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Universal source coding for complementary delivery

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Abstract— This paper deals with a universal coding problem for a certain kind of multiterminal source coding system that we call the complementary delivery coding system. Both fixed-to-fixed length and fixed-to-variable length lossless coding schemes are considered. Explicit constructions of universal codes and bounds of the error probabilities are clarified via type-theoretical and graph-theoretical analyses.

Keywords— multiterminal source coding, complementary delivery, universal coding, type of sequences, bipartite graphs

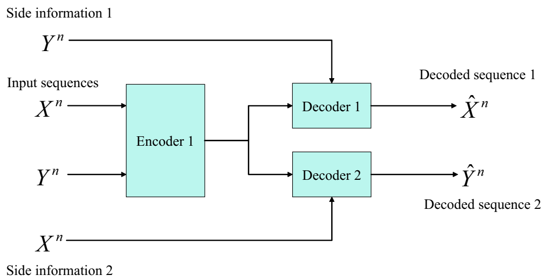

Figure 1: Complementary delivery coding system

1 Introduction

A coding problem for correlated information sources was first described and investigated by Slepian and Wolf [1], and later, various coding problems derived from that work were considered (e.g. Wyner [2], Körner and Marton [3], Sgarro [4]). Meanwhile, the problem of universal coding for these systems was first investigated by Csiszár and Körner [5]. Universal coding problems are not only interesting in their own right but also very important in terms of practical applications. Subsequent work has mainly focused on the Slepian-Wolf coding system [6, 7, 8] since it appears to be difficult to construct universal codes for most of the other coding systems. It has been shown that there are no universal codes without decoding errors for several coding systems [9].

This paper deals with a universal coding problem for a certain kind of multiterminal source coding system that we call complementary delivery coding system [10, 11]. Figure 1 shows a block diagram of the complementary delivery coding system. The encoder observes messages emitted from two correlated sources, and delivers these messages to other locations (i.e. decoders). Each decoder has access to one of two messages, and therefore wants to reproduce the other message. Although the previous articles [10, 11] considered lossy configurations, this paper considers a lossless configuration. We show an explicit construction of fixed-to-fixed length universal codes. We also clarify the upper and lower bounds of the error probabilities via type-theoretical and graph-theoretical analyses. Fixed-to-variable universal codes can be also constructed in a similar manner.

2 Preliminaries

Let $\mathcal{X}$ be a finite set, $\mathcal{B}$ be a binary set, and $\mathcal{B}^*$ be a set of all finite sequences in the alphabet $\mathcal{B}$. Let $|\mathcal{X}|$ be the cardinality of $\mathcal{X}$ and $\mathcal{I}_M = \{1, 2, \cdots, M\}$. A member of $\mathcal{X}^n$ is written as $x^n = (x_1, x_2, \cdots, x_n)$, and substrings of x^n are written as $x_i^j = (x_i, x_{i+1}, \cdots, x_j)$ for $i \le j$. When the dimension is clear from the context, vectors will be denoted by boldface letters, i.e., $\boldsymbol{x} \in \mathcal{X}^n$. $\mathcal{M}(\mathcal{X})$ denotes the set of all probability distributions on $\mathcal{X}$. Also, $\mathcal{M}(\mathcal{X}|P_Y)$ denotes the set of all probability distributions on $\mathcal{X}$ given a distribution $P_Y \in \mathcal{M}(\mathcal{Y})$, namely each member $P_{X|Y}$ of $\mathcal{M}(\mathcal{X}|P_Y)$ is characterized by $P_{XY} \in \mathcal{M}(\mathcal{X} \times \mathcal{Y})$ as $P_{XY} = P_{X|Y}P_Y$. A discrete memoryless source $(\mathcal{X}, P_X)$ is an infinite sequence of independent copies of a random variable X taking values in $\mathcal{X}$ with a generic distribution $P_X \in \mathcal{M}(\mathcal{X})$. We will denote a source $(\mathcal{X}, P_X)$ by referring to its generic distribution P_X or random variable X. For a correlated source (X, Y), $H(X|Y)$ denotes the conditional entropy of X given Y. For a generic distribution $P_Y \in \mathcal{M}(\mathcal{Y})$ and a conditional distribution $P_{X|Y} \in \mathcal{M}(\mathcal{X}|P)$, $H(P_{X|Y}|P_Y)$ also denotes the conditional entropy of X given Y. $D(P\|Q)$ denotes the Kullback-Leibler divergence between two distributions P and Q. In the following, all bases of exponentials and logarithms are set at 2.

Let $\mathcal{P}_n(\mathcal{X})$ be the set of types of sequences in $\mathcal{X}^n$, T_Q^n be the set with sequences of type $Q \in \mathcal{P}_n(\mathcal{X})$. Also, let $T_V^n(\boldsymbol{x})$ be the set of sequences with conditional type $V : \mathcal{X} \to \mathcal{Y}$ (we call this a V*-shell*) for a given $\boldsymbol{x} \in \mathcal{X}^n$, and $\mathcal{V}_n(\mathcal{Y}|Q)$ be the set of conditional types of sequences in $\mathcal{Y}^n$ for a given $Q \in \mathcal{P}_n(\mathcal{X})$ such that $T_V^n(\boldsymbol{x}) \neq \phi$ (empty set) for any $\boldsymbol{x} \in T_Q^n$.

3 Previous results

Definition 1. (Fixed-to-fixed complementary delivery (FF-CD) code)
A set $(\varphi^n, \widehat{\varphi}^n_{(1)}, \widehat{\varphi}^n_{(2)})$ of an encoder and two decoders is an FF-CD code of parameters $(n,\ M_n,\ \rho_n^{(X)},\ \rho_n^{(Y)})$ for the source (X, Y) if and only if

$$\varphi^n : \mathcal{X}^n \times \mathcal{Y}^n \to \mathcal{I}_{M_n}$$
$$\widehat{\varphi}^n_{(1)} : \mathcal{I}_{M_n} \times \mathcal{Y}^n \to \mathcal{X}^n,\ \ \widehat{\varphi}^n_{(2)} : \mathcal{I}_{M_n} \times \mathcal{X}^n \to \mathcal{Y}^n,$$

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$$\rho_n^{(X)} = \Pr\left\{X^n \neq \widehat{X}^n\right\},\ \rho_n^{(Y)} = \Pr\left\{Y^n \neq \widehat{Y}^n\right\},$$

where

$$\widehat{X}^n \overset{\text{def.}}{=} \widehat{\varphi}^n_{(1)}(\varphi^n(X^n, Y^n), Y^n),$$
$$\widehat{Y}^n \overset{\text{def.}}{=} \widehat{\varphi}^n_{(2)}(\varphi^n(X^n, Y^n), X^n).$$

Definition 2. (FF-CD-achievable rate)
R is an FF-CD-achievable rate of the source (X, Y) if and only if there exists a sequence of FF-CD codes $\{(\varphi^n, \widehat{\varphi}^n_{(1)}, \widehat{\varphi}^n_{(2)})\}_{n=1}^{\infty}$ of parameters $\{(n, M_n, \rho_n^{(X)}, \rho_n^{(Y)})\}_{n=1}^{\infty}$ for the source (X, Y) such that

$$\limsup_{n\to\infty} \frac{1}{n} \log M_n \le R,$$
$$\limsup_{n\to\infty} \rho_n^{(X)} = 0,\ \ \limsup_{n\to\infty} \rho_n^{(Y)} = 0.$$

Definition 3. (Inf FF-CD-achievable rate)

$$R_f(X, Y) = \inf\{R : R \text{ is an FF-CD-achievable rate of } (X, Y)\}.$$

Theorem 1. (Coding theorem of FF-CD code) *[12].*

$$\begin{aligned} R_f(X, Y) &= \max\{H(X|Y), H(Y|X)\} \\ &= \max\{H(P_{X|Y}|P_Y), H(P_{Y|X}|P_X)\} \end{aligned}$$

4 Code construction

This section shows an explicit construction of universal codes for the complementary delivery coding system defined by Definition 1. The coding scheme is described as follows:

1. For a given coding rate $R > 0$, select joint types $Q_{XY} \in \mathcal{P}_n(\mathcal{X} \times \mathcal{Y})$ that satisfy
$$\max\{H(W|Q_Y), H(V|Q_X)\} \le R,$$
where $Q_X \in \mathcal{P}_n(\mathcal{X})$, $Q_Y \in \mathcal{P}_n(\mathcal{Y})$, $V \in \mathcal{V}_n(\mathcal{Y}|Q_X)$ and $W \in \mathcal{V}_n(\mathcal{X}|Q_Y)$ that satisfy $Q_X V = Q_Y W = Q_{XY}$. We note that the joint type Q_{XY} specifies the conditional types V and W.
2. Create a table (henceforth we call this a *coding table*, see Figure 2 left) for each joint type Q_{XY} selected in Step 1. Each row of the coding table corresponds to a sequence $\boldsymbol{x} \in T^n_{Q_X}$, and each column corresponds to a sequence $\boldsymbol{y} \in T^n_{Q_Y}$. Codewords are determined arbitrarily for the sequence pairs whose types have been eliminated in Step 1.
3. Mark cells that correspond to sequence pairs $(\boldsymbol{x}, \boldsymbol{y}) \in T^n_{Q_{XY}}$ (see Figure 2 middle). Codewords will be given only to sequence pairs that correspond to marked cells.
4. Fill the marked cells with $\exp(nR)$ different symbols such that each symbol occurs at most once in each row and at most once in each column. An example of symbol filling is shown in Figure 2 right.

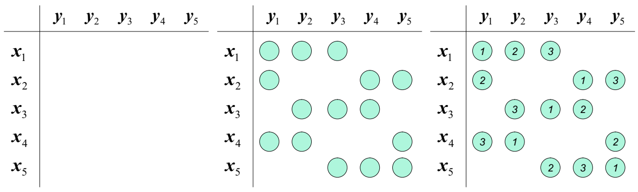

Figure 2: Example of coding scheme (left) Coding table (middle) Positions where codewords will be provided (right) Provided codewords

5. For a given pair of sequences $(\boldsymbol{x}, \boldsymbol{y}) \in \mathcal{X} \times \mathcal{Y}$, the index assigned to the joint type Q_{XY} of $(\boldsymbol{x}, \boldsymbol{y})$ is the first part of the codeword, and the symbol filled at the cell of $(\boldsymbol{x}, \boldsymbol{y})$ in the coding table of Q_{XY} is determined as the second part of the codeword.

First, we show the existence of such coding tables. To do this, we introduce the following two lemmas.

Lemma 1. *For a given coding table of a joint type $Q_{XY} \in \mathcal{P}_n(\mathcal{X} \times \mathcal{Y})$, the number of marked cells in every row of the coding table, N_y, is a constant value less than $\exp(nR)$, and the number of marked cells in every column of the coding table, N_x, is also a constant value less than $\exp(nR)$, both of which depend solely on the joint type Q_{XY}.*

Proof. We omit the details. Note that the number of marked cells in each row equals the cardinality of the V-shell $T^n_V(\boldsymbol{x})$ for the sequence $\boldsymbol{x} \in T^n_{Q_X}$ that corresponds to the row. □

Lemma 2. *For given integers m_x, m_y, n_x and n_y that satisfy $m_x \ge n_x$ and $m_y \ge n_y$, there exists a $m_x \times m_y$ table filled with* $\max(n_x, n_y)$ *different symbols such that*

- *at most n_y cells are filled with a certain symbol for each row (blank cells are possible),*
- *at most n_x cells are filled with a certain symbol for each column (blank cells are possible),*
- *each symbol occurs at most once in each row and at most once in each column.*

Proof. The table mentioned in this lemma is equivalent to a bipartite graph such that

- each node in one set corresponds to a row in the table, and each node in the other set corresponds to a column in the table,
- each edge corresponds to a cell in the table, to which a certain symbol is assigned
- $\max(n_x, n_y)$ different colors are given to edges, each of which corresponds to a symbol in the table,
- no two edges with the same color share a node in common.

Figure 3 shows an example of such a graph. Here, let us introduce the following lemma for bipartite graphs:

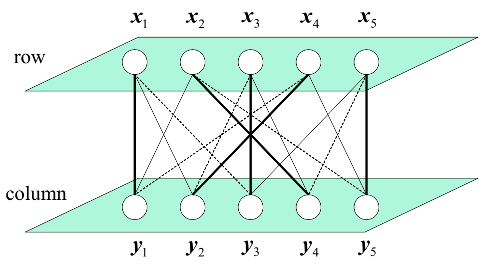

Figure 3: Example of a bipartite graph ($m_x = m_y = 5$, $n_x = n_y = 3$, equivalent to the table in Fig. 2 right)

Lemma 3. (König [13])
If a graph G is bipartite, the minimum number of colors necessary for edge coloring of the graph G equals the maximum degree of G.

Lemma 3 ensures the existence of the above bipartite graph. This concludes the proof of Lemma 2. □

From Lemmas 1 and 2, we can easily show the existence of coding tables by setting $m_x = |T^n_{Q_X}|$, $m_y = |T^n_{Q_Y}|$, $n_x = |T^n_W(\boldsymbol{y})|$ and $n_y = |T^n_V(\boldsymbol{x})|$ in Lemma 2.

5 Coding theorems

We can obtain the following theorem for the universal FF-CD codes constructed in Section 4. The proof is similar to that of the theorem of universal coding for a single source.

Theorem 2. *For a given real number $R > 0$, there exists a sequence of universal FF-CD codes of parameters $\{(n,\, M_n,\, \rho_n^{(X)},\, \rho_n^{(Y)})\}_{n=1}^{\infty}$ such that for any integer $n \geq 1$ and any source (X, Y) with a generic distribution $P_{XY} \in \mathcal{M}(\mathcal{X} \times \mathcal{Y})$*

$$\begin{aligned}\frac{1}{n}\log M_n &\leq R + \frac{1}{n}|\mathcal{X}\times\mathcal{Y}|\log(n+1),\\ \rho_n^{(X)} + \rho_n^{(Y)} &\leq 2(n+1)^{|\mathcal{X}\times\mathcal{Y}|}\\ &\quad \exp\left\{-n \min_{Q_{XY}\in\mathcal{S}_n(R)} D(Q_{XY}\|P_{XY})\right\},\end{aligned}$$

where

$$\begin{aligned}\mathcal{S}_n(R) = \{&Q_{XY} \in \mathcal{P}_n(\mathcal{X}\times\mathcal{Y}):\\ &\max\{H(V|Q_X), H(W|Q_Y)\} > R,\\ &Q_X \in \mathcal{P}_n(\mathcal{X}), Q_Y \in \mathcal{P}_n(\mathcal{Y}),\\ &V \in \mathcal{V}_n(\mathcal{Y}|Q_X), W \in \mathcal{V}_n(\mathcal{X}|Q_Y),\\ &Q_{XY} = Q_X V = Q_Y W\}.\end{aligned}$$

We can see that for any real value $R \geq R_f(X, Y)$ we have $P_{XY} \notin \mathcal{S}_n(R)$, and therefore

$$\min_{Q_{XY}\in\mathcal{S}_n(R)} D(Q_{XY}\|P_{XY}) \quad > \quad 0.$$

This implies that any real value $R \geq R_f(X, Y)$ is *a universal FF-CD achievable rate* of (X, Y), namely, there exists a sequence of universal FF-CD codes of parameters $\{(n,\, M_n,\, \rho_n^{(X)},\, \rho_n^{(Y)})\}_{n=1}^{\infty}$ such that

$$\begin{aligned}\limsup_{n\to\infty}\frac{1}{n}\log M_n &\leq R,\\ \limsup_{n\to\infty}\rho_n^{(X)} = \limsup_{n\to\infty}\rho_n^{(Y)} &= 0.\end{aligned}$$

The following converse theorem indicates that the error exponent obtained in Theorem 2 is tight.

Theorem 3. *Any sequence of FF-CD codes of parameters $\{(n, M_n, \rho_n^{(X)}, \rho_n^{(Y)})\}_{n=1}^{\infty}$ for the source (X, Y) must satisfy*

$$\begin{aligned}\rho_n^{(X)} + \rho_n^{(Y)} &\geq \frac{1}{2}(n+1)^{-|\mathcal{X}\times\mathcal{Y}|}\\ &\quad \exp\left\{-n \min_{Q_{XY}\in\mathcal{S}_n(R+\epsilon_n)} D(Q_{XY}\|P_{XY})\right\}\end{aligned}$$

for any integer $n \geq 1$ and a given coding rate $R = 1/n \log M_n > 0$, where $\epsilon_n \to 0$ $(n \to \infty)$.

The following corollary is directly derived from Theorems 2 and 3.

Corollary 1. *For a given real number $R > 0$, there exists a sequence of universal FF-CD codes of parameters $\{(n, M_n, \rho_n^{(X)}, \rho_n^{(Y)})\}_{n=1}^{\infty}$ such that for any source (X, Y)*

$$\begin{aligned}&\limsup_{n\to\infty}\frac{1}{n}\log M_n \leq R,\\ &\lim_{n\to\infty} -\frac{1}{n}\log(\rho_n^{(X)} + \rho_n^{(Y)}) = \min_{Q_{XY}\in\mathcal{S}(R)} D(Q_{XY}\|P_{XY}),\end{aligned}$$

where

$$\begin{aligned}\mathcal{S}(R) = \{&Q_{XY} \in \mathcal{M}(\mathcal{X}\times\mathcal{Y}):\\ &\max\{H(V|Q_X), H(W|Q_Y)\} > R,\\ &Q_X \in \mathcal{M}(\mathcal{X}), Q_Y \in \mathcal{M}(\mathcal{Y}),\\ &V \in \mathcal{M}(\mathcal{Y}|Q_X), W \in \mathcal{M}(\mathcal{X}|Q_Y),\\ &Q_{XY} = Q_X V = Q_Y W\}.\end{aligned}$$

6 Variable-length coding

6.1 Formulation

Definition 4. (Fixed-to-variable complementary delivery (FV-CD) code)
A set $(\varphi^n, \widehat{\varphi}^n_{(1)}, \widehat{\varphi}^n_{(2)})$ of an encoder and two decoders is an FV-CD code for the source (X, Y) if and only if

$$\begin{aligned}\varphi^n &: \mathcal{X}^n \times \mathcal{Y}^n \to \mathcal{B}^*\\ \widehat{\varphi}^n_{(1)} &: \varphi^n(\mathcal{X}^n, \mathcal{Y}^n) \times \mathcal{Y}^n \to \mathcal{X}^n,\\ \widehat{\varphi}^n_{(2)} &: \varphi^n(\mathcal{X}^n, \mathcal{Y}^n) \times \mathcal{X}^n \to \mathcal{Y}^n,\\ \rho_n^{(X)} &= \Pr\left\{X^n \neq \widehat{X}^n\right\} = 0,\\ \rho_n^{(Y)} &= \Pr\left\{Y^n \neq \widehat{Y}^n\right\} = 0,\end{aligned}$$

where

$$\widehat{X}^n \overset{\text{def.}}{=} \widehat{\varphi}^n_{(1)}(\varphi^n(X^n, Y^n), Y^n),$$

$$\widehat{Y}^n \stackrel{\text{def.}}{=} \widehat{\varphi}^n_{(2)}(\varphi^n(X^n, Y^n), X^n),$$

and the image of φ^n is a prefix set.

Definition 5. (FV-CD-achievable rate)
R is an FV-CD-achievable rate of the source (X, Y) if and only if there exists a sequence of FV-CD codes $\{(\varphi^n, \widehat{\varphi}^n_{(1)}, \widehat{\varphi}^n_{(2)})\}_{n=1}^{\infty}$ for the source (X, Y) such that

$$\limsup_{n\to\infty} \frac{1}{n} E\left[l(\varphi^n(X^n, Y^n))\right] \leq R,$$

where $l(\cdot) : \mathcal{B}^* \to \mathcal{R}$ is a length function.

Definition 6. (Inf FV-CD-achievable rate)

$$R_v(X, Y) = \inf\{R : R \text{ is an FV-CD-achievable rate of } (X, Y)\}.$$

6.2 Code construction

We can construct universal FV-CD codes in a similar manner to universal FF-CD codes. Note that the coding rate is fixed beforehand for fixed-length coding, whereas the coding rate depends on the type of the sequence pair to be encoded. The coding scheme is described as follows:

1. Create a coding table for each joint type $Q_{XY} \in \mathcal{P}_n(\mathcal{X} \times \mathcal{Y})$ in the same way as Step 2 of Section 4.
2. Mark cells that correspond to sequence pairs $(\boldsymbol{x}, \boldsymbol{y}) \in T_{Q_{XY}}$.
3. Fill the marked cells on the coding table with $\max\{|T^n_V(\boldsymbol{x})|, |T^n_W(\boldsymbol{y})|\}$ different symbols such that each symbol occurs at most once in each row and at most once in each column, where $\boldsymbol{x} \in T^n_{Q_X}$, $\boldsymbol{y} \in T^n_{Q_Y}$.
4. For a given pair of sequences $(\boldsymbol{x}, \boldsymbol{y}) \in \mathcal{X}^n \times \mathcal{Y}^n$, the number (index) assigned to the joint type Q_{XY} of $(\boldsymbol{x}, \boldsymbol{y})$ is the first part of the codeword, and the symbol filled at the cell of $(\boldsymbol{x}, \boldsymbol{y})$ in the coding table of Q_{XY} is determined as the second part of the codeword.

6.3 Coding theorems

We begin by showing a theorem for (non-universal) variable-length coding, which indicates that the inf coding rate of variable-length coding is the same as that of fixed-length coding.

Theorem 4. (Coding theorem of FV-CD code)

$$R_v(X, Y) = R_f(X, Y) = \max\{H(X|Y), H(Y|X)\}.$$

The following direct theorem for universal coding indicates that the coding scheme presented in the previous subsection can achieve the inf coding rate. The theorem can be easily derived in almost the same way as that for a single source.

Theorem 5. *There exists a sequence of universal FV-CD codes $\{(\varphi^n, \widehat{\varphi}^n_{(1)}, \widehat{\varphi}^n_{(2)})\}_{n=1}^{\infty}$ such that for any integer $n \geq 1$ and any source (X, Y), the overflow probability is bounded as*

$$\begin{aligned}\overline{e}_n(R) &\stackrel{\text{def.}}{=} \Pr\{l(\varphi^n(X^n, Y^n)) > n(R + \epsilon_n)\} \\ &\leq (n+1)^{|\mathcal{X}\times\mathcal{Y}|} \\ &\quad \exp\left\{-n \min_{Q_{XY}\in\mathcal{S}_n(R)} D(Q_{XY}\|P_{XY})\right\},\end{aligned}$$

where $\epsilon_n \to 0$ $(n \to \infty)$. This implies that there exists a sequence of universal FV-CD codes $\{(\varphi^n, \widehat{\varphi}^n_{(1)}, \widehat{\varphi}^n_{(2)})\}_{n=1}^{\infty}$ that satisfies

$$\limsup_{n\to\infty} \frac{1}{n} l(\varphi^n(X^n, Y^n)) \leq R_v(X, Y) \quad a.s.$$

The converse theorem for variable-length coding can be easily obtained in the same way as Theorem 3.

Theorem 6. *Any sequence of FV-CD codes $\{(\varphi^n, \widehat{\varphi}^n_{(1)}, \widehat{\varphi}^n_{(2)})\}_{n=1}^{\infty}$ for the source (X, Y) must satisfy*

$$\begin{aligned}\overline{e}_n(R) &\geq (n+1)^{-|\mathcal{X}\times\mathcal{Y}|} \\ &\quad \exp\left\{-n \min_{Q_{XY}\in\mathcal{S}_n(R+\epsilon_n)} D(Q_{XY}\|P_{XY})\right\}.\end{aligned}$$

for a given real number $R > 0$ and any integer $n \geq 1$, where $\epsilon_n \to 0$ $(n \to \infty)$.

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