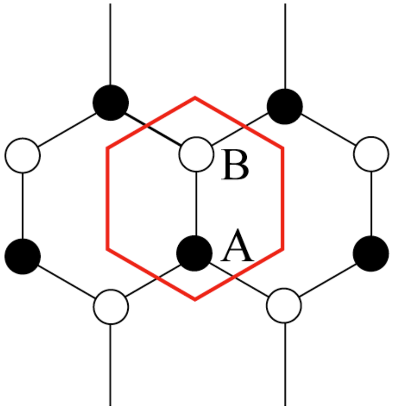
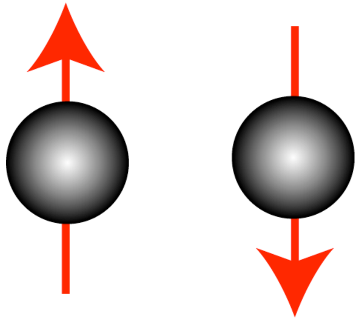
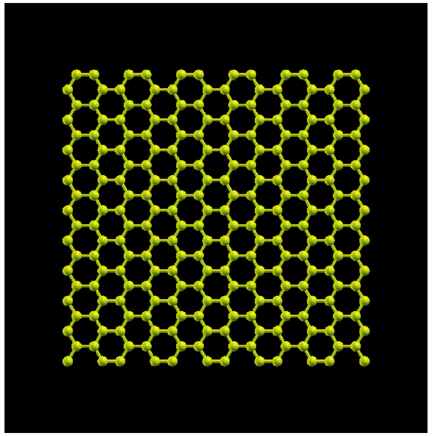
Gauge field for the edge states in graphene



pseudospin



realspin

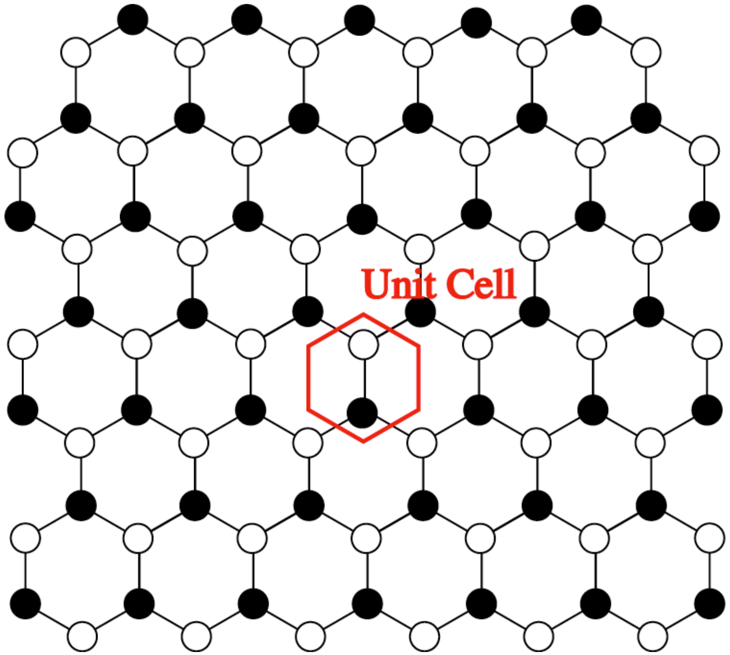


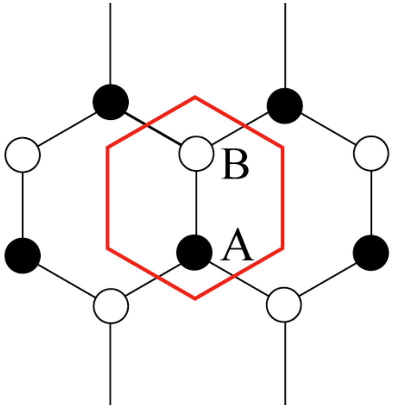
edge state

K. Sasaki[1], S. Murakami[2] and R. Saito[3]

[1]Department of Quantum Matter, Hiroshima University, Japan
[2]Department of Physics, Tokyo Institute of Technology, Japan
[3]Department of Physics, Tohoku University, Japan

What is the pseudospin?





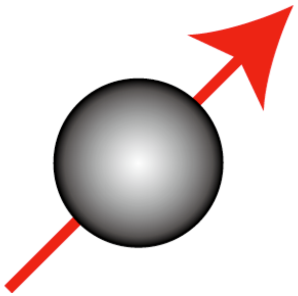
Two atoms in the unit cell:
A-atom and B-atom

Wave function of a unit cell

$$\Psi(\mathbf{r}) = \begin{pmatrix} \Psi_{\mathrm{A}}(\mathbf{r}) \\ \Psi_{\mathrm{B}}(\mathbf{r}) \end{pmatrix}$$

---

real spin



$$\Psi_{\uparrow} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

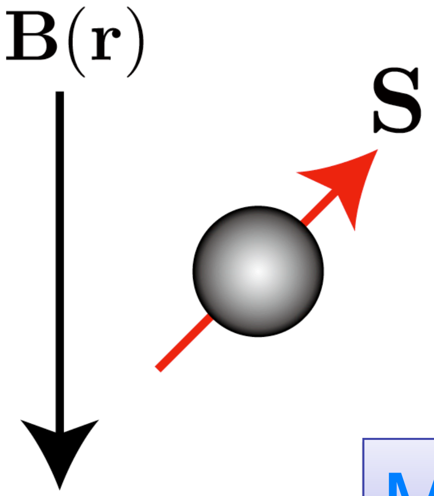
$$\Psi_{\downarrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\Psi_{\mathrm{spin}} = C_{\uparrow} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_{\downarrow} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\Psi(\mathbf{r}) = \Psi_{\mathrm{A}}(\mathbf{r}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \Psi_{\mathrm{B}}(\mathbf{r}) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

pseudo spin

Magnetic field for a real spin

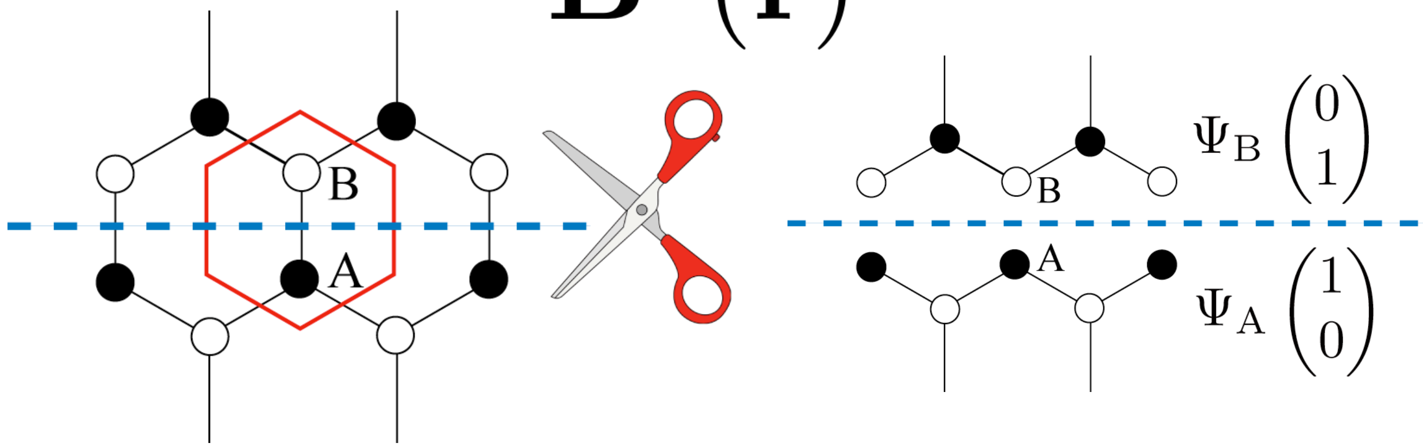


$$\frac{-e}{m_e c}\mathbf{S}\cdot\mathbf{B}(\mathbf{r})$$

Zeeman term

Magnetic field for pseudospin

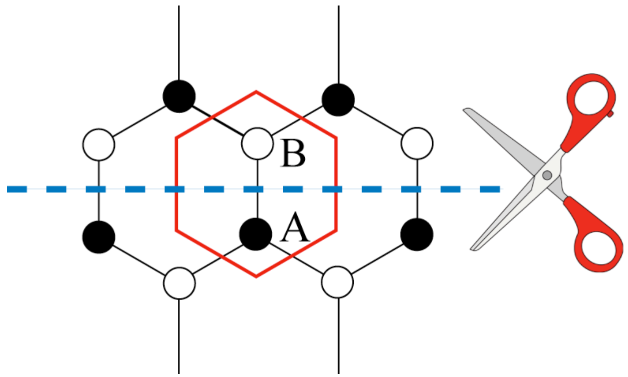
$$\mathbf{B}^{\mathrm{q}}(\mathbf{r})$$



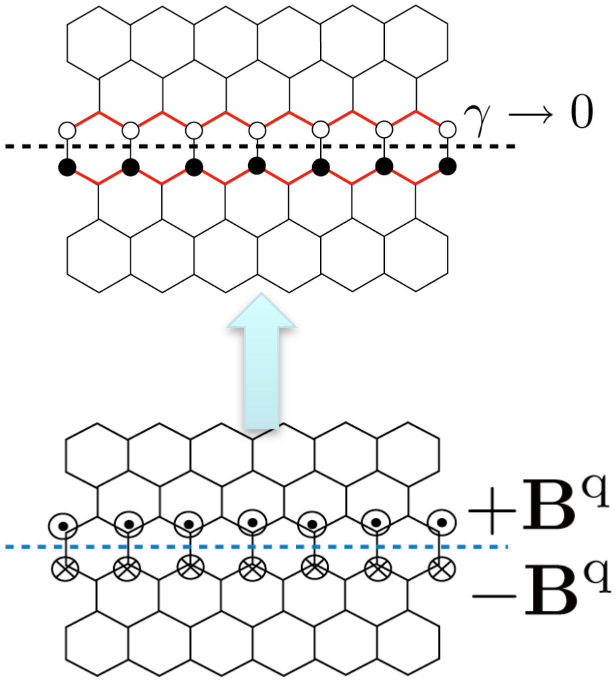
$$\Psi(\mathbf{r}) = \Psi_{\mathrm{A}}(\mathbf{r})\begin{pmatrix}1\\0\end{pmatrix} + \Psi_{\mathrm{B}}(\mathbf{r})\begin{pmatrix}0\\1\end{pmatrix}$$

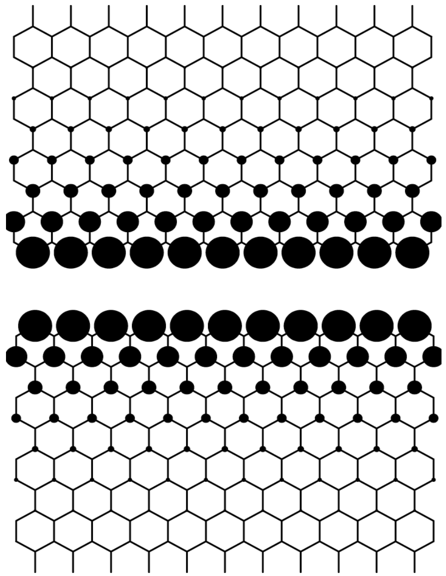
Edge states

Fujita *et al.*, J. Phys. Soc. Jpn. **65** (1996)



$$\Psi(\mathbf{r}) = \Psi_{\mathrm{A}}(\mathbf{r}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \Psi_{\mathrm{B}}(\mathbf{r}) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$





B-atom

A-atom

pseudospin down

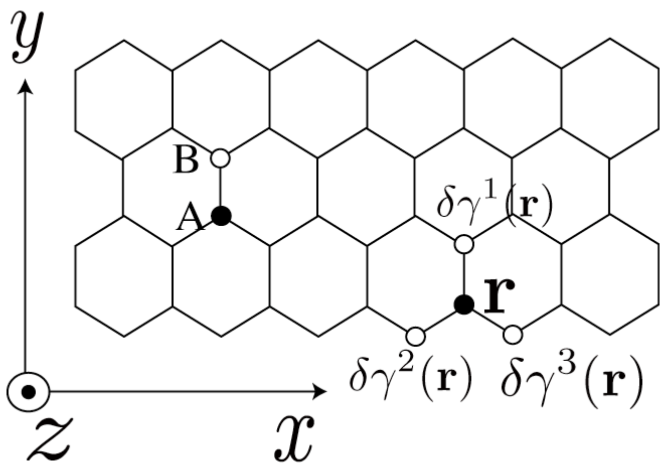
$$\Psi \propto e^{-\frac{|y|}{\xi}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\Psi \propto e^{-\frac{|y|}{\xi}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

pseudospin up

Edge state is a pseudospin polarized state

Gauge field for lattice deformation

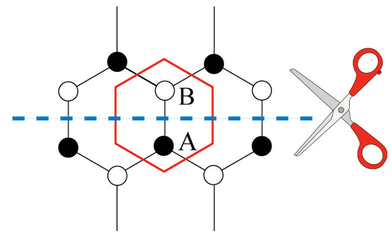


$$-\gamma + \delta\gamma^i(\mathbf{r})$$

Local modulation of hopping integral
(lattice distortion)

$$\mathcal{H}_{\mathrm{K}} = v_F \sigma \cdot (\hat{\mathbf{p}} + \mathbf{A}^{\mathrm{q}}(\mathbf{r}))$$

Weyl equation



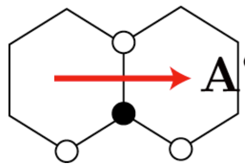
Deformation-induced gauge field

$$v_F A_x^{\mathrm{q}}(\mathbf{r}) = \delta\gamma^1(\mathbf{r}) - \frac{1}{2}\left(\delta\gamma^2(\mathbf{r}) + \delta\gamma^3(\mathbf{r})\right)$$

$$v_F A_y^{\mathrm{q}}(\mathbf{r}) = \frac{\sqrt{3}}{2}\left(\delta\gamma^2(\mathbf{r}) - \delta\gamma^3(\mathbf{r})\right)$$

Sasaki *et al.*, Prog. Theo. Phys. **113** (2005)

$\delta\gamma^1(\mathbf{r})$



$$\mathbf{A}^{\mathrm{q}}(\mathbf{r}) = (A_x^{\mathrm{q}}(\mathbf{r}), 0)$$

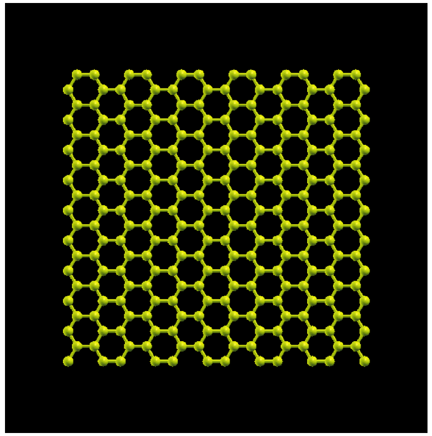
For a uniform deformation

Kane and Mele, Phys. Rev. Lett. **78** (1997)

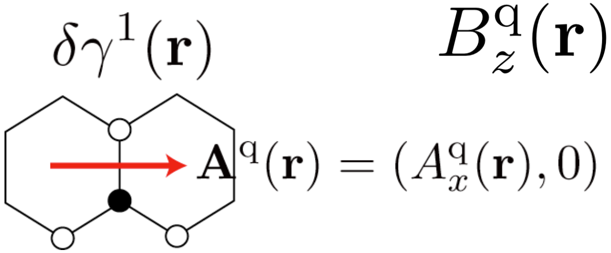
$$v_F A_x^{\mathrm{q}}(\mathbf{r}) = \delta\gamma^1(\mathbf{r})$$

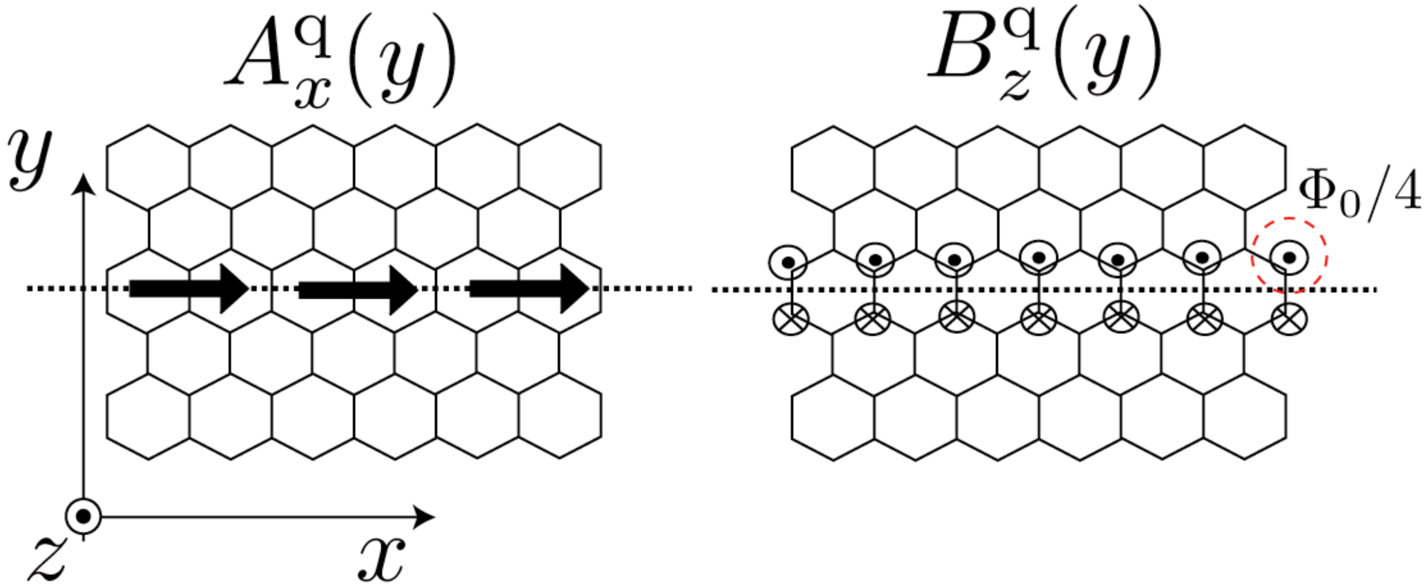
$$v_F A_y^{\mathrm{q}}(\mathbf{r}) = 0$$

Deformation-induced magnetic field



$$B_z^{\mathrm{q}}(\mathbf{r}) = \frac{\partial A_y^{\mathrm{q}}(\mathbf{r})}{\partial x} - \frac{\partial A_x^{\mathrm{q}}(\mathbf{r})}{\partial y}$$

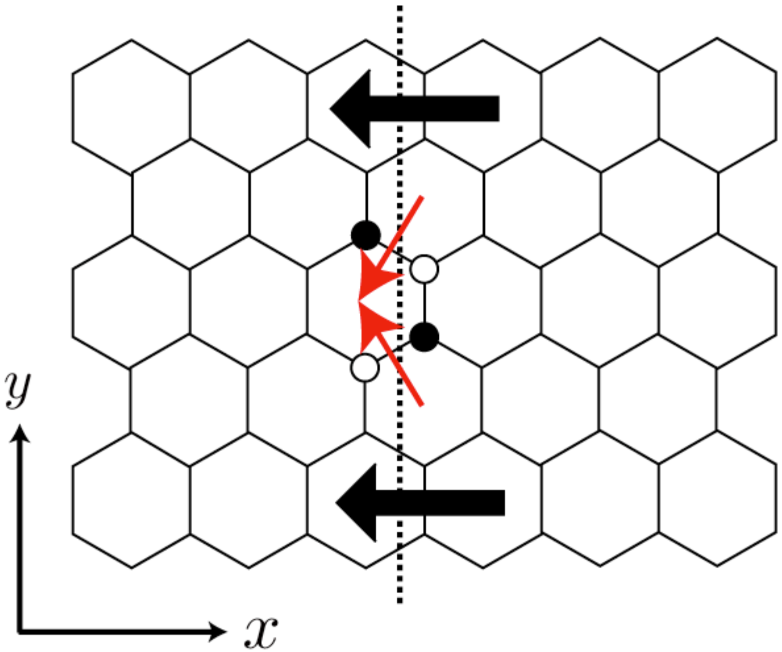




The Weyl equation with the gauge field reproduces the known properties of the edge states.

Sasaki *et al*., J. Phys. Soc. Jpn.**75** (2006)

Armchair edge

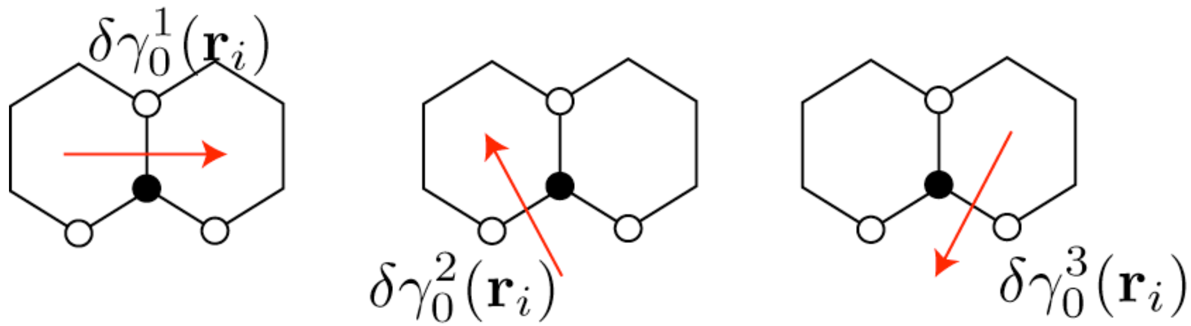


$$v_F A_x^{\mathrm{q}}(\mathbf{r}) = \delta\gamma^1(\mathbf{r}) - \frac{1}{2}\left(\delta\gamma^2(\mathbf{r}) + \delta\gamma^3(\mathbf{r})\right)$$

$$v_F A_y^{\mathrm{q}}(\mathbf{r}) = \frac{\sqrt{3}}{2}\left(\delta\gamma^2(\mathbf{r}) - \delta\gamma^3(\mathbf{r})\right)$$

$$A_x^{\mathrm{q}}(x)$$

Direction of the gauge field



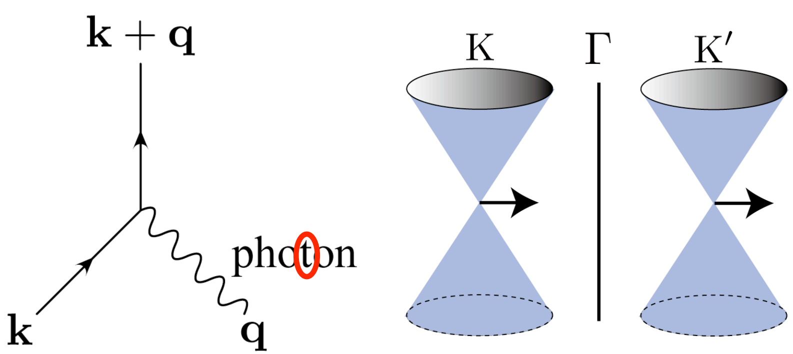
$$B_z^{\mathrm{q}}(\mathbf{r}) = \frac{\partial A_y^{\mathrm{q}}(\mathbf{r})}{\partial x} - \frac{\partial A_x^{\mathrm{q}}(\mathbf{r})}{\partial y}$$

$$B_z^{\mathrm{q}}(\mathbf{r}) = 0$$

No pseudospin polarization

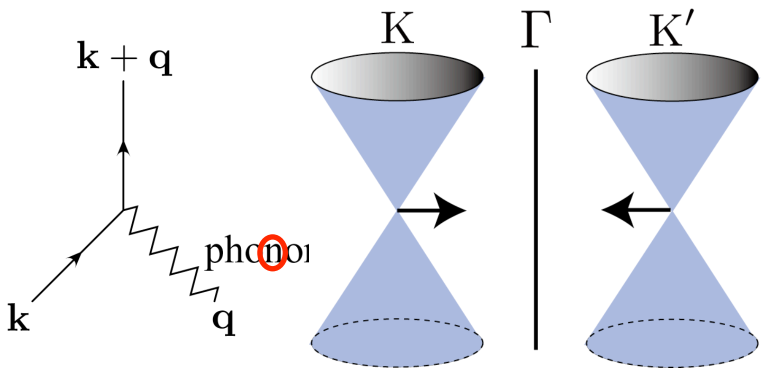
Two gauge fields

Electro-magnetic gauge field



breaks time-reversal symmetry

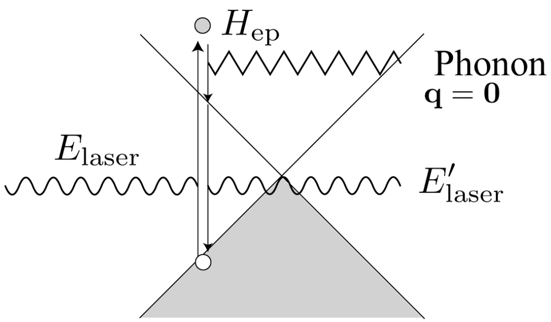
Deformation-induced gauge field



K-point $\hat{\mathbf{p}} \longrightarrow \hat{\mathbf{p}} - e\mathbf{A}(\mathbf{r})$ $\qquad \hat{\mathbf{p}} \longrightarrow \hat{\mathbf{p}} + \mathbf{A}^{\mathrm{q}}(\mathbf{r})$

K'-point $\hat{\mathbf{p}} \longrightarrow \hat{\mathbf{p}} - e\mathbf{A}(\mathbf{r})$ $\qquad \hat{\mathbf{p}} \longrightarrow \hat{\mathbf{p}} - \mathbf{A}^{\mathrm{q}}(\mathbf{r})$

Sasaki *et al.*, Prog. Theo. Phys. **113** (2005)



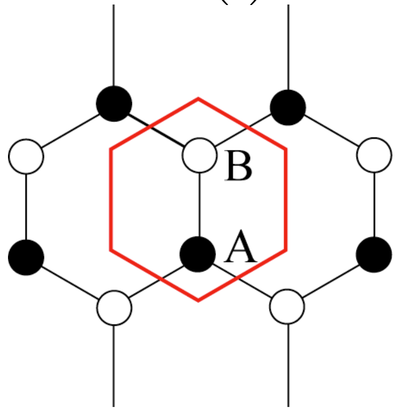
$$\mathcal{H}_{\mathrm{K}} = v_F \sigma \cdot (\hat{\mathbf{p}} + \mathbf{A}^{\mathrm{q}}(\mathbf{r}) - e\mathbf{A}(\mathbf{r}))$$

Raman spectroscopy of carbon nanotubes

K. S, R. Saito, G. Dresselhaus, M.S. Dresselhaus, H. Farhat, and J. Kong, PRB**77**, 245441 (2008); PRB**78**, 235405 (2008)

Summary

$$\Psi(\mathbf{r}) = \Psi_{\mathrm{A}}(\mathbf{r})\begin{pmatrix}1\\0\end{pmatrix} + \Psi_{\mathrm{B}}(\mathbf{r})\begin{pmatrix}0\\1\end{pmatrix}$$



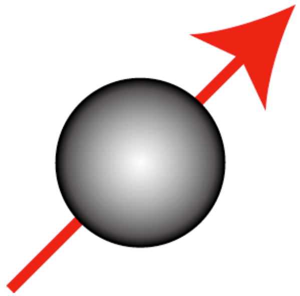
pseudo spin

$$\mathbf{B}^{\mathrm{q}}(\mathbf{r})$$

$$\mathbf{B}^{\mathrm{q}}(\mathbf{r}) = \nabla \times \mathbf{A}^{\mathrm{q}}(\mathbf{r})$$

Deformation-induced gauge field

$$\mathbf{B}(\mathbf{r})$$

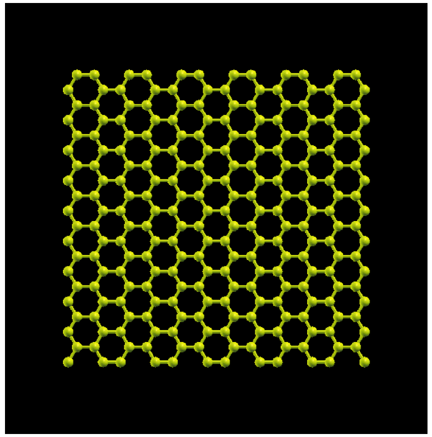


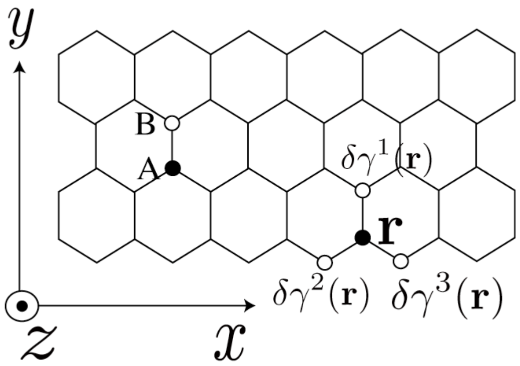
real spin

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$$

$$v_F A_x^{\mathrm{q}}(\mathbf{r}) = \delta\gamma^1(\mathbf{r}) - \frac{1}{2}\left(\delta\gamma^2(\mathbf{r}) + \delta\gamma^3(\mathbf{r})\right)$$

$$v_F A_y^{\mathrm{q}}(\mathbf{r}) = \frac{\sqrt{3}}{2}\left(\delta\gamma^2(\mathbf{r}) - \delta\gamma^3(\mathbf{r})\right)$$



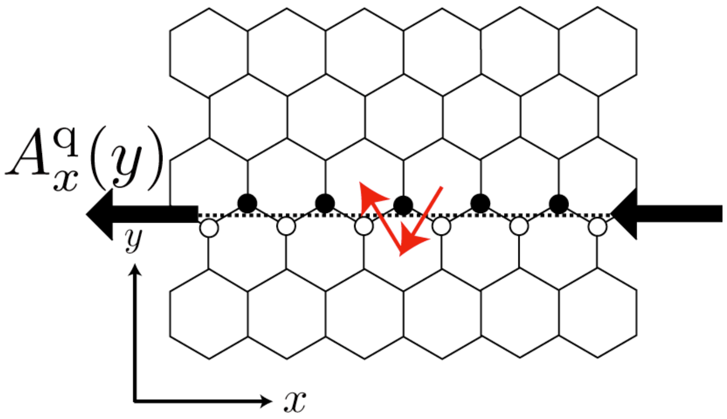


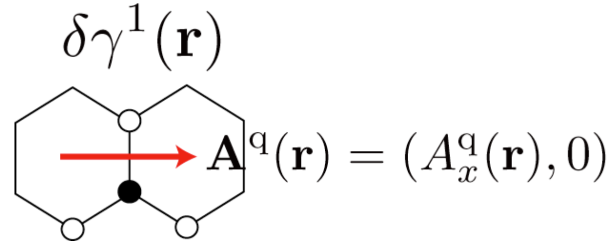
$$\mathcal{H}_{\mathrm{K}} = v_F \sigma \cdot (\hat{\mathbf{p}} + \mathbf{A}^{\mathrm{q}}(\mathbf{r}))$$

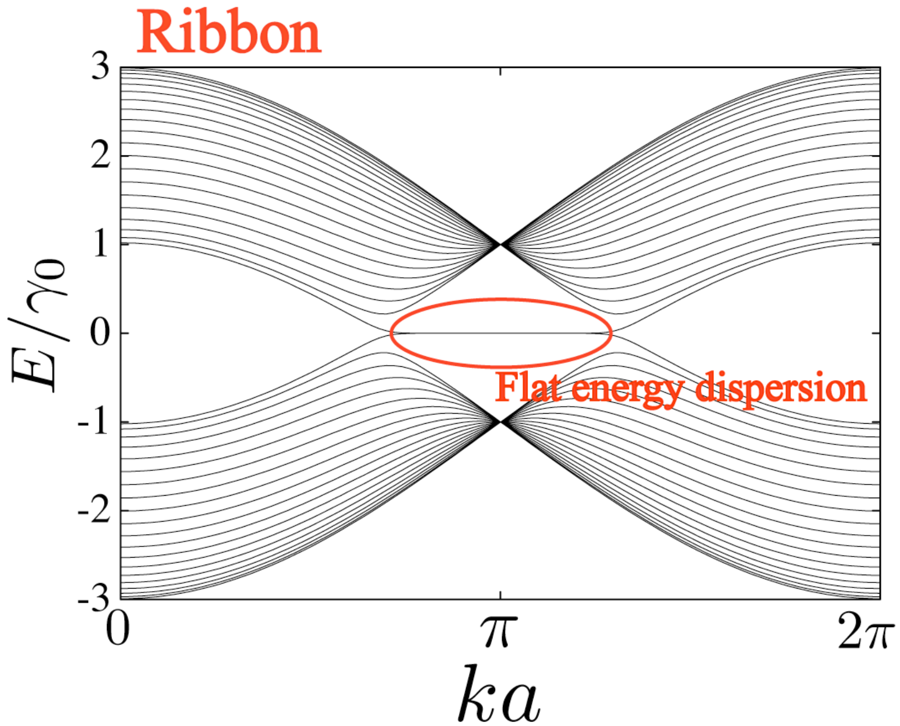
$$\mathcal{H}_{\mathrm{K}'} = v_F \sigma' \cdot (\hat{\mathbf{p}} - \mathbf{A}^{\mathrm{q}}(\mathbf{r}))$$

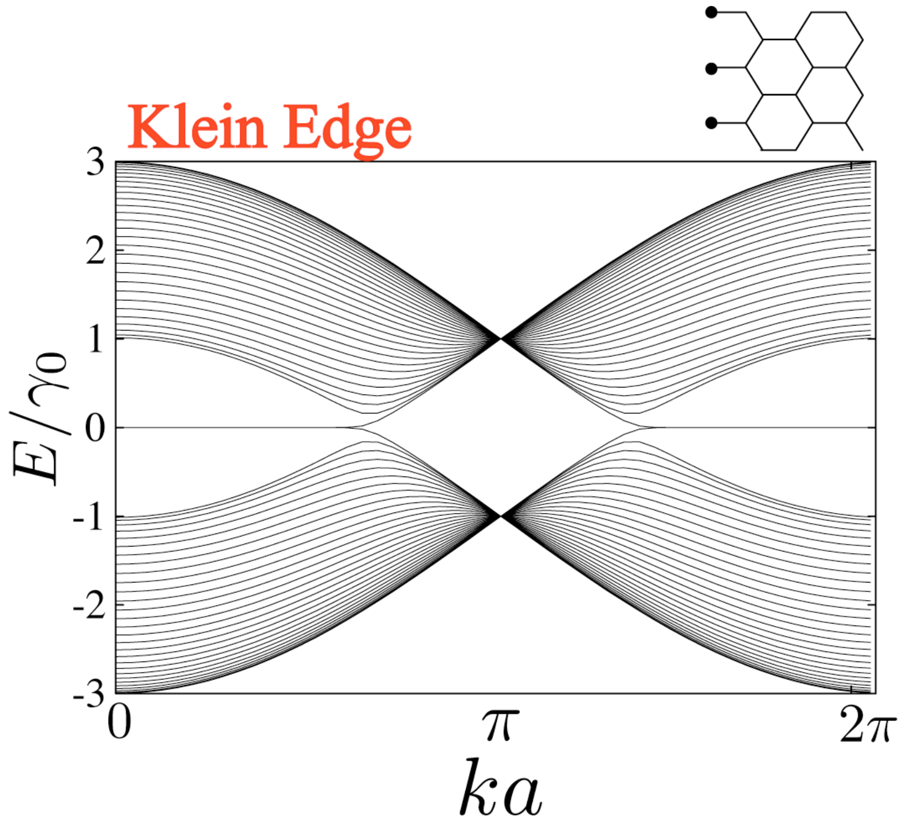
Review article on pseudospin in graphene:
Sasaki and Saito, Prog. Theo. Phys. Suppl. **176** (2008)

Zigzag and Klein edges







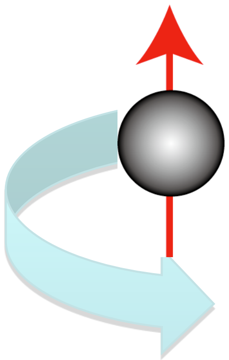


Similarities between Spin & Pseudospin

a minus sign under 2π rotation

real spin

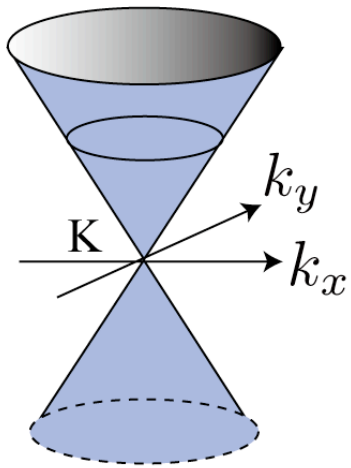
$$R(2\pi)\Psi_{\rm spin} = -\Psi_{\rm spin}$$

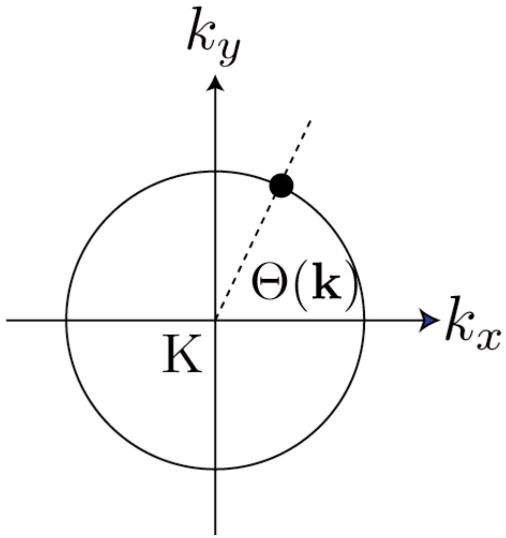


pseudo spin

Bloch states

$$\Psi(\mathbf{k}) = \begin{pmatrix} \Psi_{\rm A}(\mathbf{k}) \\ \Psi_{\rm B}(\mathbf{k}) \end{pmatrix}$$





$$\Psi(\mathbf{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\Theta(\mathbf{k})/2} \\ e^{+i\Theta(\mathbf{k})/2} \end{pmatrix}$$

$$R(2\pi)\Psi(\mathbf{k}) = -\Psi(\mathbf{k})$$

Absence of backward scattering
(Berry's phase)

Ando *et al*., JPSJ**67** (1998)