Universal coding for correlated sources with generalized complementary delivery

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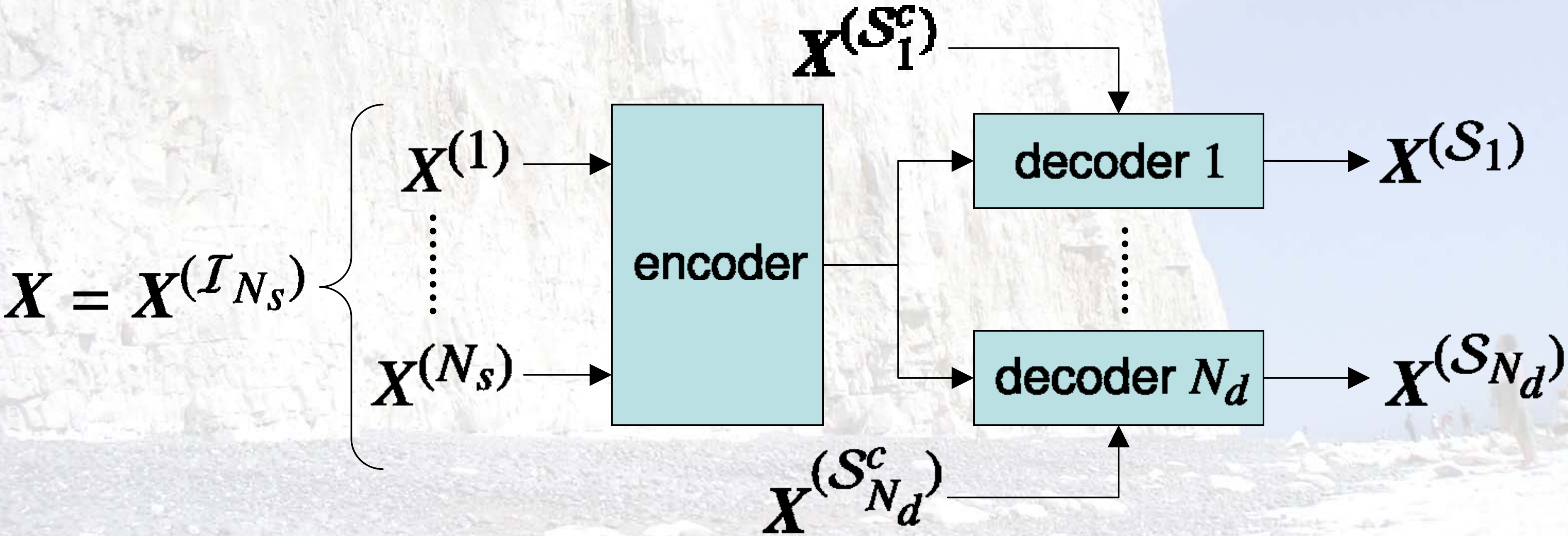
Abstract

- Universal source coding problem for multiterminal coding system
- Explicit constructions of universal codes
  - Fixed-length & variable-length lossless codes



Background

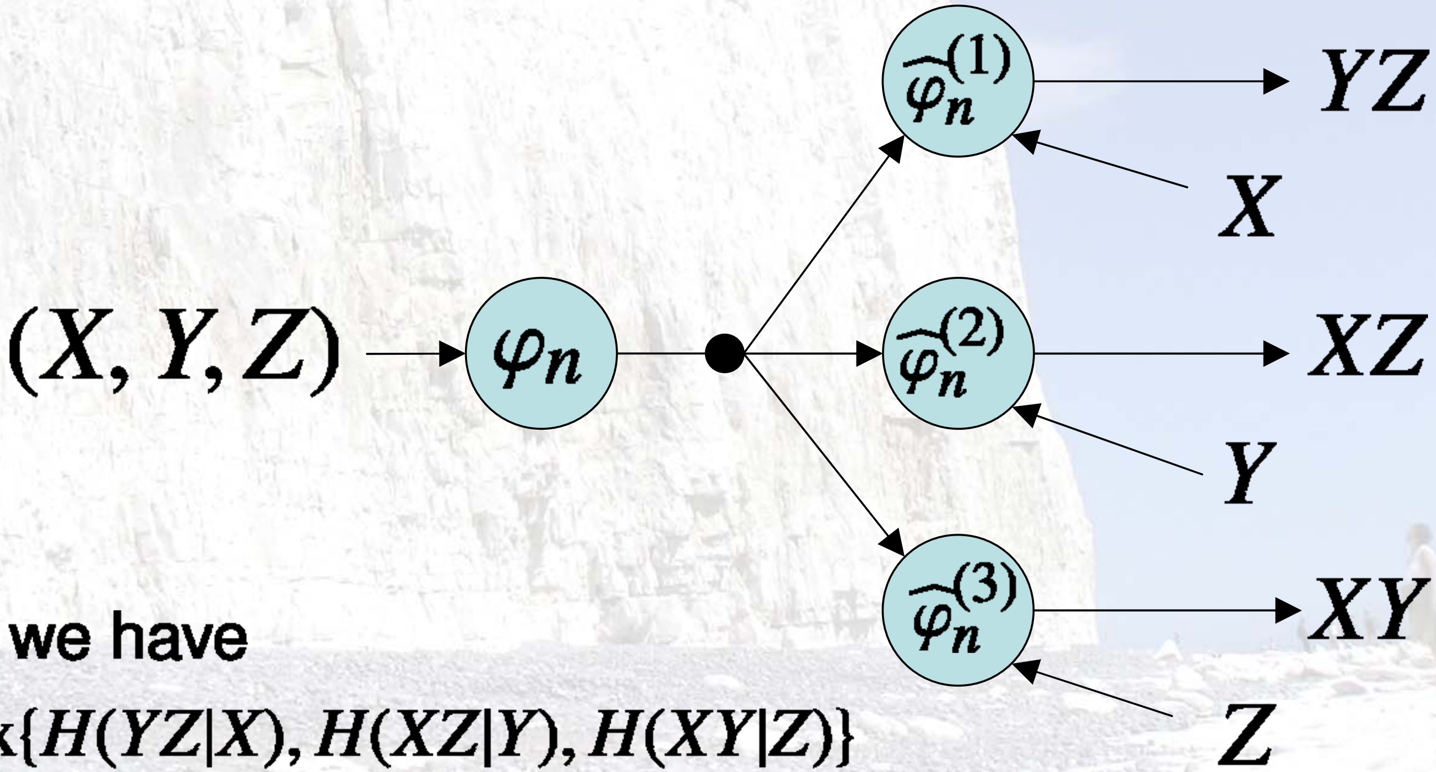
- Slepian-Wolf (1973):
  - beginning of multiterminal source coding
- Csiszar-Korner (1980):
  - first investigated problem of universal coding for Slepian-Wolf coding system
  - showed existence of universal codes

- Subsequent research focused on Slepian-Wolf coding system
  - Csiszar (1982), Ahlswede-Dueck (1982), Oohama (1996), Uyematsu (2001)

Minimum achievable rate

Theorem (Coding theorem of fixed-length GCD codes).

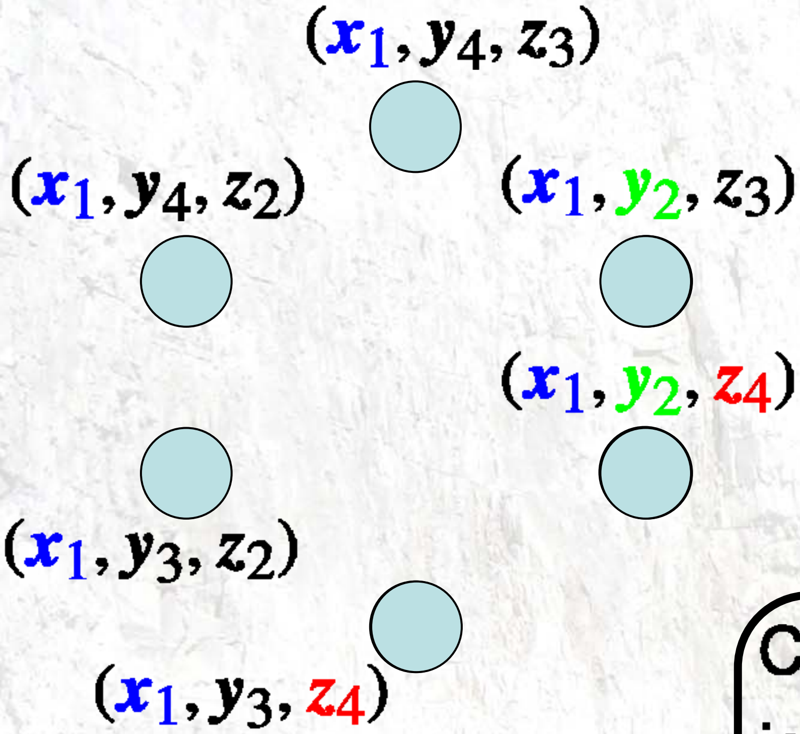
$$R_f(X) = \max_{1 \le j \le N_d} H(X^{(S_j)} | X^{(S_j^c)})$$



In this case, we have

$$R_f(X) = \max\{H(YZ|X), H(XZ|Y), H(XY|Z)\}$$

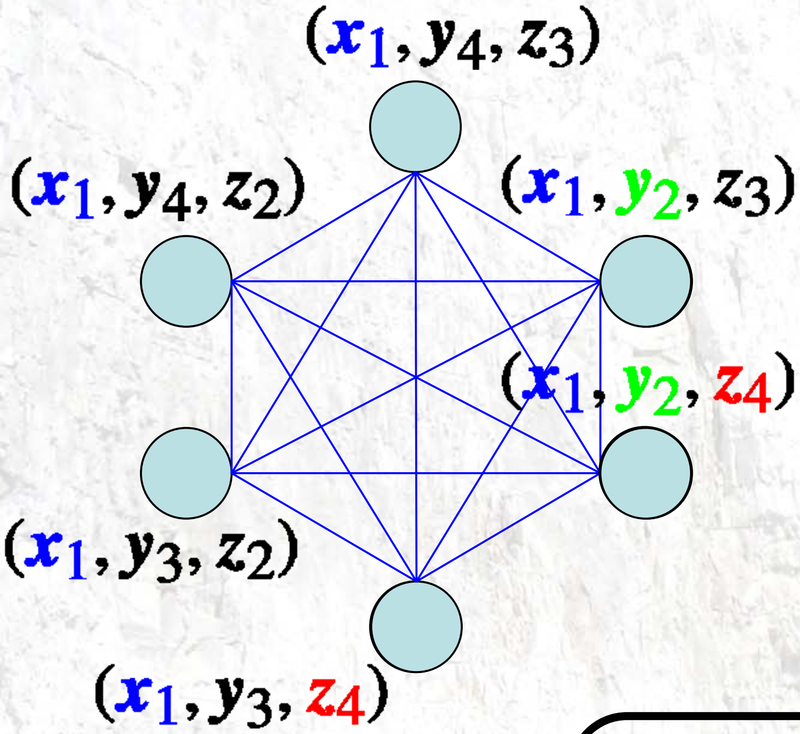
Code construction (1)



Create a coding graph for each joint type $Q_{XYZ} \in \mathcal{P}_n(\mathcal{X} \times \mathcal{Y} \times \mathcal{Z})$

- each node corresponds to a sequence set $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \in T^n_{Q_{XYZ}}$

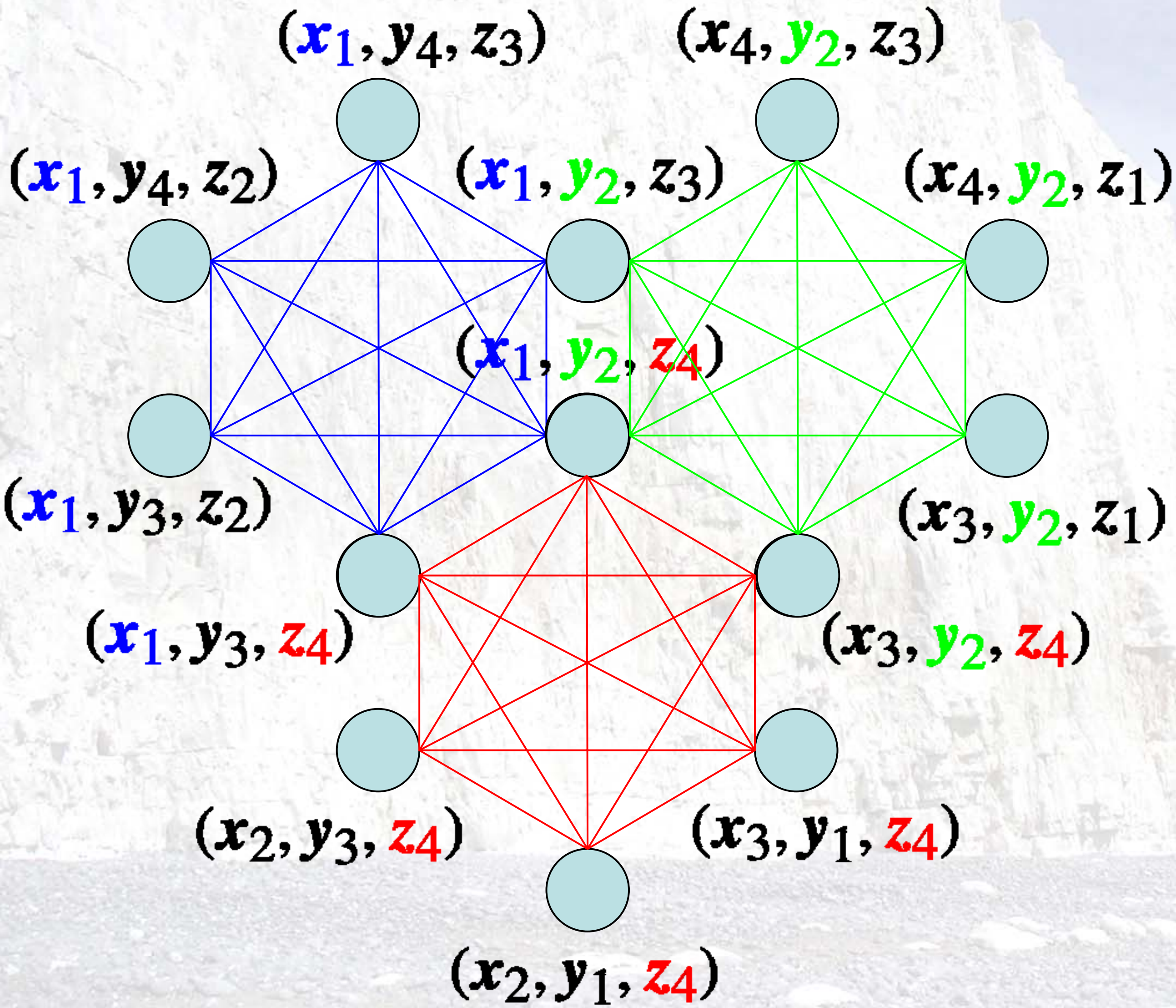
Code construction (2)



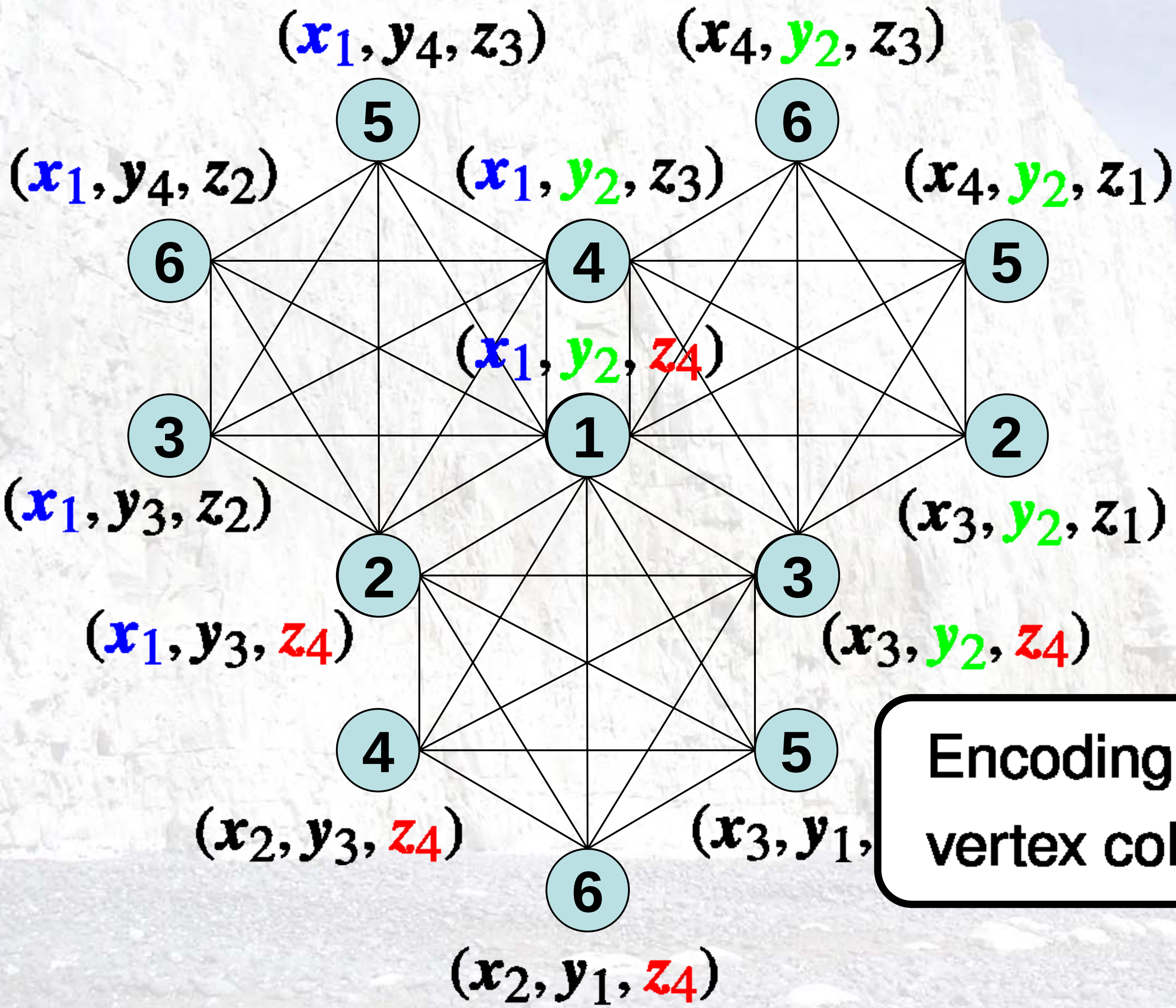
$$\left(\begin{array}{l} Q_{XYZ} = Q_X V_{YZ|X}, \\ Q_X \in \mathcal{P}_n(\mathcal{X}), \\ V_{YZ|X} \in \mathcal{V}_n(\mathcal{Y} \times \mathcal{Z} | Q_X). \end{array} \right)$$

For a given $\boldsymbol{x}_i$, we must distinguish each $(\boldsymbol{y}_j, \boldsymbol{z}_k)$ such that $(\boldsymbol{x}_i, \boldsymbol{y}_j, \boldsymbol{z}_k) \in T^n_{Q_{XYZ}}$.
(about $\exp\{nH(V_{YZ|X}|Q_X)\}$ sequences)

Code construction (3)



Code construction (4)



Encoding is equivalent to vertex coloring of the graph

Code construction (5)

Lemma (Brooks). *For any graph G except a complete graph and an odd cycle, the chromatic number (i.e. the number of symbols needed for coloring) is bounded*

$$\chi(G) \leq \Delta(G).$$

Lemma . *For a given joint type $Q_{\mathbf{X}} \in \mathcal{P}_n(\mathcal{X}^{(\mathcal{I}_{N_s})})$, the coding graph $G(Q_{\mathbf{X}})$ has a degree*

$$\Delta(G(Q_{\mathbf{X}})) \leq \sum_{j \in \mathcal{I}_{N_d}} \exp\{nH(V_j|Q_j)\},$$

where $Q_{\mathbf{X}} = Q_j V_j, Q_j \in \mathcal{P}_n(\mathcal{X}^{(\mathcal{S}_j)}), V_j \in \mathcal{V}_n(\mathcal{X}^{(\mathcal{S}_j^c)}|Q_j)$。

Code construction (6)

- Fixed-length coding:

Create a table for each joint type $Q_X \in \mathcal{T}_n(R)$.

$$\begin{aligned}\mathcal{T}_n(R) \;=\; \{Q_X \in \mathcal{P}_n(\mathcal{X}^{(\mathcal{I}_{N_s})}) :\; & \max_{j \in \mathcal{I}_{N_d}} H(V_j|Q_j) \le R, \quad Q_{XY} = Q_j V_j, \\ & Q_j = \mathcal{P}_n(\mathcal{X}^{(\mathcal{S}_j^c)}), V_j \in \mathcal{V}_n(\mathcal{X}^{(\mathcal{S}_j)}|Q_j)\},\end{aligned}$$

- Variable-length coding

Create a table for every joint type Q_X. The codeword length of an input sequence pair depends on its type.

Coding theorems (1)

Theorem (Coding theorem for universal f-GCD codes).

For any real number $R > R_f(X)$, there exists a universal f-GCD code for the system $\boldsymbol{S}$ such that for any source $\boldsymbol{X}$

$$\limsup_{n\to\infty} \frac{1}{n}\log M_n \le R,$$

$$\lim_{n\to\infty} -\frac{1}{n}\log \sum_{j\in\mathcal{I}_{N_d}} e_n^{(j)} = \min_{Q_{\boldsymbol{X}}\in\mathcal{T}^c(R)} D(Q_{\boldsymbol{X}}\|P_{\boldsymbol{X}}) > 0,$$

$$\limsup_{n\to\infty} -\frac{1}{n}\log\Bigl(1 - \sum_{j\in\mathcal{I}_{N_d}} e_n^{(j)}\Bigr) \le \min_{Q_{\boldsymbol{X}}\in\mathcal{T}(R)} D(Q_{\boldsymbol{X}}\|P_{\boldsymbol{X}}),$$

$$e_n^{(j)} = \Pr\left\{\{X^{(S_j)}\}^n \ne \{\widehat{X}^{(S_j)}\}^n\right\}.$$

Coding theorems (2)

Theorem (Coding theorem for universal v-GCD codes).
There exists a universal v-GCD code such that for any source $\boldsymbol{X}$

$$\lim_{n\to\infty} \frac{1}{n} l(\varphi_n(\boldsymbol{X}^n)) = R_v(\boldsymbol{X}) = R_f(\boldsymbol{X}) \quad \text{a.s.}$$

$$\lim_{n\to\infty} -\frac{1}{n} \log \overline{\rho}_n(R) = \min_{Q_{\boldsymbol{X}} \in \mathcal{T}^c(R)} D(Q_{\boldsymbol{X}} \| P_{\boldsymbol{X}}) > 0$$

$$\limsup_{n\to\infty} -\frac{1}{n} \log \underline{\rho}_n(R) \le \min_{Q_{\boldsymbol{X}} \in \mathcal{T}(R)} D(Q_{\boldsymbol{X}} \| P_{\boldsymbol{X}})$$

$$\overline{\rho}_n(R) = \Pr\{l(\varphi_n(\boldsymbol{X}^n)) > R\}, \quad \underline{\rho}_n(R) = \Pr\{l(\varphi_n(\boldsymbol{X}^n)) < R\}.$$