1. Introduction

Since the concept of the Kondo effect was established in 1964 by Professor Kondo,\(^1\) a number of experimental and theoretical studies have been performed on various kinds of bulk metals to explore the many-body effects in the condensed matter physics. The interest in the Kondo effect was renewed when the Kondo effect was observed in 1998 for semiconductor quantum dot devices.\(^2\) These devices have enabled studies on the Kondo effect associated with a single isolated magnetic impurity or a spin-half electron trapped in a quantum dot, instead of a large ensemble of magnetic impurities in bulk metal. In addition, various parameters that influence the many-body effects are handled in a controlled manner in quantum dot devices. These advantages of quantum dots have opened up new approaches to the Kondo physics, resulting in a number of novel findings.

The Kondo effect in quantum dots arises from the singlet coupling between a localized electron spin in a dot and Fermi seas of the tunnel-coupled contact leads.\(^3\) Figure 1(a) shows an energy diagram of such a quantum dot having one spin-up electron at the uppermost level whose energy is \(E_0\). The total spin \(S = 1/2\) for this dot. The coupling of the electron spin to Fermi seas in the contact leads has tunnel rates, \(\Gamma_L\) and \(\Gamma_R\), for the left and right barriers, respectively. \(E_0\) is adjusted so that \(E_0 < E_F < E_0 + U\), where \(E_F\) is the Fermi energy and \(U\) is the charging energy. Then, the first-order tunneling through the dot is inhibited because of \(U\) (Coulomb blockade). However, when the temperature is lowered to \(\sim T_K\) (Kondo temperature), the electron in the dot forms a singlet coupling with a spin-down electron in the lead, and concurrent tunneling of these two electrons contributes to a net electron transfer between the source and drain leads as depicted in Fig. 1(a). At the same time, a Kondo resonance peak appears in the local density of states (DOS) at the Fermi energy. Therefore, the conductance starts to increase at \(T \sim T_K\). This behaviour is qualitatively opposite to that in bulk metal because the current path is only through the “magnetic impurity” in the case of the quantum dot. The Kondo temperature, \(T_K\), is given as

\[
T_K = \frac{\sqrt{\Gamma U}}{2} \exp[-\pi(E_F - E_0)(U - E_F + E_0)/\Gamma U],
\]

where \(\Gamma = \Gamma_L + \Gamma_R\). The above second order tunneling occurs coherently and tends to screen the initial localized magnetic moment in the dot through the dot-lead spin singlet formation. Figure 1(b) shows the Kondo effect theoretically.
predicted for Coulomb oscillations in a quantum dot.\textsuperscript{3)} Regular Coulomb oscillations, which appear at temperature \(T > T_K\), are significantly modified as the temperature becomes comparable to or lower than \(T_K\). In the region of the conductance valley (Coulomb valley), the number of electrons, \(N\), in the dot is fixed to an integer. However, the valley conductance for odd \(N\) with \(S = 1/2\) increases and finally reaches the unitary limit (linear conductance = \(N\) electrons).

are schematically shown in Fig. 2(a). The electrochemical potential \(\mu\) is tuned, usually takes a total spin, \(S\), of \(N\) electrons, \(N\), and \(1/2\) for filling the dot, and found that the electronic configuration is significantly modified for the filling of nearly degenerate orbital states. The orbital degeneracy can be adjusted as a function of magnetic field \(B\). This allows us to investigate the Kondo effect for tunable electronic configurations.

Here we assume two orbital states crossing with each other at a magnetic field of \(B = B_0\). By taking into account spin degeneracy, these orbital states are consecutively filled by four electrons in total. If these are the electrons from the orbital state with the parallel spins in a single orbital state for \(B\) far away from \(B_0\), however, close to the point of \(B = B_0\), the GS is a spin triplet state \((S = 1)\) having parallel spins in two different orbital states, following Hund’s rule.\textsuperscript{10,11)} The triplet GS is a spin singlet state \((S = 0)\) having two anti-parallel spins in a single orbital state for \(B\) far away from \(B_0\). However, close to the point of \(B = B_0\), the GS is a spin triplet state \((S = 1)\) having parallel spins in two different orbital states, following Hund’s rule.\textsuperscript{10,11)} The triplet GS is a spin singlet state \((S = 0)\) having two anti-parallel spins in a single orbital state for \(B\) far away from \(B_0\). However, close to the point of \(B = B_0\), the GS is a spin triplet state \((S = 1)\) having parallel spins in two different orbital states, following Hund’s rule.\textsuperscript{10,11)}

Fig. 2. (a) Schematic magnetic field dependence of the electrochemical potential for electron numbers from \(N + 1\) (odd) to \(N + 4\) (even) occupying two crossing orbitals. A triplet state \((S = 1)\) appears around the level crossing field at \(N + 2\). (b) Fock–Darwin states calculated with \(\hbar \omega_0 = 1\) meV. The circle denotes crossing orbital state \((n, l) = (0, -1)\) (dotted line) and \((0, 7)\) (thick solid line) where the Kondo effect for \(N = 15\) and 16 are studied in detail.
3. The Kondo Effect Enhanced by State Degeneracy

We use a technique of manipulating S–T and D–D degeneracies described in the preceding section to study the Kondo effect in our quantum dot device. Here we also use a vertical quantum dot but having a much stronger coupling between the dot and contact leads. The tunnel barriers are made from two 7-nm-thick AlGaAs barriers for this dot device is 400μeV for the first electron entering the dot, and gradually increases as N increases. As described before, the values of S and N can be unambiguously determined in this quantum dot. All the transport measurements were performed in a dilution refrigerator with a base temperature of ≈60 mK, using a standard lock-in technique with an ac excitation voltage between source and drain of 3μV. This excitation voltage is much smaller than $k_B T_K$.

Figure 4 shows a gray-scale plot of the linear conductance, $G$, as a function of $V_g$ and $B$ at the base temperature. White stripes indicate Coulomb peaks between $N = 0$ and 21. The overall features observed here such as shell filling and spin pairing are consistent with those in Fig. 3. The magnetic evolutions of the peak pairs are assigned to successive filling of FD states\(^{13}\) by spin-up and spin-down electrons.

On the other hand, marked difference from Fig. 3 is observed for $V_g > -1.0$ V. We see many white vertical lines connecting neighboring Coulomb peaks in the regions where pairs of peaks become close to each other. These vertical lines indicate lifting of Coulomb blockade due to the Kondo effect. There are more such lines as $V_g$ increases, because in

![Fig. 3. Evolution of current peaks for $N = 1$ to 14 with magnetic field measured for $V_{sd} = 120\mu$V. The dotted line indicates the magnetic field of the filling factor $\nu = 2$. Dotted ovals indicate the regions where two crossing Fock–Darwin states are consecutively filled by four electrons. The four peak lines to the top in each oval correspond to electrochemical potentials of $\mu(N + 1)$ to $\mu(N + 4)$, respectively.](image)

and evolution of neighboring peaks in pairs with magnetic field due to consecutive anti-parallel spin filling of a single orbital state.

The eigenstates confined by a 2D harmonic potential in the presence of a magnetic field perpendicular to the 2D plane of the dot are the Fock–Darwin (FD) states with energies $E_{n,l}^{13}$ is shown in Fig. 2(b);

$$E_{n,l} = \frac{l}{2} \hbar \omega_l + \left( n + \frac{1}{2} + \frac{1}{2} |l| \right) \hbar \sqrt{4\omega_0^2 + \omega_l^2}, \quad (2)$$

where $n = 0, 1, 2, \ldots$ is the radial quantum number and $l = 0, \pm 1, \pm 2, \ldots$ is the angular quantum number. $\hbar \omega_0$ is the lateral confinement energy and $\hbar \omega_l = eB/m^*$. Zeeman splitting is neglected, so each state is two-fold degenerate. The evolutions of the paired lines in Fig. 3 are well reproduced by the FD diagram. Note crossing of the FD states, or orbital and spin degeneracy, is lifted by the interaction effect in Fig. 3. So the wiggles or anti-crossings between pairs of peaks correspond to the crossings of FD states. Modifications to the simple pairing of peaks are observed in each dashed oval connecting pairs of peaks along the dashed line at non-zero field. This oval indicates the four-electron filling at the crossing of two FD states, and the four peak lines inside the oval are well reproduced by our electrochemical potential model of Fig. 2(a).

![Fig. 4. Gray-scale plot of the linear conductance as a function of the magnetic field and gate voltage, or $B$–$V_g$ diagram. Black corresponds to zero conductance, and white to $G = 50 \mu$S. All the electrons occupy the first Landau level on the right hand side of the dashed line (filling factor $\nu = 2$). The magnetic field induced Kondo effect is observed within region A. Spin flip transitions occur within region B until completely spin polarized $\nu = 1$ state is reached. The inset shows a schematic diagram of the dot structure made from AlGaAs/InGaAs/AlGaAs double barrier tunnel-diode.](image)
our quantum dot device, $\Gamma$ gradually increases with increasing $V_g$.

Figure 5 shows detailed measurement conducted in region A marked in Fig. 4. The last orbital crossings occur between $E_{nl}$ states with $(n, l) = (0, 1)$ and $(0, l)$ $(l > 1)$ on the dotted line, and all the electrons occupy the ground Landau level at higher $B$ ($v = 2$). A spin triplet state is observed at $N = 16$ and $B \simeq 1.2$ T, where states $(n, l) = (0, 1) - (0, 7)$ are occupied by electrons having parallel spins [see the circle in Fig. 2(b)]. When we compare the white lines in Fig. 5 with the electrochemical potentials vs $B$ in Fig. 2(a), we find that the white vertical lines in Fig. 5 fall onto the dotted and dash-dotted lines in Fig. 2(a). For example, in the $N = 16$ Coulomb valley, the conductance is enhanced at $B \simeq 1.1$ T and $B \simeq 1.3$ T corresponding to the dotted lines in Fig. 2(a) where the singlet and triplet GS are degenerate. These are both assigned to the S–T Kondo effect. The Kondo temperature, $T_{K}^{S-T}$, is considerably higher than the conventional $S = 1/2$ Kondo temperature, $T_{K}^{0}$, because of the larger degeneracy.\(^9\) As for $N = 15$ and 17, the conventional $S = 1/2$ Kondo effect is expected. However, the conductance enhancement in the Coulomb valleys is not clearly observed except in the regions corresponding to the dash-dotted lines in Fig. 2(a), where two $S = 1/2$ states with different total angular momentum, $M$, are degenerate. When such an orbital degeneracy is present for odd $N$, a total of four states, i.e., $M = M_1, M_2$ $(M_1 \neq M_2)$, $S_z = \pm 1/2$, are involved in forming the Kondo singlet state if the Zeeman splitting is negligible. Then, an enhancement of $T_{K}$ is expected reflecting this four-fold degeneracy as in the S–T Kondo effect. We refer this type of Kondo effect for odd $N$ to “doublet–doublet” (D–D) Kondo effect.\(^12\) Because $T_{K}^{0}$ is much lower than the D–D Kondo temperature, $T_{K}^{D-D}$, only a slight conductance enhancement is observed in the odd $N$ Coulomb valleys when there is no orbital degeneracy. Since many orbital crossings occur before the system enters the $v = 2$ regime, a honeycomb pattern is formed in a $B$–$N$ diagram by the high conductance region, provided $T_{K}^{0} < T < T_{K}^{S-T}, T_{K}^{D-D}$. Such honeycomb pattern is clearly captured in Fig. 5, due to the S–T and D–D Kondo effects that occur consecutively for different orbital crossings.

Our honeycomb pattern is different from “chessboard pattern” discussed in a lateral quantum dot.\(^14-17\) In a vertical quantum dot, one can assume that the orbital quantum numbers are conserved in tunnel processes between the dot and leads due to their same rotational symmetry. Hence we expect “two channels” of conduction electrons in the leads when two orbitals are relevant in the quantum dot; each channel couples to only one of the two orbitals. On the other hand, a “single channel” in the leads preferentially couples to the outer orbital in the case of a lateral quantum dot. Then, high-conductance Coulomb valleys appear alternately in $B$–$N$ diagram due to the Kondo effect involving electrons in the outer orbital.

Figures 6(a) and 6(b) show temperature dependence of the differential conductance $dI/dV_{sd}$ vs $V_{sd}$ for the S–T ($N = 16$) and D–D ($N = 15$) Kondo effect, respectively. The gate voltage is fixed in the center of the respective Coulomb valley (solid and open triangle in Fig. 5). A clear Kondo peak at $V_{sd} = 0$ V is observed at low temperatures, whose height decreases with increasing temperature as expected for the Kondo effect.

After a background subtraction of non-Kondo cotunneling component,\(^18\) the above temperature dependence of the Kondo peak height is fitted to the function

$$\frac{G}{G_0} = \left( \frac{T}{T_{K}^{0}} \right)^{1/2},$$

where $G_0$ is the low temperature limit conductance and $T_{K}^{0}$ does not reach the unitary limit conductance of $2e^2/h$.\(^19,20\) $G_{0}$ does not reach the unitary limit conductance of $2e^2/h$ probably because of the asymmetry in the two tunnel barriers; it is impossible to tune $\Gamma$ to the upper and lower leads separately in our vertical quantum dot because $\Gamma$ is pre-determined by the growth parameter of the

![Fig. 5. Detailed measurement conducted in region A in Fig. 4. The gray-scale is same as in Fig. 4. S, T and D denote states with $S = 0$ (singlet), $S = 1$ (triplet) and $S = 1/2$ (doublet). The enhanced conductance due to the Kondo effect is observed in the Coulomb valley corresponding to the dotted and dash-dotted lines in Fig. 2(a).](image)

![Fig. 6. Temperature dependence of the differential conductance $dI/dV_{sd}$ vs $V_{sd}$ from $T = 60 \text{ mK}$ (thick solid line) to 1.5 K (dotted line) for (a) S–T Kondo effect, (b) D–D Kondo effect and (c) $S = 1/2$–$S = 3/2$ degeneracy.](image)
material. The $T^s_{K}^{S-T}$ and $T^D_{K}^{D-D}$ estimated from the curve fitting are 700 mK and 490 mK, respectively. The higher $T_K$ for the S–T Kondo effect may be due to the larger $\Gamma$, and it is difficult to experimentally determine which is larger, $T^s_{K}^{S-T}$ or $T^D_{K}^{D-D}$, for the same $\Gamma$. The fitted values of the parameter $s$ are 0.8 for the S–T Kondo effect and 1.1 for the D–D Kondo effect, much larger than $s \approx 0.2$ for a conventional spin 1/2 system. However, the estimation of $s$ is less reliable because it changes substantially with the chosen fitting range. The expected Zeeman splitting of $\Delta B \approx 30 \mu$eV at $B \approx 1.2$ T is smaller than $T_K$ estimated above. Therefore, a Zeeman splitting in the $d/dV_{sd}$ Kondo peak is not resolved, and we are allowed to treat all four S–T and D–D states as quasi-degenerate.

Figures 7(a) and 7(b) show a gray-scale plot of $dI/dV_{sd}$ in $B$–$V_{sd}$ plane for the S–T Kondo effect ($N = 16$) and for the D–D Kondo effect ($N = 15$), respectively, with $V_s$ fixed in the center of the respective Coulomb valley. Conductance peaks at $V_{sd} = 0$ V are observed near the degeneracy field, $B_0$ ($B_0 = 1.255$ T for D–D, 1.12 and 1.25 T for S–T). The two zero-bias S–T Kondo peaks in Fig. 7(a) correspond to the two conductance maxima in the $N = 16$ Coulomb valley (see Fig. 5). Because the S–T or D–D degeneracy is lifted as $|\Delta B| = |B - B_0|$ increases, the Kondo effect is broken and the zero-bias peak is suppressed. At large $|\Delta B|$, a peak or step is observed at $eV_{sd} = \pm \Delta$ where the brightness suddenly changes. Here, $\Delta$ is the $B$-dependent energy difference between the singlet and triplet states, or between the two doublet states. This peak/step is due to cotunneling associated with the two states separated by $\Delta$. The observed D–D Kondo peak conductance drops to zero suddenly on the singlet side. At large $|\Delta B|$, the Kondo effect is broken and the zero-bias peak is suppressed. At large $|\Delta B|$, a peak or step is observed at $eV_{sd} = \pm \Delta$ where the brightness suddenly changes. Here, $\Delta$ is the $B$-dependent energy difference between the singlet and triplet states, or between the two doublet states. This peak/step is due to cotunneling associated with the two states separated by $\Delta$. The observed D–D Kondo peak conductance drops to zero suddenly on the singlet side.

Figure 7 shows $V_{sd}$ values of the conductance peak/step as a function of $\Delta B$. Peak/step positions for both the S–T ($B_0 = 1.12$ T) and the D–D ($B_0 = 1.25$ T) Kondo effect almost coincide, indicating that they involve the same orbital states, namely $(n, l) = (0, -1)$ and $(0, 7)$.

Figure 8(a) compares the relative conductance, $\Delta G$, measured from the degeneracy ($\Delta B = 0$) at $V_{sd} = 0$ V, as a function of $\Delta B$. The scaling calculation $8,21$ has shown that, in the S–T Kondo effect, $T_K(\Delta) \approx (T_K(0)/\Delta^{\gamma})$ with $\gamma = 2 + \sqrt{5}$ on the triplet side, whereas $T_K(\Delta)$ drops to zero suddenly on the singlet side. The observed S–T Kondo peak conductance drops more quickly on the singlet side ($\Delta B < 0$) than on the triplet side ($\Delta B > 0$) reflecting the asymmetric behavior of $T_K$ given by the scaling calculation.

Another scaling calculation was recently applied to the D–D Kondo effect, $12,22$ and $\gamma = 1$ was obtained in the case of an equivalent dot-lead coupling for the two orbitals involved. The observed D–D Kondo peak conductance
drops more slowly and symmetrically compared to the S−T Kondo case, i.e., $T_{K_{SD}}^0$ is more robust against degeneracy lifting than $T_{K_{SD}}^0$, in qualitative agreement with the theory. Please note that, in Fig. 5, the width of the enhanced conductance region, or a "bridge" between the Coulomb peaks, is consistently larger for the D−D Kondo effect [the dotted-dashed line in Fig. 2(a)] than for the S−T Kondo effect [the dotted line in Fig. 2(a)] at other $N$ as well.

When all the electrons occupy the ground Landau level ($v = 2$), the spin state is doublet (singlet) for odd (even) $N$. The system then enters a spin flip region ("B" in Fig. 4) when $B$ further increases, and finally reaches a totally spin polarized state ($v = 1$), or maximum density droplet (MDD). The first spin-flip transition occurs at $B \approx 2.0$ T from $S = 0$ to 1 for even $N$, and from $S = 1/2$ to 3/2 (quadruplet) for odd $N$ within region B. This spin transition in the GS is clearly signaled by the kink in the Coulomb peak evolution with $B$ as well as by the Kondo effect; the S−T Kondo effect is observed for even $N$ as the enhanced conductance in the Coulomb valley suggests. A similar conductance enhancement is observed in the Coulomb valley for the doublet–quadruplet (D−Q) degeneracy with odd $N$, suggesting a D−Q Kondo effect. However, a zero-bias peak is not observed in the differential conductance characteristic shown in Fig. 6(c). On the contrary, a zero-bias "dip" develops at low temperatures. In this particular case of D−Q degeneracy at $N = 23$, the transition occurs from a doublet state where $(n_0, l_0) = (0, 11)$ state is occupied by an unpaired electron, to a quadruplet state where $(n_0, l_0) = (0, 10), (0, 11)$ and $(0, 12)$ states are each occupied by an unpaired electron. Figure 7(c) shows the $B$-dependence of the differential conductance characteristic, where the D−Q degeneracy occurs at $B = 1.91$ T and $N = 21$. Temperature dependence of the differential conductance characteristic similar to Fig. 6(c) is observed for this electron number as well, although the overall feature is more asymmetric. Unlike the two-stage Kondo effect reported in a lateral quantum dot, no peak is found at $V_{sd} = 0$ V even when $B$ is scanned across the transition point, and the D−Q degeneracy remains as a saddle point in the $B$–$V_{sd}$ diagram. The peaks in the differential conductance in Fig. 6(c) roughly corresponds to three times the Zeeman splitting at this magnetic field. Although the peaks may be related to the Zeeman splitting between $S_z = 3/2$ and $-3/2$ states, we have no clear explanation as yet for the above observed features.

4. Conclusion

In conclusion, we have observed a strong Kondo effect in a two-dimensional harmonic quantum dot when a magnetic field induces state degeneracies between a spin singlet and triplet states and between two spin doublet states. The estimated Kondo temperature is comparable between the two kinds of degeneracies but much higher than that for a standard spin-half Kondo effect, indicating that a total of four-fold spin and orbital degeneracy for both cases accounts for the similar enhancement of the Kondo temperature. In addition, for degeneracy between a spin doublet and quadruplet states, we have observed enhanced conductance in the Coulomb valley but a zero bias dip. This can be due to the Zeeman splitting, but the physics is not yet clear.

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