Multiterminal source coding for cascading and feedback refinemet systems

Akisato Kimura[*1,*2], Tomohiko Uyematsu[*2]

[*1] Media Information Laboratory,

NTT Communication Science Laboratories,

NTT Corporation

[*2] Department of Communications and Integrated Systems,

Graduate School of Science and Engineering,

Tokyo Institute of Technology

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Abstract

- Coding problems for correlated information sources are investigated for communication systems in the presence of
  - cascading information channels
  - feedback information channels
  - cascading and feedback information channels

  from decoders so as to refine reproduction messages.
- Outer and inner bounds of achievable rate-distortion regions for those problems are obtained.

Introduction

Scalable coding problems has attracted considerable attention

- Successive refinement
  (e.g. [Koshelev, 1980, Equitz and Cover, 1991])
- Multiresolution coding
  (e.g. [Effros, 2001, Effros and Dugatkin, 2004])
- Sequential coding [Viswanathan and Berger, 2000]

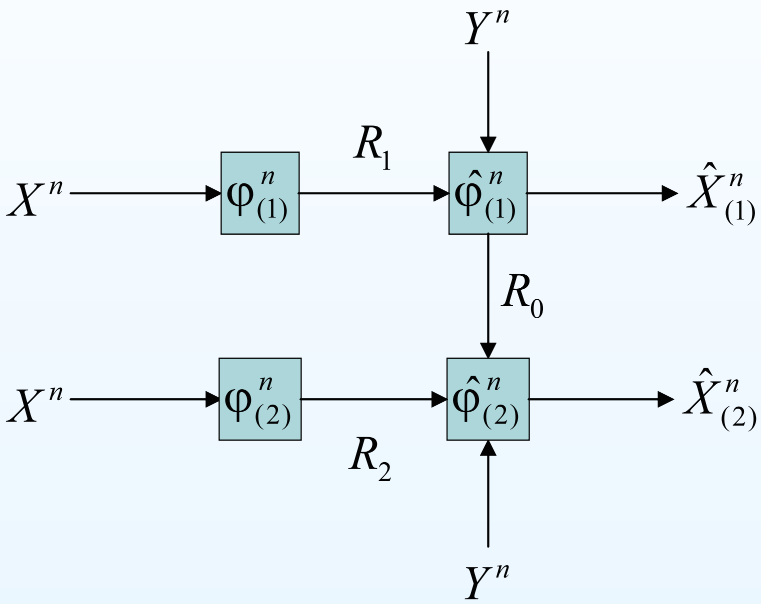
Some extended coding problems have also been investigated

- Successive refinement with side information
  [Steinberg and Merhav, 2004]
- Hierarchical source-channel coding
  [Steinberg and Merhav, 2006]

$\Longrightarrow$ We consider other types of scalable coding structures

- involve cascading and/or feedback transmission

Refinement with cascading channel



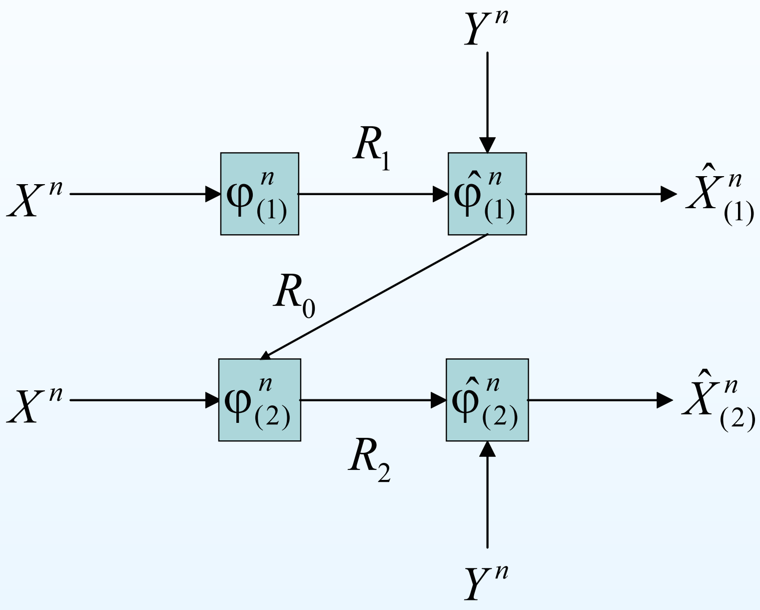
Related coding problems

- Cascading and branching coding systems [Yamamoto, 1981]
- Triangular coding system [Yamamoto, 1996]

This coding problem is an extended version of the simple trianglular coding system.

- Correlated side information Y is additionally provided to the decoders.

Refinement with feedback channel



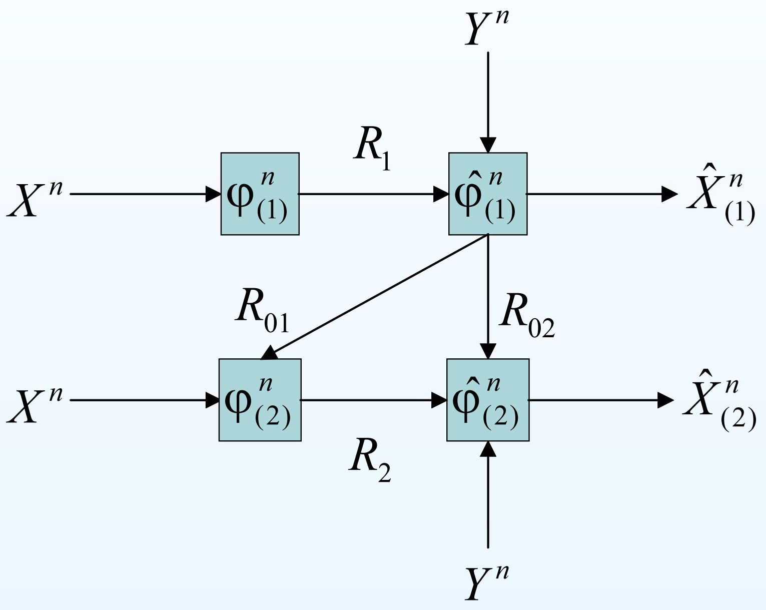
Related coding problems

- Channel coding with a feedback channel
  (e.g. [Schalkwijk and Bluestein, 1967])
- Source coding with zero-rate feedback and side information
  [Yang et al., 2005]

This coding problem is an extended version of Yang's work.

- lossy coding problems are considered
- non-zero rate transmission is admissible on the feedback channel (namely, $R_0 > 0$ is admissible)

Refinement with cascading and feedback channels



Related coding problems

- Hierarchical source-channel coding [Steinberg and Merhav, 2006]

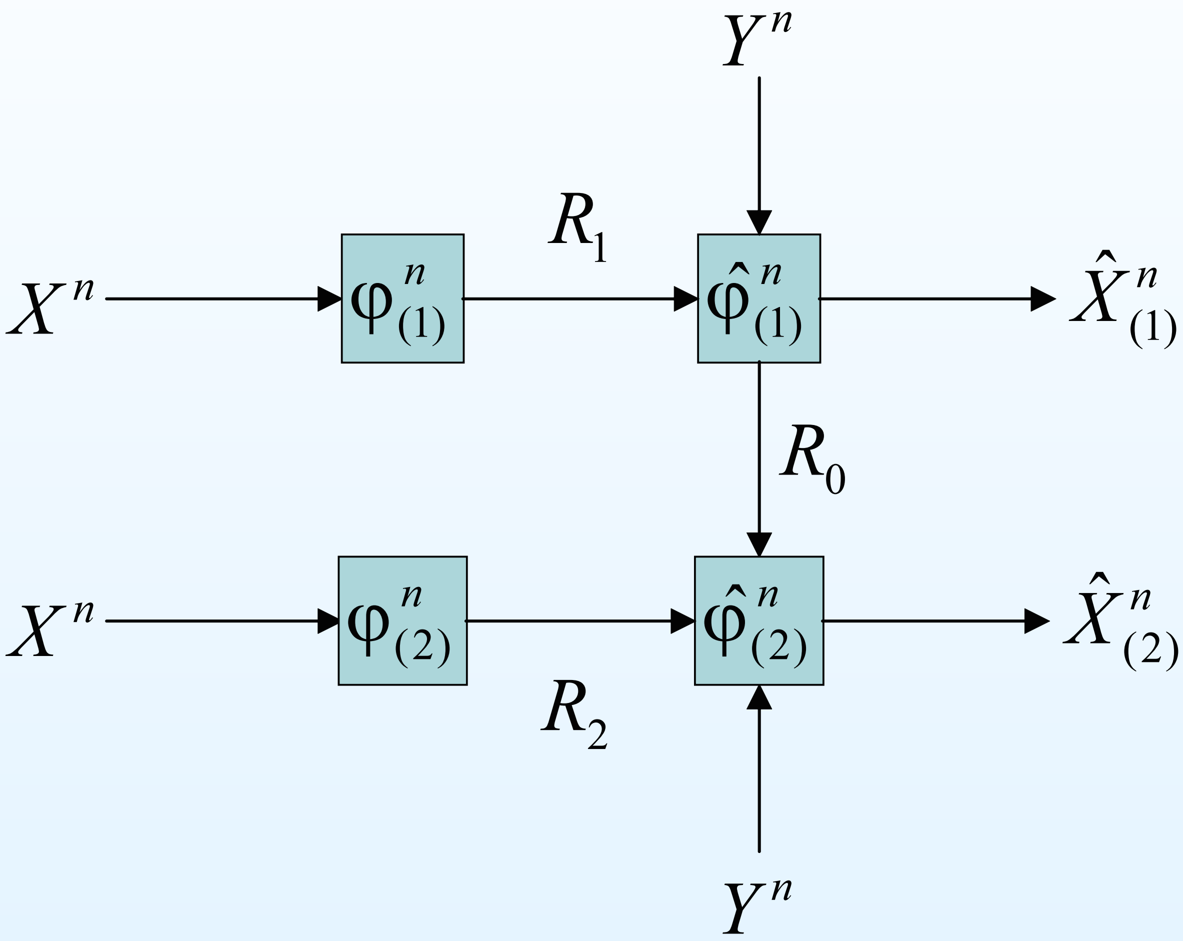
This system models a certain type of information retrieval with index structures

- Details will be presented at SITA2006.

Preliminaries

- $\mathcal{X}, \mathcal{Y}, \widehat{\mathcal{X}}$: finite alphabets, $\mathcal{R}$: the set of all real values,
  $\mathcal{X}^* = \cup_{n \geq 0} \mathcal{X}^n$: the set of all finite-lendth sequences,
  $|\mathcal{X}|$: the cardinality of $\mathcal{X}$, $\mathcal{I}_M = \{1, 2, \cdots, M\}$
- $x^n = (x_1, \cdots, x_n) \in \mathcal{X}^n$, $x_i^j = (x_i, \cdots, x_j) \in \mathcal{X}^{j-i+1}$ $(i \leq j)$
- $\mathcal{M}(\mathcal{X})$: the set of all probability distributions (PDs) on $\mathcal{X}$,
  $\mathcal{M}(\mathcal{X}|P_Y)$: the set of all conditional PDs on $\mathcal{X}$ given a PD $P_Y \in \mathcal{M}(\mathcal{Y})$
- $\boldsymbol{X} = \{X_i\}_{i=1}^{\infty}$: source to be encoded with a PD P_X (i.i.d.),
  $\boldsymbol{Y} = \{Y_i\}_{i=1}^{\infty}$: decoder side information with a PD P_Y (i.i.d.),
  $\widehat{\boldsymbol{X}}_{(j)} = \{\widehat{X}_{i(j)}\}_{i=1}^{\infty}$ $(j = 1, 2)$: outputs of the j-th decoder
- $H(X)$: entropy, $I(X;Y)$: mutual information
- $\Delta : \mathcal{X} \times \widehat{\mathcal{X}} \to \mathcal{R}$: a distortion function,
  $\Delta^n : \mathcal{X}^n \times \widehat{\mathcal{X}}^n \to \mathcal{R}$: additive

Cascading refinement system



Cascading refinement system

Definition 1. (CR (Cascading Refinement) code)
A set $\left(\varphi_{(0)}^n, \varphi_{(1)}^n, \varphi_{(2)}^n, \widehat{\varphi}_{(1)}^n, \widehat{\varphi}_{(2)}^n\right)$ of encoders and decoders is a CR code $\left(n, M_n^{(0)}, M_n^{(1)}, M_n^{(2)}, \rho_n^{(1)}, \rho_n^{(2)}\right)$ for the source (X, Y) if and only if

$$\varphi_{(1)}^n : \mathcal{X}^n \to \mathcal{I}_{M_n^{(1)}}, \quad \varphi_{(2)}^n : \mathcal{X}^n \to \mathcal{I}_{M_n^{(2)}}, \quad \varphi_{(0)}^n : \mathcal{Y}^n \times \mathcal{I}_{M_n^{(1)}} \to \mathcal{I}_{M_n^{(0)}},$$
$$\widehat{\varphi}_{(1)}^n : \mathcal{I}_{M_n^{(1)}} \times \mathcal{Y}^n \to \widehat{\mathcal{X}}^n, \quad \widehat{\varphi}_{(2)}^n : \mathcal{I}_{M_n^{(0)}} \times \mathcal{I}_{M_n^{(2)}} \times \mathcal{Y}^n \to \widehat{\mathcal{X}}^n,$$

where

$$\rho_n^{(i)} = E\left[\Delta^n(X^n, \hat{X}_{(i)}^n)\right], \quad (i = 1, 2)$$
$$A_n^{(1)} = \varphi_{(1)}^n(X^n), \quad A_n^{(0)} = \varphi_{(0)}^n(A_n^{(1)}, Y^n), \quad A_n^{(2)} = \varphi_{(2)}^n(X^n),$$
$$\widehat{X}_{(1)}^n = \widehat{\varphi}_{(1)}^n(A_n^{(1)}, Y^n), \quad \widehat{X}_{(2)}^n = \widehat{\varphi}_{(2)}^n(A_n^{(0)}, A_n^{(2)}, Y^n).$$

Achievable rate region

Definition 2. (CR-achievable rate triad)
(R_0, R_1, R_2) is a CR-achievable rate triad of the source (X, Y) for a given distortion pair (D_1, D_2) if and only if there exists a sequence of CR codes $\{(n, M_n^{(0)}, M_n^{(1)}, M_n^{(2)}, \rho_n^{(1)}, \rho_n^{(2)})\}_{n=1}^{\infty}$ for the source (X, Y) such that

$$\limsup_{n\to\infty} \frac{1}{n} \log M_n^{(i)} \leq R_i, \quad (i = 0, 1, 2)$$

$$\limsup_{n\to\infty} \rho_n^{(j)} \leq D_j. \quad (j = 1, 2)$$

Definition 3. (CR-achievable rate region)

$$\mathcal{R}_c(X, Y | D_1, D_2) = \{(R_0, R_1, R_2) :$$

(R_0, R_1, R_2) is a CR-achievable rate triad of (X, Y) for $(D_1, D_2)\}$.

Main Theorem

Theorem 1. (Coding theorem of CR code)

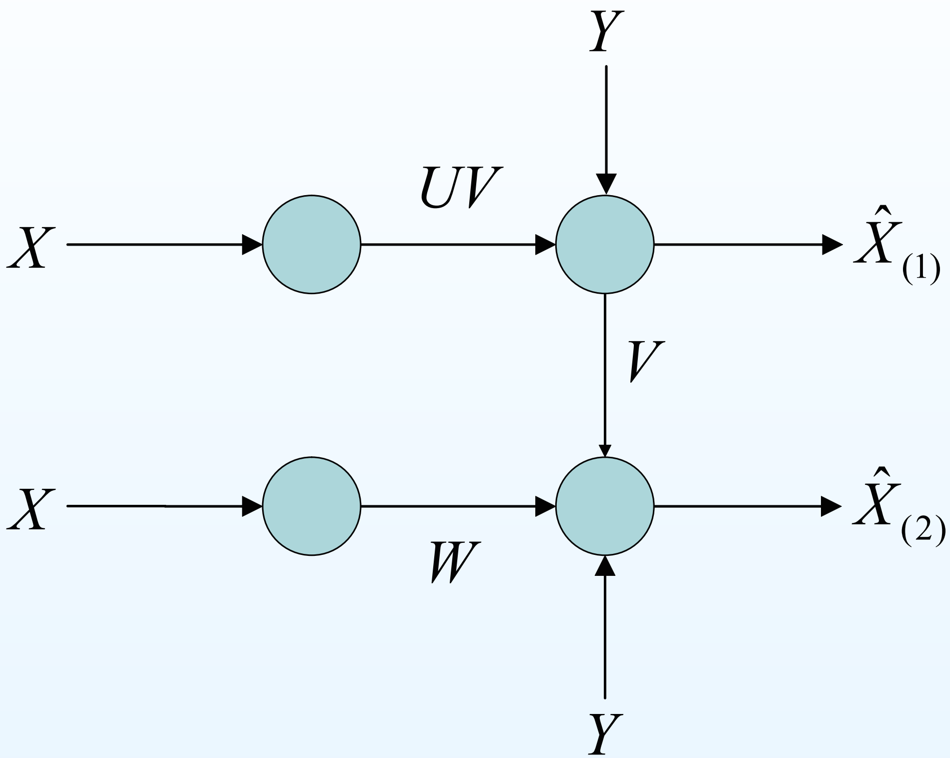
$$\mathcal{R}_c(X,Y|D_1,D_2) \subseteq \Big\{(R_0,R_1,R_2): \quad \text{(outer bound)}$$
$$R_1 \geq I(X;UV|Y), \quad R_0 \geq I(X;V|Y), \quad R_2 \geq I(X;W|VY)\Big\}$$

where the random variables $U \in \mathcal{U}$, $V \in \mathcal{V}$ *and* $W \in \mathcal{W}$ *are selected s.t.*

- *the alphabet sizes are bounded as* $|\mathcal{U}| \leq |\mathcal{X}|+1,\ |\mathcal{W}| \leq |\mathcal{U}\times\mathcal{X}|+1,\ |\mathcal{V}| \leq |\mathcal{U}\times\mathcal{W}\times\mathcal{X}\times\mathcal{Y}|+4$,
- $UW \to X \to Y$ *forms a Markov chain,*
- *there exist functions* $\phi_{(1)} : \mathcal{U}\times\mathcal{V}\times\mathcal{Y} \to \widehat{\mathcal{X}}$ *and* $\phi_{(2)} : \mathcal{V}\times\mathcal{W}\times\mathcal{Y} \to \widehat{\mathcal{X}}$ *which safisfy*
$$D_1 \geq E\left[\Delta(X,\phi_{(1)}(U,V,Y))\right],\ D_2 \geq E\left[\Delta(X,\phi_{(2)}(V,W,Y))\right].$$

An inner bound is obtained with the same functional forms, while the Markov chain is replaced as $UVW \to X \to Y$.

Intuitive interpretation



$$\mathcal{R}_c(X,Y|D_1,D_2) = \big\{(R_0,R_1,R_2) :$$
$$R_1 \geq I(X;UV|Y), \quad R_0 \geq I(X;V|Y), \quad R_2 \geq I(X;W|VY)\big\}$$

$UW \to X \to Y$ (Outer bound), $UVW \to X \to Y$ (Inner bound)

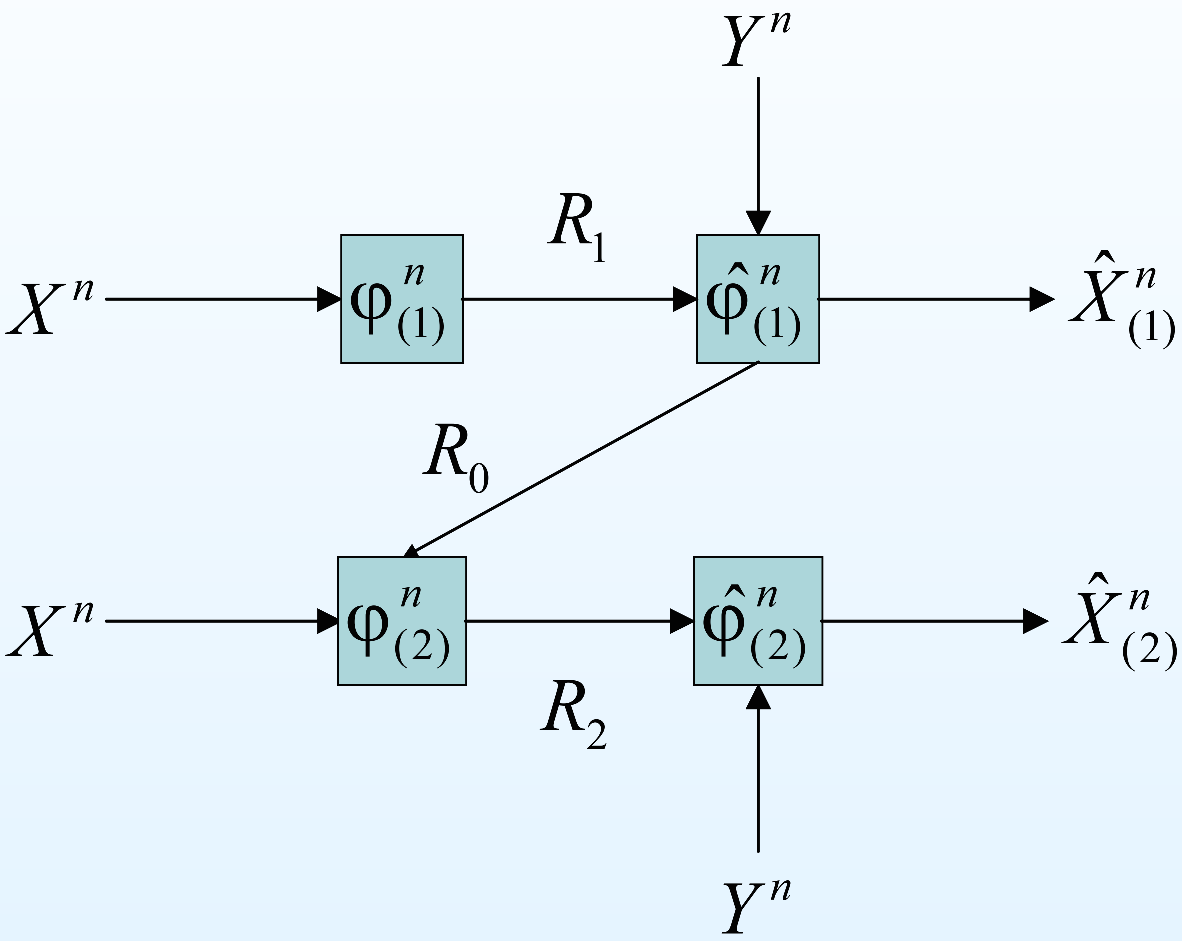
Corollary

Corollary 1. (Compatibility with a known result)
If side information Y is not available at both of two decoders, the outer bound indicated in Theorem 1 coincides with the inner bound, i.e.

$$\mathcal{R}_c(X|D_1, D_2) = \Big\{(R_0, R_1, R_2) : \\ R_1 \geq I(X; \widehat{X}_{(1)}V), \quad R_0 \geq I(X;V), \quad R_2 \geq I(X; \widehat{X}_{(2)}|V)\Big\}$$

where the random variable $V \in \mathcal{V}$ is selected such that $|\mathcal{V}| \leq |\mathcal{X}| + 2$. This rate region coincides with the one indicated by Yamamoto [Yamamoto, 1996].

Feedback refinement system



Feedback refinement system

Definition 4. (FR (Feedback Refinement) code)
A set $\left(\varphi^n_{(0)}, \varphi^n_{(1)}, \varphi^n_{(2)}, \widehat{\varphi}^n_{(1)}, \widehat{\varphi}^n_{(2)}\right)$ of encoders and decoders is an FR code $\left(n, M_n^{(0)}, M_n^{(1)}, M_n^{(2)}, \rho_n^{(1)}, \rho_n^{(2)}\right)$ for the source (X, Y) if and only if

$$\varphi^n_{(1)} : \mathcal{X}^n \to \mathcal{I}_{M_n^{(1)}}, \quad \varphi^n_{(0)} : \mathcal{I}_{M_n^{(1)}} \times \mathcal{Y}^n \to \mathcal{I}_{M_n^{(0)}},$$
$$\varphi^n_{(2)} : \mathcal{I}_{M_n^{(0)}} \times \mathcal{X}^n \to \mathcal{I}_{M_n^{(2)}},$$
$$\widehat{\varphi}^n_{(1)} : \mathcal{I}_{M_n^{(1)}} \times \mathcal{Y}^n \to \widehat{\mathcal{X}}^n, \quad \widehat{\varphi}^n_{(2)} : \mathcal{I}_{M_n^{(2)}} \times \mathcal{Y}^n \to \widehat{\mathcal{X}}^n,$$

where

$$\rho_n^{(i)} = E\left[\Delta^n(X^n, \widehat{X}^n_{(i)})\right], \quad (i = 1, 2)$$
$$A_n^{(1)} = \varphi^n_{(1)}(X^n), \quad A_n^{(0)} = \varphi^n_{(0)}(A_n^{(1)}, Y^n), \quad A_n^{(2)} = \varphi^n_{(2)}(A_n^{(0)}, X^n),$$
$$\widehat{X}^n_{(1)} = \widehat{\varphi}^n_{(1)}(A_n^{(1)}, Y^n), \quad \widehat{X}^n_{(2)} = \widehat{\varphi}^n_{(2)}(A_n^{(2)}, Y^n).$$

Achievable region

Definition 5. (FR-achievable rate triad)
(R_0, R_1, R_2) is an FR-achievable rate triad of the source (X, Y) for a given distortion pair (D_1, D_2) if and only if there exists a sequence of FR codes $\{(n, M_n^{(0)}, M_n^{(1)}, M_n^{(2)}, \rho_n^{(1)}, \rho_n^{(2)})\}_{n=1}^{\infty}$ for the source (X, Y) such that

$$\limsup_{n \to \infty} \frac{1}{n} \log M_n^{(i)} \leq R_i, \quad (i = 0, 1, 2)$$

$$\limsup_{n \to \infty} \rho_n^{(j)} \leq D_j. \quad (j = 1, 2)$$

Definition 6. (FR-achievable rate region)

$$\mathcal{R}_f(X, Y | D_1, D_2) = \{(R_0, R_1, R_2) :$$
$$(R_0, R_1, R_2) \text{ is an FR-achievable rate triad of } (X, Y) \text{ for } (D_1, D_2)\}.$$

Main Theorem

Theorem 2. (Coding theorem of FR code)

$$\mathcal{R}_f(X,Y|D_1,D_2) \subseteq \Big\{(R_0,R_1,R_2): \quad \text{(outer bound)}$$
$$R_1 \geq I(X;U|Y), \quad R_0 \geq I(Y;V|UX), \quad R_2 \geq I(X;W|VY)\Big\}$$

where the random variables $U \in \mathcal{U}$, $V \in \mathcal{V}$ *and* $W \in \mathcal{W}$ *are selected s.t.*

- *the alphabet sizes are bounded as* $|\mathcal{U}| \leq |\mathcal{X}| + 2$, $|\mathcal{V}| \leq |\mathcal{U} \times \mathcal{Y}| + 1$, $|\mathcal{W}| \leq |\mathcal{U} \times \mathcal{V} \times \mathcal{X}| + 1$,
- *the following Markov chains are satisfied:* $U \to X \to Y$, $V \to UY \to X$, $W \to VX \to UY$,
- *there exist functions* $\phi_{(1)} : \mathcal{U} \times \mathcal{Y} \to \widehat{\mathcal{X}}$ *and* $\phi_{(2)} : \mathcal{W} \times \mathcal{Y} \to \widehat{\mathcal{X}}$ *which safisfy*

$$D_1 \geq E\left[\Delta(X,\phi_{(1)}(U,Y))\right], \quad D_2 \geq E\left[\Delta(X,\phi_{(2)}(W,Y))\right].$$

Main Theorem (Contd.)

Theorem 2. Contd.
An inner bound is obtained in the same functional forms, while the Markov chains are replaced as
$U \to X \to Y,\ V \to Y \to UX,\ W \to VX \to Y.$

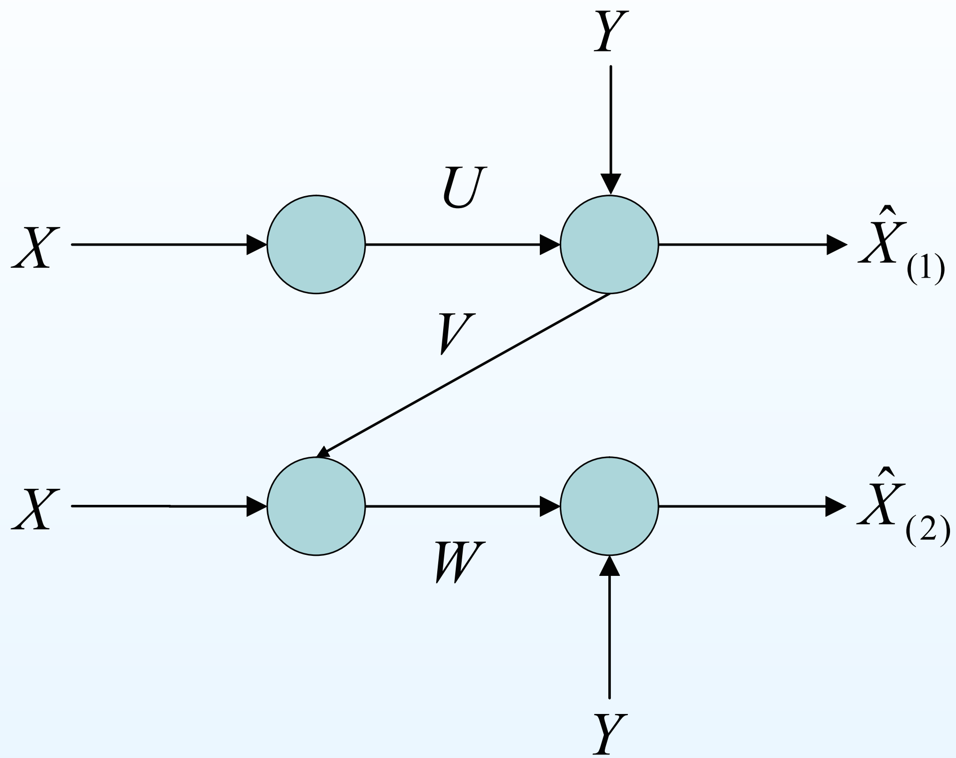
Remark .
The following equation is equivalent to the Markov conditions of the outer bound:

$$\begin{aligned} & P_{UVWXY}(u,v,w,x,y) \\ = \quad & P_{XY}(x,y)P_{U|X}(u|x)P_{V|UY}(v|u,y)P_{W|VX}(w|v,x). \end{aligned}$$

Also, the following equation is equivalent to the Markov conditions of the inner bound:

$$\begin{aligned} & P_{UVWXY}(u,v,w,x,y) \\ = \quad & P_{XY}(x,y)P_{U|X}(u|x)P_{V|Y}(v|y)P_{W|VX}(w|v,x). \end{aligned}$$

Intuitive interpretation



$$\mathcal{R}_f(X,Y|D_1,D_2) = \Big\{(R_0,R_1,R_2):$$
$$R_1 \geq I(X;U|Y), \quad R_0 \geq I(Y;V|UX), \quad R_2 \geq I(X;W|VY)\Big\}$$

$U \to X \to Y$, $V \to UY \to X$, $W \to VX \to UY$ (Outer bound)
$U \to X \to Y$, $V \to Y \to UX$, $W \to VX \to UY$ (Inner bound)

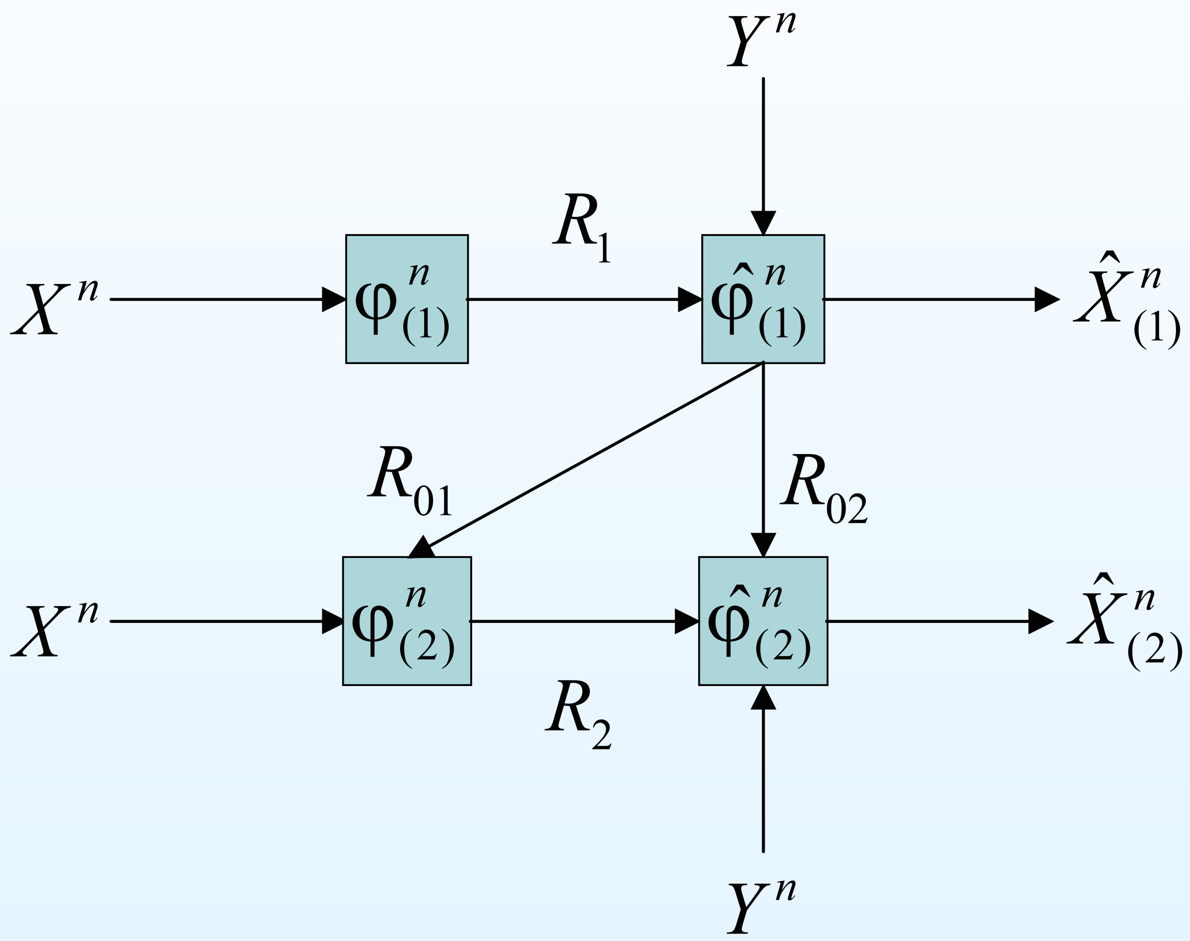
Corollary

Corollary 2. (Coding theorem for some special cases)
If side information Y is not available at both of two decoders, the outer bound indicated in Theorem 2 coincides with the inner bound, i.e.

$$\mathcal{R}_f(X|D_1, D_2) = \Big\{(R_0, R_1, R_2) : \\ R_1 \geq I(X; \widehat{X}^{(1)}), \quad R_0 \geq 0, \quad R_2 \geq I(X; \widehat{X}^{(2)})\Big\}.$$

This shows that feedback information is of no use for refining in the absence of side information.

Cascasing and feedback refinement system



Cascading and feedback refinement system

Definition 7. (CFR (Cascading and Feedback Refinement) code)
A set $\left(\varphi^n_{(01)}, \varphi^n_{(02)}, \varphi^n_{(1)}, \varphi^n_{(2)}, \widehat{\varphi}^n_{(1)}, \widehat{\varphi}^n_{(2)}\right)$ of encoders and decoders is a CFR code $\left(n, M_n^{(01)}, M_n^{(02)}, M_n^{(1)}, M_n^{(2)}, \rho_n^{(1)}, \rho_n^{(2)}\right)$ for the source (X, Y) if and only if

$$\varphi^n_{(1)} : \mathcal{X}^n \to \mathcal{I}_{M_n^{(1)}}, \quad \varphi^n_{(2)} : \mathcal{I}_{M_n^{(01)}} \times \mathcal{X}^n \to \mathcal{I}_{M_n^{(2)}},$$

$$\varphi^n_{(01)} : \mathcal{I}_{M_n^{(1)}} \times \mathcal{Y}^n \to \mathcal{I}_{M_n^{(01)}}, \quad \varphi^n_{(02)} : \mathcal{I}_{M_n^{(1)}} \times \mathcal{Y}^n \to \mathcal{I}_{M_n^{(02)}}$$

$$\widehat{\varphi}^n_{(1)} : \mathcal{I}_{M_n^{(1)}} \times \mathcal{Y}^n \to \widehat{\mathcal{X}}^n, \quad \widehat{\varphi}^n_{(2)} : \mathcal{I}_{M_n^{(02)}} \times \mathcal{I}_{M_n^{(2)}} \times \mathcal{Y}^n \to \widehat{\mathcal{X}}^n,$$

where

$$\rho_n^{(i)} = E\left[\Delta^n(X^n, \widehat{X}^n_{(i)})\right] \quad (i = 1, 2), \quad A_n^{(1)} = \varphi^n_{(1)}(X^n),$$

$$A_n^{(i)} = \varphi^n_{(i)}(A_n^{(1)}, Y^n) \quad (i = 01, 02), \quad A_n^{(2)} = \varphi^n_{(2)}(A_n^{(01)}, X^n),$$

$$\widehat{X}^n_{(1)} = \widehat{\varphi}^n_{(1)}(A_n^{(1)}, Y^n), \quad \widehat{X}^n_{(2)} = \widehat{\varphi}^n_{(2)}(A_n^{(01)}, A_n^{(2)}, Y^n).$$

Achievability

Definition 8. (CFR-achievable rate quadruplet)
$(R_{01}, R_{02}, R_1, R_2)$ is a CFR-achievable rate quadruplet of the source (X, Y) for a given distortion pair (D_1, D_2) if and only if there exists a sequence of CFR codes $\{(n, M_n^{(01)}, M_n^{(02)}, M_n^{(1)}, M_n^{(2)}, \rho_n^{(1)}, \rho_n^{(2)})\}_{n=1}^{\infty}$ for the source (X, Y) such that

$$\limsup_{n\to\infty} \frac{1}{n} \log M_n^{(i)} \le R_i, \quad (i = 01, 02, 1, 2)$$

$$\limsup_{n\to\infty} \rho_n^{(j)} \le D_j. \quad (j = 1, 2)$$

Definition 9. (CFR-achievable rate region)

$$\mathcal{R}_{cf}(X, Y | D_1, D_2) = \{(R_{01}, R_{02}, R_1, R_2) :$$

$(R_{01}, R_{02}, R_1, R_2)$ is a CFR-achievable rate quadruplet of (X, Y) for $(D_1, D_2)\}$.

Main Theorem

Theorem 3. (Coding theorem of CFR code)

$$\mathcal{R}_{cf}(X,Y|D_1,D_2) \subseteq \Big\{(R_{01},R_{02},R_1,R_2): \quad \text{(outer bound)}$$
$$R_1 \geq I(X;UV^{(2)}|Y),\ R_{01} \geq I(Y;V^{(1)}|UV^{(2)}X),$$
$$R_{02} \geq I(X;V^{(2)}|Y),\ R_2 \geq I(X;W|V^{(1)}V^{(2)}Y)\Big\}$$

where the random variables $U \in \mathcal{U}$, $V^{(1)} \in \mathcal{V}^{(1)}$, $V^{(2)} \in \mathcal{V}^{(2)}$ and $W \in \mathcal{W}$ are selected s.t.

- *the alphabet sizes are bounded (See the proceedings),*
- *the following Markov chains are satisfied: $U \to X \to Y$, $V^{(1)} \to UV^{(2)}Y \to X$, $W \to V^{(1)}V^{(2)}X \to UY$,*
- *there exist functions* $\phi_{(1)}: \mathcal{U} \times \mathcal{V}^{(2)} \times \mathcal{Y} \to \widehat{\mathcal{X}}$ *and* $\phi_{(2)}: \mathcal{W} \times \mathcal{V}^{(2)} \times \mathcal{Y} \to \widehat{\mathcal{X}}$ *s.t.*
  $$D_1 \geq E\left[\Delta(X,\phi_{(1)}(U,V^{(2)},Y))\right],\ D_2 \geq E\left[\Delta(X,\phi_{(2)}(W,V^{(2)},Y))\right].$$

Main Theorem (Contd.)

Theorem 3. Contd.
An inner bound is obtained in the same functional forms, while the Markov chains are replaced as
$UV^{(2)} \to X \to Y,\ V^{(1)} \to V^{(2)}Y \to UX,\ W \to V^{(1)}V^{(2)}X \to UY.$

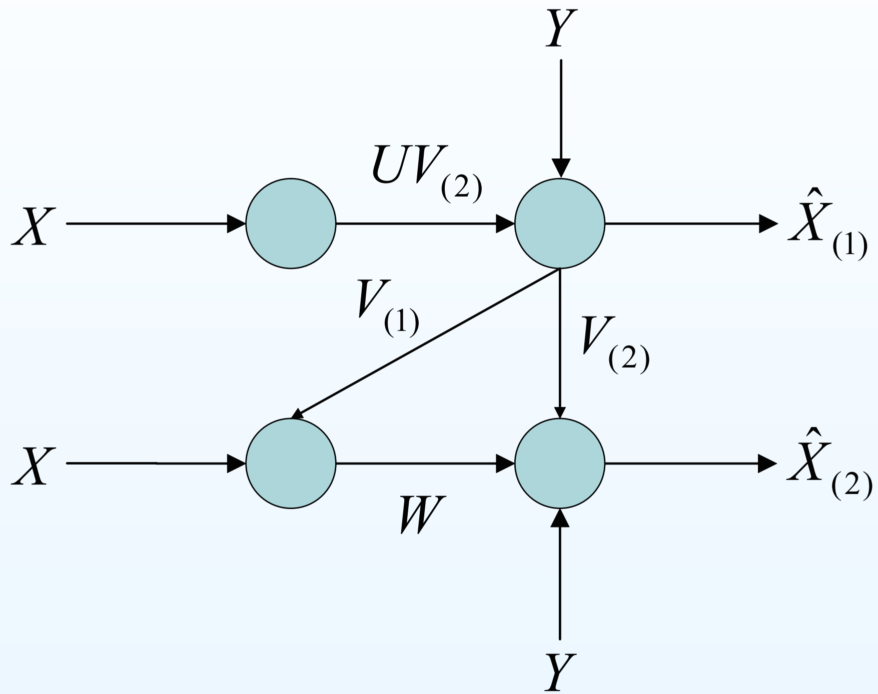
Remark .
The following equation is equivalent to the Markov conditions of the outer bound:

$$\begin{aligned} & P_{UV^{(1)}V^{(2)}WXY} \\ & \quad = \quad P_{XY}P_{U|X}P_{V^{(2)}|UXY}P_{V^{(1)}|UV^{(2)}Y}P_{W|V^{(1)}V^{(2)}X}. \end{aligned}$$

Also, the following equation is equivalent to the Markov conditions of the inner bound:

$$\begin{aligned} & P_{UV^{(1)}V^{(2)}WXY} \\ & \quad = \quad P_{XY}P_{U|X}P_{V^{(2)}|UXY}P_{V^{(1)}|V^{(2)}Y}P_{W|V^{(1)}V^{(2)}X}. \end{aligned}$$

Intuitive interpretation



$$\mathcal{R}_{cf}(X,Y|D_1,D_2) = \{(R_{01},R_{02},R_1,R_2) :$$
$$R_1 \geq I(X;UV^{(2)}|Y),\ R_{01} \geq I(Y;V^{(1)}|UV^{(2)}X),$$
$$R_{02} \geq I(X;V^{(2)}|Y),\ R_2 \geq I(X;W|V^{(1)}V^{(2)}Y)\}$$

$U \to X \to Y$, $V^{(1)} \to UV^{(2)}Y \to X$, $W \to V^{(1)}V^{(2)}X \to UY$ (Outer)
$UV^{(2)} \to X \to Y$, $V^{(1)} \to V^{(2)}Y \to UX$, $W \to V^{(1)}V^{(2)}X \to UY$ (Inner)

Concluding remarks

- Coding problems for correlated information sources were investigated for communication systems that we call
  - the cascading refimenet system,
  - the feedback refinement system,
  - the cascading and feedback refinement system.
- Outer and inner bounds of achievable rate-distortion regions for those problems were clarified.

[Future works]

- Find special cases where the inner bounds coincide with the outer bounds
- Computing explicit outer and inner bounds
- Expanding the results to more general cases (e.g. sources, alphabets, distortion measures)

Thank you

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Some materials will be available at *http://www.brl.ntt.co.jp/people/akisato/*