JOB OPENING

- PD position in Quantum Computation and Information Project, ERATO-SORST,JST
- Theorists (1-2, working in Tokyo, Japan)
  - Quantum computing: HSP, communication complexity, interacting proof, …
  - Quantum Cryptography
- Experimentalist (1, working in Tsukuba, Japan)
  - entangled photons, quantum cryptography,…
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Self-teleportation and its applications to LOCC state estimation and cloning

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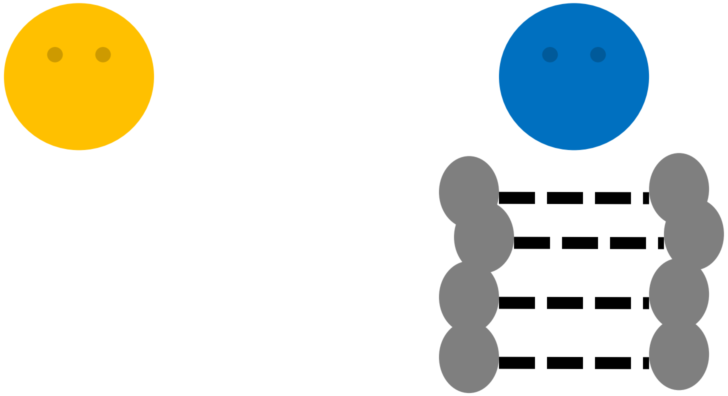


Contents

- Self-teleportation and estimation of entangled bipartite pure states
- LOCC estimation of tensor product states
- Self-teleport-concentration and local copying
- Information spectrum approach to non-i.i.d entanglement theory

Self-Teleportation
&
LOCC estimation of entangled pure bipartite states

Motivation, Background

Characterization of quantum non-locality by
the best LOCC vs. collective operations
in the efficiency of state estimation

Known: non-entangled states can be non-local

1. Holevo, Belavkin etc (1970s): State detection $\rho^{\otimes n}$ vs $\sigma^{\otimes n}$
2. Bennett et al. **"nonlocality without entanglement"**, 1999

A set of pure orthogonal separable states with non-zero detection error

Message of the talk

entangled pure states are not nonlocal, at all

separable states are nonlocal, but small exceptions

The Challenge

There is no good characterization of LOCC

Optimization is awfully hard

Past researches

1.restrict to special case

2. weak statements

lowerboud only,

perfect detection only

☹

This talk : ☺

1. arbitrary pure state family

2.optimality

3. *n* copies of the same state. ($n \gg 10$)

LOCC Estimation

- Given n-copies of unknown bipartite pure state, shared by **A** and **B** with $n \fallingdotseq 20$ or more.

  $\{|\phi_\theta\rangle\}$: a parameterized family of pure states

- Error measure: mean distance

  $$E(D(|\phi_\theta\rangle, |\phi_{\theta\,\mathrm{est}}\rangle)^2) = a/n + b/n^{3/2} + c/n^2 + \ldots$$

want to minimize a, b, c, …. except for exponentially small order.

Question: Can we do as good as global measurement?

YES for entangled state,

No for separable state (some exceptions)

Self-teleportation

1. **A** and **B** share given ***n*** copies of an unknown pure entangled state.

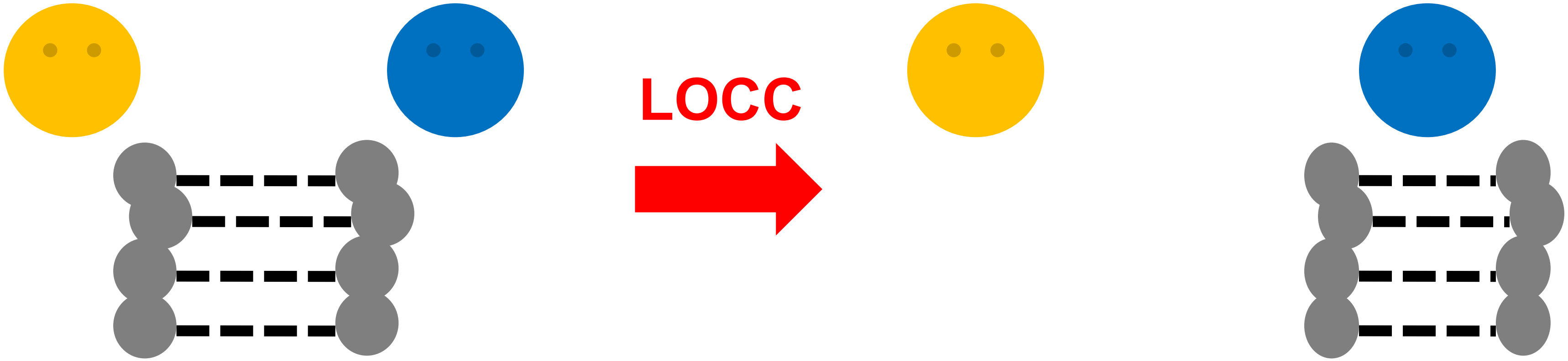
$$|\phi\rangle = \sum_{i=1}^{d} \sqrt{p_i}\, |i\rangle_A |i\rangle_B$$

$$p_1 \geq p_2 \geq \cdots \geq p_d$$

2. By LOCC, **A** sends her quantum info to **B**.

   No quantum channel, No extra entangled states

   Without sacrificing any of pairs **Error: p_1^n**



LOCC

$|\phi\rangle_{AB}^{\otimes n}$

$|\phi\rangle_{BB}^{\otimes n} + \varepsilon$

Difference from teleportation, remote state preparation, and entanglement swapping

- **Teleportation, remote state preparation** requires additional entanglement other than the state teleported
- **Self-teleportation** uses its own entanglement to teleport oneself.

Caution : Measurement based protocol does not work!

1. **A** and **B** measure the state, compute the estimate $|\phi_{est}\rangle$
2. **B** locally fabricate $|\phi_{est}\rangle^{\otimes n}$

$$|\langle \phi_{est}|\phi\rangle|^2 \leqq 1-O(1/n)$$

$$\therefore\ |\langle \phi_{est}|\phi\rangle|^{2n} \leqq \text{const.} <1$$ very bad

You have to do something non-trivial.

LOCC estimation by Self-teleportation

1. Self-teleportation:

$$|\phi_\theta\rangle_{AB}^{\otimes n} \Rightarrow |\phi_\theta\rangle_{BB}^{\otimes n} + \varepsilon$$

If entangled, $|\varepsilon| = O(p_1^n)$: exponentially small

If not entangled, $p_1=1$: $|\varepsilon|=1$ Totally fail

2. If succeed,

B does globally optimal measurement.

Separable states suffers from quantum non-locality. (Counter intuitive)

A standard form [MH00]

Total angular Momentum, if d=2

$$\lambda = (\lambda_1, \ldots, \lambda_d), \qquad \lambda_i \geq \lambda_{i+1} \geq 0, \sum_{i=1}^{d} \lambda_i = n$$

Representation of GL

Representation of permutation

$\mathbf{A}^{\cdot n}$

$$H^{\otimes n} = \bigoplus_{\lambda} Y_{\lambda} \otimes \varsigma_{\lambda}$$

|ϕ> |ϕ'>

|ϕ''>

(3,0)

$|000\rangle$

$|001\rangle + |010\rangle + |100\rangle$

$|011\rangle + |101\rangle + |110\rangle$

$|111\rangle$

$$\dim \mathcal{U}_{(3,0)} = 4$$
$$\dim \mathcal{V}_{(3,0)} = 1$$

(2,1)

$|010\rangle - |100\rangle$

$|011\rangle - |101\rangle$

$\frac{1}{2}(|100\rangle + |010\rangle) - |001\rangle$

$\frac{1}{2}(|011\rangle + |101\rangle) - |110\rangle$

$$\dim \mathcal{U}_{(2,1)} = 2$$
$$\dim \mathcal{V}_{(2,1)} = 2$$

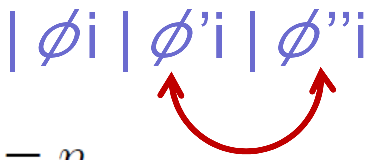
A standard form [MH00]

$$H^{\otimes n} = \bigoplus_{\lambda} \mathrm{Y}_{\lambda} \otimes \varsigma_{\lambda}$$

Decoherence free subspace,

Representation of permutation

Representation of GL $A^{\bullet n}$


$|\phi\mathrm{i} \; |\phi'\mathrm{i} \; |\phi''\mathrm{i}$

$$\lambda = (\lambda_1, \ldots, \lambda_d), \quad \lambda_i \geq \lambda_{i+1} \geq 0, \sum_{i=1}^{d} \lambda_i = n$$

$$|\phi\mathrm{i}^{\otimes n} = \bigoplus_{\lambda} a_{\lambda} |\phi^{\lambda}\mathrm{i} \otimes |\Phi_{\lambda}\mathrm{i}$$

$\mathrm{Y}_{\lambda,A} \otimes \mathrm{Y}_{\lambda,B}$ $\quad$ $\varsigma_{\lambda,A} \otimes \varsigma_{\lambda,B}$

Depends on $|\phi\mathrm{i}$

Independent of $|\phi\mathrm{i}$
Max. ent

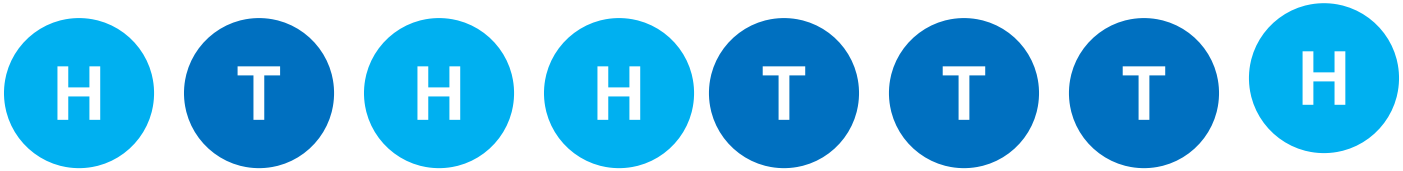
dim $\mathrm{Y}_{\lambda} \leqq \mathrm{poly}(n)$

dim ς_{λ} : Typically exponential

Necessary quantum information for estimation is negligibly small

Why necessary quantum information for estimation is negligibly small ?

- Consider n-times (biased) coin flip.
- Want to estimate prob. of tail.
- For that, we only need the frequency of tail, and can forget when tail occurred.
- # of tails is 0,1,2, …,n,
- Information = log(n+1) bits = o(n)-bits
- Similar for quantum case.



Self-teleportation Protocol

1. **A** & **B** project onto $A_n := \{\lambda : \dim \mathcal{U}_\lambda \leq \dim \mathcal{V}_\lambda\}$
2. **A** measures by basis

$$|\{U_\lambda\}\rangle := \bigoplus_{\lambda \in A_n} \sqrt{\dim \mathcal{V}_\lambda} \sum_{i=1}^{\dim \mathcal{U}_\lambda} \underset{\mathcal{U}_{\lambda,A}}{|e_{\lambda,i}\rangle} U_\lambda \underset{\mathcal{V}_{\lambda,A}}{|f_{\lambda,i}\rangle}$$

3. **A** sends $\{U_\lambda\}$
4. **B** does $\bigoplus_{\lambda \in A_n} U_\lambda$ upon $\mathcal{V}_{\lambda,B}$

$$\bigoplus_{\lambda \in A_n} a_\lambda | \phi^\lambda i$$

5. **B** creates max. ent locally

$$\bigoplus_{\lambda \in A_n} a_\lambda | \phi^\lambda i \otimes |\Phi_\lambda i \doteqdot | \phi i^{\otimes n}$$

Failure probability

Failure Prob = $\sum_{\lambda \notin A_n} a_\lambda^2 \doteqdot p_1^n$

p_1 :the largest Schmidt coefficient

$p_1 < 1$ for entangled state
$p_1 = 1$ for separable state

So long as the state is not separable,
Failure prob vanishes exponentially fast!

LOCC estimation of multi-partite tensor product states

Estimation of $\rho_\theta = \rho_\theta^A \otimes \rho_\theta^B$ by LOCC

1. Seemingly easy.
2. However, recall the state family studied by Bennett et al. **"nonlocality without entanglement"**, 1999 are in this form. Can be highly non-local.
3. Self-teleportation fails with certainty

Message:

In most cases, this state family is Highly non-local.

Ex. Anit-copy

$|\phi_\theta\rangle = |\alpha\rangle\,|\alpha^*\rangle$ $\quad$ $\alpha = \theta^1 + \sqrt{-1}\theta^2$

$|\alpha\rangle$: coherent state

Global optimal: θ^1 by $X_A + X_B$

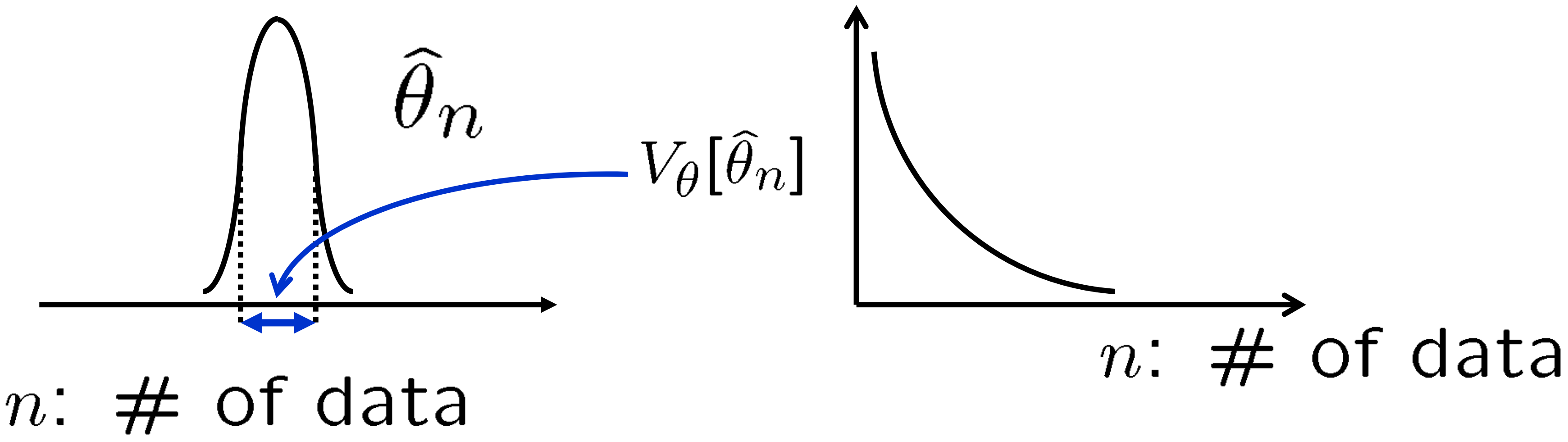
θ^2 by $Y_A - Y_B$ (commute)

Locally : θ^1 by X_A, X_B at each site

θ^2 by Y_A, Y_B at each site

Estimation is harder for separable states!

First order asymptotic theory of probability distributions



Asymptotic Cramer-Rao:

$$\min V_\theta[\widehat{\theta}_n] = \frac{1}{n} J_\theta^{-1} + o\left(\frac{1}{n}\right)$$

Fisher Information:

$$J_{\theta,i,j} = \int p_\theta(x) l_{\theta,i}(x) l_{\theta,j}(x) \mathrm{d}x \quad l_{\theta,i} := \partial_i \log p_\theta$$

Quantum : non-collective measurement

Theorem [Nagaoka 1989, GillMassar 2000,HM 1998]

$$\min_{\underline{M^n}:\text{non-collective}} \mathbf{Tr}\, G V_\theta\left[M^n\right] = \frac{1}{n} C_\theta(G) + o\left(\frac{1}{n}\right)$$

1. **Measurement on n-copy,**
2. **Construction is independent of θ**

$$C_\theta(G) = \min_{\underline{M}} \mathbf{Tr}\, G\left(J_\theta{}^{M}\right)^{-1}$$

Cramer-Rao-type bound

1. **Measurement on single copy,**
2. **Construction depends on θ**

J_θ^M : Fisher Information of $p_\theta(i) = \mathrm{tr}\, \rho_\theta M_i$

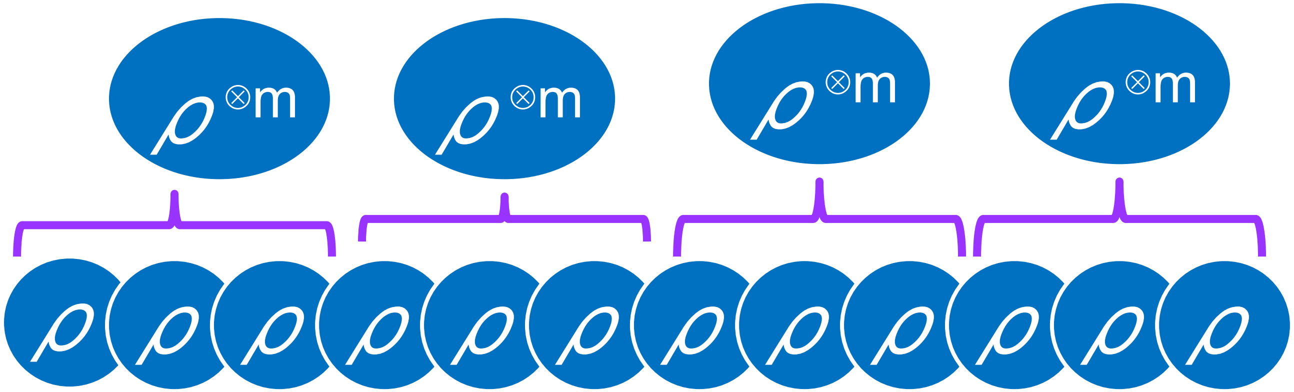
Quantum: Collective measurement

Theorem [HM 1998]

$$\inf_{M^n:\text{collective}} \mathbf{Tr}\, G V_\theta\left[M^n\right] = \frac{1}{n} C_\theta^Q(G) + o\left(\frac{1}{n}\right)$$

$$C_\theta^Q(G) = \lim_{m\to\infty} m C_\theta^m(G)$$

$$C_\theta^m(G) : \text{CR - type bound of } \left\{\rho_\theta^{\otimes m}\right\}$$



tensor-product (mixed or pure) state,
A-B LOCC & collective measurement

A-B Between copies

$$\min_{M^n:\mathrm{LOCC}} \mathbf{Tr} G V_\theta\left[M^n\right] = \frac{1}{n} C_\theta^{Q,LOCC}(G) + o\left(\frac{1}{n}\right)$$

$$C_\theta^{Q,LOCC}(G) := \lim_{m\to\infty} m C_\theta^{m,LOCC}(G)$$

$$C_\theta^{m,LOCC}(G) := \min_{M^m:M_A{}^m \otimes M_B{}^m} \mathbf{Tr} G\left(J_\theta{}^{M^m}\right)^{-1}$$

- minimization is only over LO.
- Can by-pass characterization of LOCC
- Doesn’t mean asymptotically optimal protocol is LO. (corresponding protocol requires 2 times of 2 way communications)

Cor. $\rho_\theta = \rho_\theta^A \otimes \rho_\theta^B$

Can be mixed states

$$C_\theta^{Q,LOCC}(G) = C_\theta^{Q}(G), \quad \forall G > 0$$

$\Leftrightarrow$ ρ_θ^A's and ρ_θ^B's tangent space have to has the same structure

$$\mathrm{tr}\, \rho_\theta^A L^A_{\theta,i} L^A_{\theta,j} \underset{\uparrow}{=} c\, \mathrm{tr}\, \rho_\theta^B L^B_{\theta,i} L^B_{\theta,j}$$

independent of *i, j*

$L^A_{\theta,i}$ $L^B_{\theta,i}$ **(SLD): defined as a solution to:**

$$\frac{\partial \rho_\theta^\alpha}{\partial \theta^i} = \frac{1}{2}\left(L^\alpha_{\theta,i}\rho_\theta^\alpha + \rho_\theta^\alpha L^\alpha_{\theta,i}\right)$$

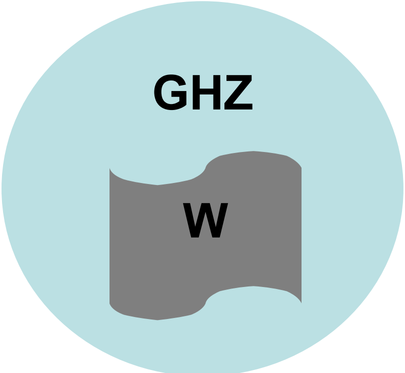
$\fallingdotseq \rho_\theta^A = \rho_\theta^B$

∴ Typically, LOCC estimation < global operation

On 3-partite entanglement

GHZ-type : $|\phi\rangle = A \otimes B \otimes C |GHZ\rangle$

A, B, C: a 2x2 matrix



Given: $|\phi\rangle^{\otimes n}$

LOCC + zero-rate quantum information transmission

Can merge the state to Alice's local state without knowing A,B,C