tau_{e-ph}\,\mathrm{max}\left\{1, (\bar{\omega}_{ph}\tau_{e-ph})^{-2/3}\right\}$$

Typical phonon frequency

In some cases the electron-phonon interaction can compete with thermal noise as a mechanism of decoherence.

At $\bar{\omega}_{ph}\tau_{e-ph} \ll 1$ phonons play mainly constructive role enhancing backscattering. Thus weal localization corrections can survive in clean systems.

Dynamic disorder

Any realistic systems contains defects, fluctuators, which can randomly switch between two metastable states.

In mesoscopic systems they create so-called random telegraph signals (RTS), i.e. switching between 2 states.

In larger systems RTS merge into noise with 1/f spectrum.

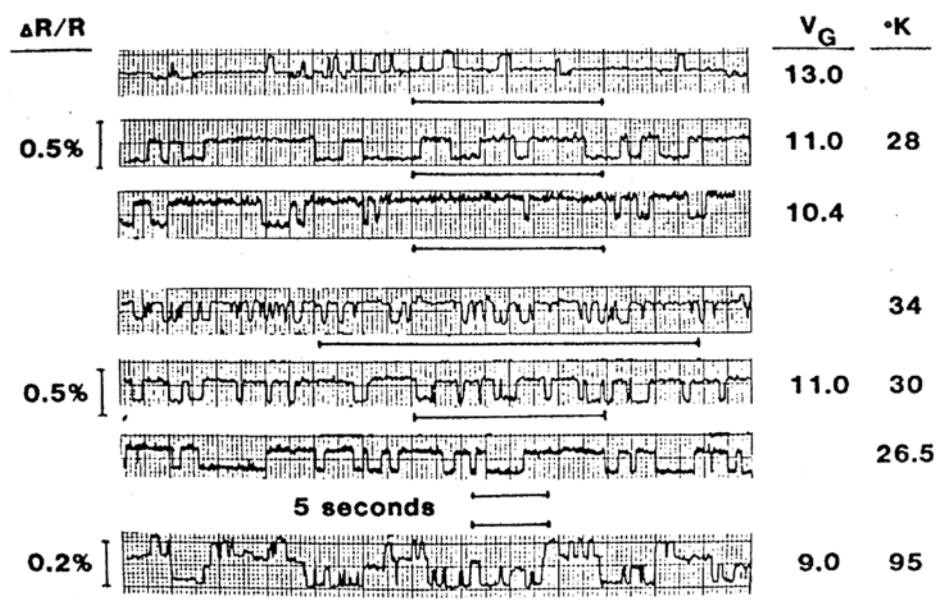
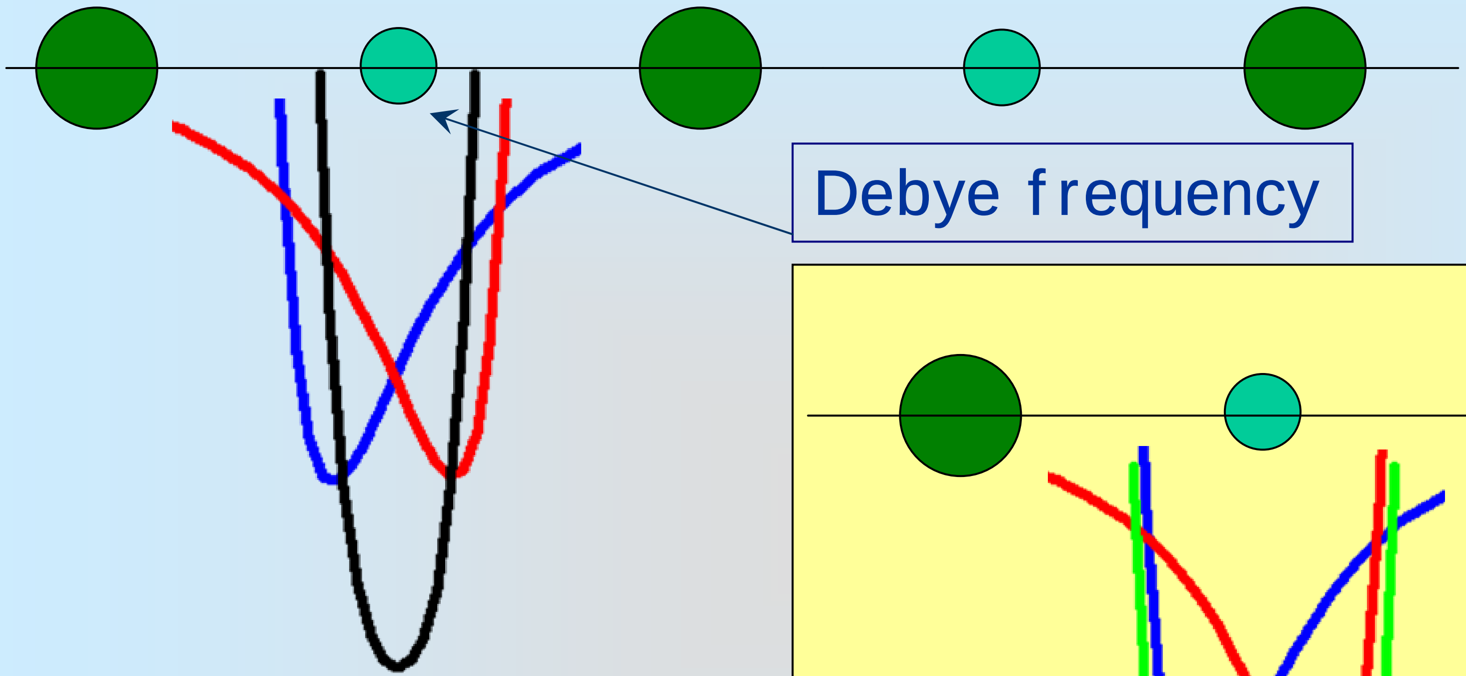
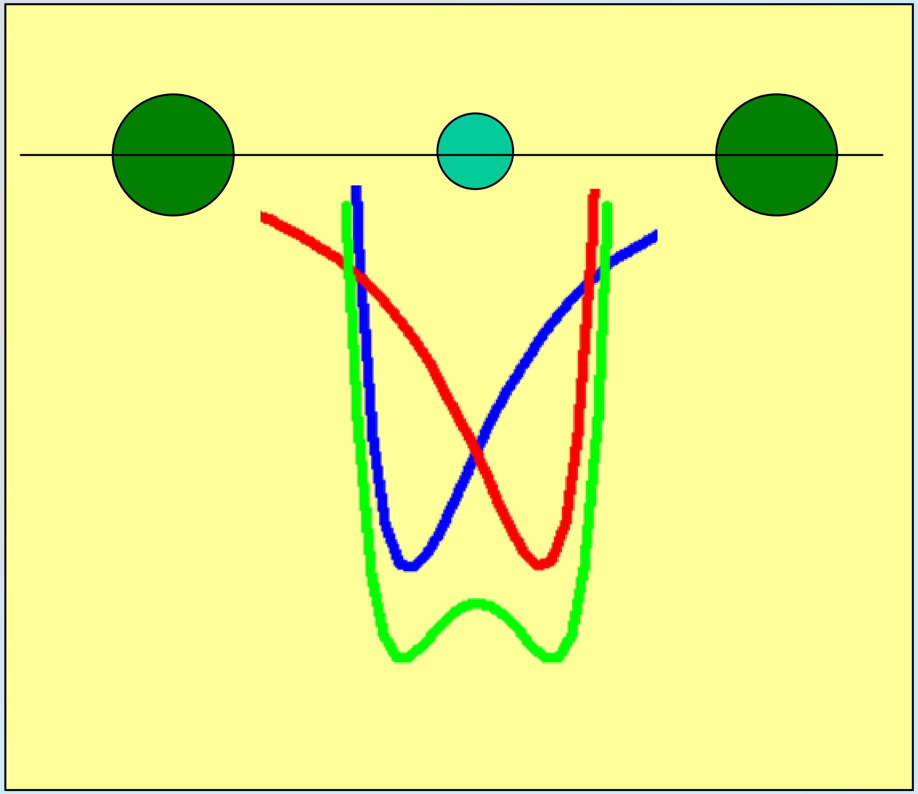


FIG. 5. Two-state switching in the voltage on small gated Si resistors, observed by Ralls *et al.* (1984). V_G is the gate voltage, and the right-hand column gives the temperature. Notice that at least two such switching sites are needed to explain the bottom trace.

From review by M. Weissman

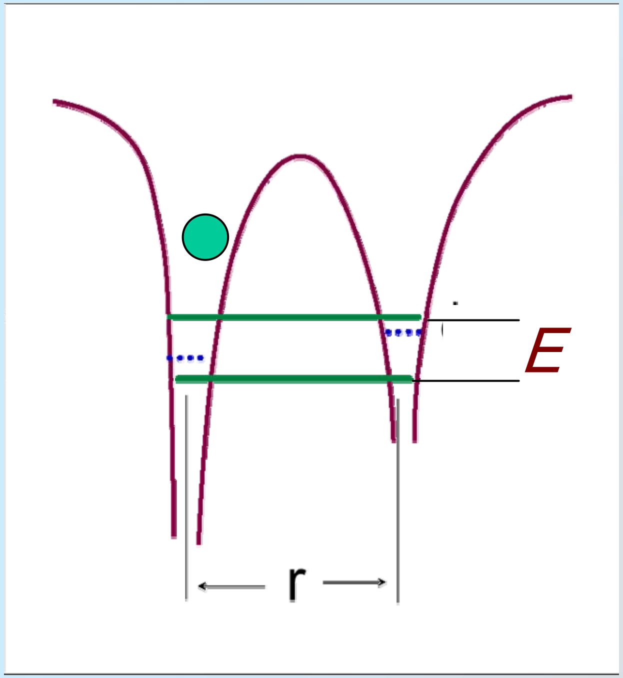
Dynamic defect in a one-dimensional chain – crude model





Strain creates defects with two metastable states

An example of phase wandering – coupling to a dynamic defect (two-level system)



Resonant interaction – direct transitions between the states due to inelastic electron scattering

for $kT \geq E$, $f(t) \approx \cos(Et/\hbar)$

Phase jumps:

$$t_0 \geq \tau_1 \gg \hbar/E, \qquad \tau_\varphi \approx \tau_1$$

Phase wandering: $t_0 \leq \hbar/E$

$$\Rightarrow \quad 1 - f(2t - t_0) \approx E^2(2t - t_0)^2/2\hbar^2$$

$$\Rightarrow \quad \overline{(\Delta\varphi)^2} \sim E^2 t_0^3/\hbar^2$$

Result for the resonant mechanism:

$$\boxed{\tau_\varphi = \max\{\tau_1, \tau_1^{1/3}(E/\hbar)^{2/3}\}}$$

Another mechanism is due to environment-induced relaxational dynamics of a TLS providing time-dependent scattering amplitude for electrons.

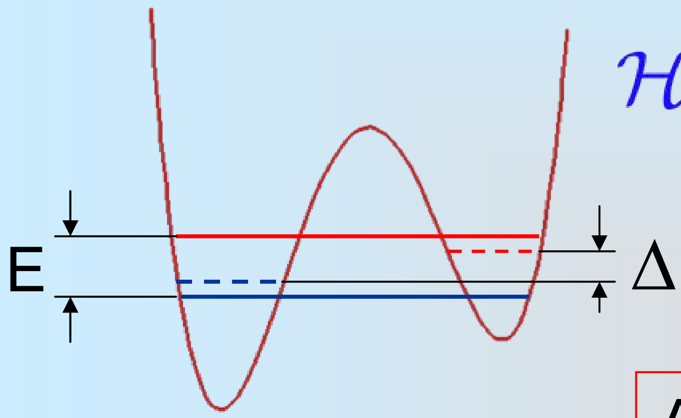
In this case,

$$f(t) - e^{-2\gamma|t|} \;\Rightarrow\; \overline{(\Delta\varphi)^2} \sim \gamma t_0^2/\tau_2$$

where γ is the typical defect hopping rate, while τ_2 is the partial relaxation time.

$$\boxed{\tau_\varphi = \max\{\tau_2, \tau_2^{1/2}(\tau_2/\gamma)^{1/2}\}}$$

Dynamics of fluctuators



$$\mathcal{H}_0 = \frac{1}{2}\begin{pmatrix} \Delta & 0 \\ 0 & -\Delta \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 0 & \Lambda \\ \Lambda & 0 \end{pmatrix}$$

$$\Lambda = \frac{\hbar\omega_0}{\pi} e^{-\lambda}$$

Attempt frequency

Tunneling action

Level splitting: $$E = \sqrt{\Delta^2 + \Lambda^2}$$

Interaction: $$\mathcal{H}_{\mathrm{int}} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \sum_{\mathbf{p},\mathbf{p}'} V_{\mathbf{pp}'} c^{+}_{\mathbf{p}'} c_{\mathbf{p}}$$

Scattering amplitudes in “left” and “right” states are different

Now we can diagonalize H_0: $\frac{\Delta}{2}\sigma_z - \frac{\Lambda}{2}\sigma_x \to \frac{E}{2}\sigma_z$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Pauli matrices

$$\mathcal{H}_{\mathrm{int}} \to \frac{V}{2}\left(\boxed{\frac{\Delta}{E}\sigma_z} - \boxed{\frac{\Lambda}{E}\sigma_x}\right), \quad V \equiv \sum_{\boldsymbol{p},\boldsymbol{p}'} V_{\boldsymbol{p}\boldsymbol{p}'} c^{+}_{\boldsymbol{p}'} c_{\boldsymbol{p}}$$

Change of the level spacing (wandering)

Inter-level transitions

Inter-level transition rate: $\gamma \propto \left(\frac{\Lambda}{E}\right)^2 \propto e^{-2\lambda}$

Maximal rate for given E and T

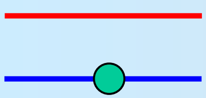
$$\gamma = \boxed{\gamma_0} \left(\frac{\Delta}{E}\right)^2$$

Fluctuations in the upper level's population, $n(t)$:

$$\frac{\partial n}{\partial t} = -\gamma(n - n_0), \;\; n_0 = \frac{1}{\exp(E/kT) - 1}$$

$$\Longrightarrow \langle(\delta n)^2\rangle_\omega = \boxed{\langle(\delta n)^2\rangle_0} \frac{\gamma}{\omega^2 + \gamma^2}$$

E > kT



$$\propto \frac{1}{kT} n_0(1 - n_0) = \frac{1}{4kT\cosh(E/2kT)}$$

In a macroscopic system there are many fluctuators with random interaction strengths, *v*, level splittings, *E*, and transition rates, γ, characterized by their distribution function, $\mathcal{P}(v, E, \gamma)$

Smooth distribution of tunneling parameters, λ, results in exponentially broad distribution of relaxation rates

$$\mathcal{P}(\lambda) = \text{const} \quad \to \quad \mathcal{P}(\gamma) \propto \boxed{1/\gamma}$$

The noise spectrum turns out to be of 1/f-type

$$S(\omega) \sim \int_0^{\gamma_0} \frac{\boxed{\mathcal{P}(\gamma)}\,\gamma\, d\gamma}{\omega^2 + \gamma^2} \sim \frac{1}{\omega} \text{ for } \omega \ll \gamma_0$$

Dynamic disorder produces 1/f noise in macroscopic systems.

Consequently, decoherence and 1/f noise are inter-related.

Spin-induced effects

Paths corresponding to different spin states can interfere -> spin flips destroy coherent motion

Situation with spin-orbit interaction is more complicated since spin-orbit interaction itself does not destroy the time-reversal symmetry.

Hikami, Larkin, Nagaoka, 1980

Anatoly I. Larkin

Spin-orbit interaction leads to deeply non-trivial effects since it changes symmetry of the problem

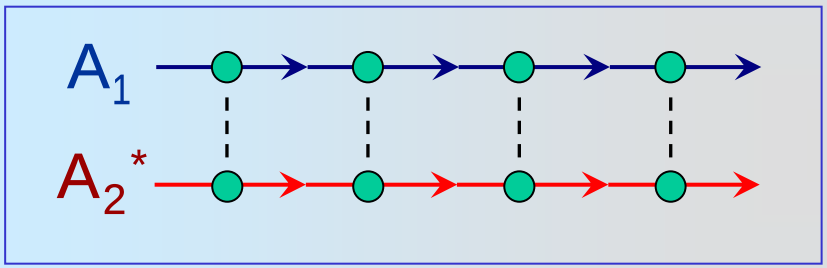
Spin-dependent scattering amplitude

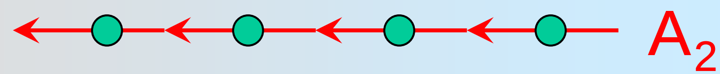
$$f_{\alpha\beta} = v\delta_{\alpha\beta} - iv_{so}(\boldsymbol{p} \times \boldsymbol{p}')\boldsymbol{\sigma}_{\alpha\beta} + v_s \boldsymbol{S}_m \cdot \boldsymbol{\sigma}_{\alpha\beta}$$

Spin indices

Spin-orbit

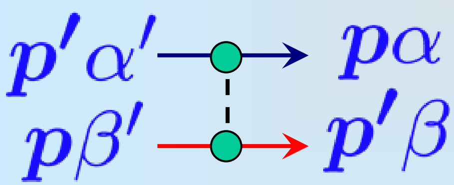
Spin-flip by *mth* magn. imp.





$$A_1 A_2^*$$

Building block



$$\Gamma^{(0)}_{\alpha\alpha'\beta\beta'} = n_i \overline{f_{\alpha\alpha'} f_{\beta\beta'}}$$

Average over impurity positions

After averaging over impurity positions, electron moments and impurity spins:

3D case

$$\overline{\Gamma^{(0)}}_{\alpha\alpha'\beta\beta'} = u^2\delta_{\alpha\alpha'}\delta_{\beta\beta'} + \frac{1}{3}(u_s^2 - u_{so}^2)\vec{\boldsymbol{\sigma}}_{\alpha\alpha'}\cdot\vec{\boldsymbol{\sigma}}_{\beta\beta'}$$

$n_i|v|^2$ $\quad$ $n_i|v_s|^2\langle S_m^2\rangle$ $\quad$ $n_i|v_{so}|^2\overline{(\boldsymbol{p}\times\boldsymbol{p}')^2}$

Total interference contribution has the same spin structure

$$\bar{\Gamma}_{\alpha\alpha'\beta\beta'} = A\left(\frac{3}{4}\delta_{\alpha\alpha'}\delta_{\beta\beta'} + \vec{\boldsymbol{\sigma}}_{\alpha\alpha'}\cdot\vec{\boldsymbol{\sigma}}_{\beta\beta'}\right) \leftarrow \text{triplet}$$

$$+B\left(\frac{1}{4}\delta_{\alpha\alpha'}\delta_{\beta\beta'} - \vec{\boldsymbol{\sigma}}_{\alpha\alpha'}\cdot\vec{\boldsymbol{\sigma}}_{\beta\beta'}\right) \leftarrow \text{singlet}$$

We have decomposed the interference contribution into two modes with different spin symmetry.

Singlet is unaffected by spin-orbit coupling, its decoherence time is determined just by spin-flip scattering from magnetic impurities, the characteristic rate being

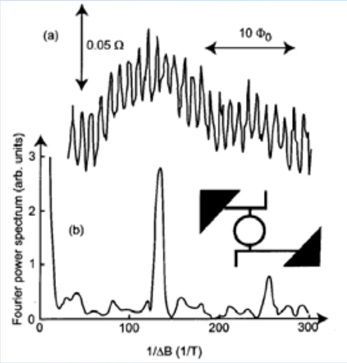
$$2/\tau_s = 4\pi N_m |u_s|^2$$

For the triplet component, the spin rotation along the time-reversed trajectory occurs in opposite sequence and opposite direction. Because of that there is positive contribution to magnetoresistance.

Conclusion to the first part

Coherent transport is extremely sensitive to

- external magnetic field
- inelastic scattering
- time-dependent fields
- spin-flip scattering



Weak localization in combination with non-equilibrium effects is a powerful tool revealing various quantum effects in electron transport