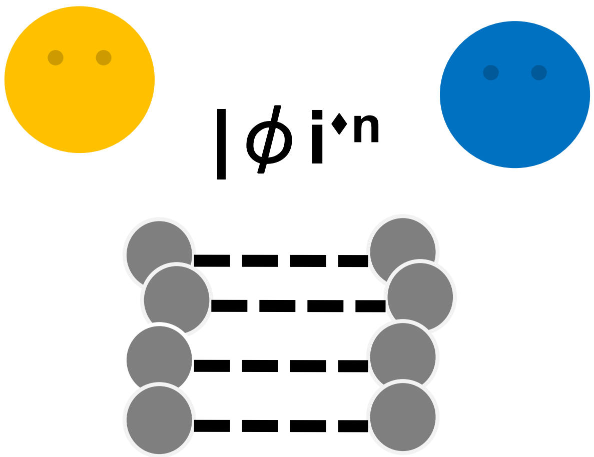
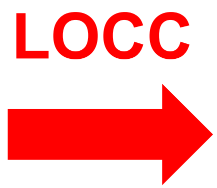
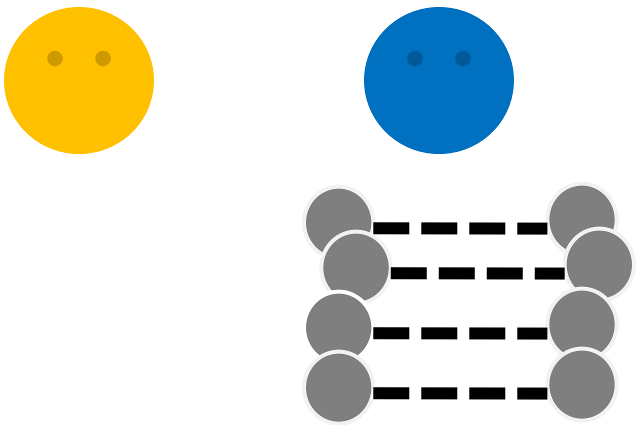
Self-teleport-concentration
&
Local copying

Self- teleprot-concentration







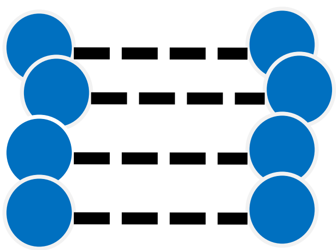
$|\varphi\rangle$: unknown

$|\varphi\rangle^{\otimes n} + \varepsilon$

$|\varepsilon| = O(\log^2 n / n)$

c.f. Horodezki et.al's state merging

1. $|\varphi\rangle$: known
2. use zero-rate quantum channel



Bell pairs
$nE(|\varphi\rangle) + o(n)$

Self-teleport-concentration

- Intrinsic maximally entangled state : O(n)
- Quantum information to be teleported : O(log n)

Therefore, without damaging intrinsic maximally entanglement, we can teleport

All necessary quantum information!

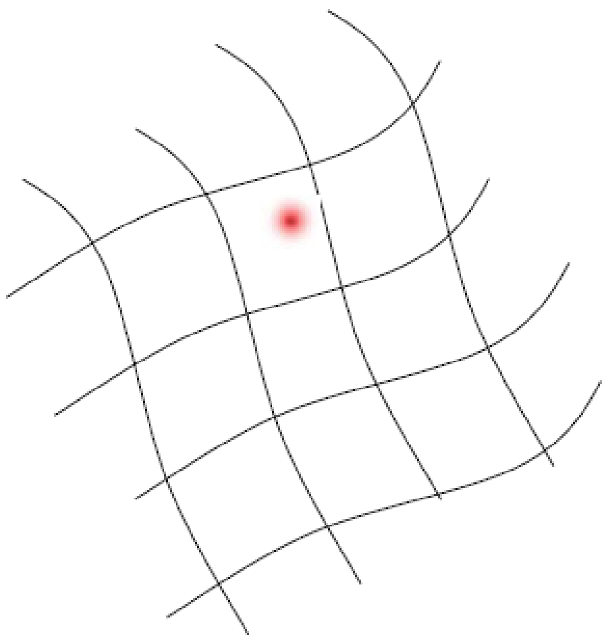
If Schmidt coefficient of $|\phi>$ is known, $F= 1-O(e^{-n})$.

If unknown, estimate it by measuring $|\phi>$, without damaging them too much

$F=1-O(\log^2 n /n)$

Gentle tomogra

("Tender measurement" from Keiji … s talk)



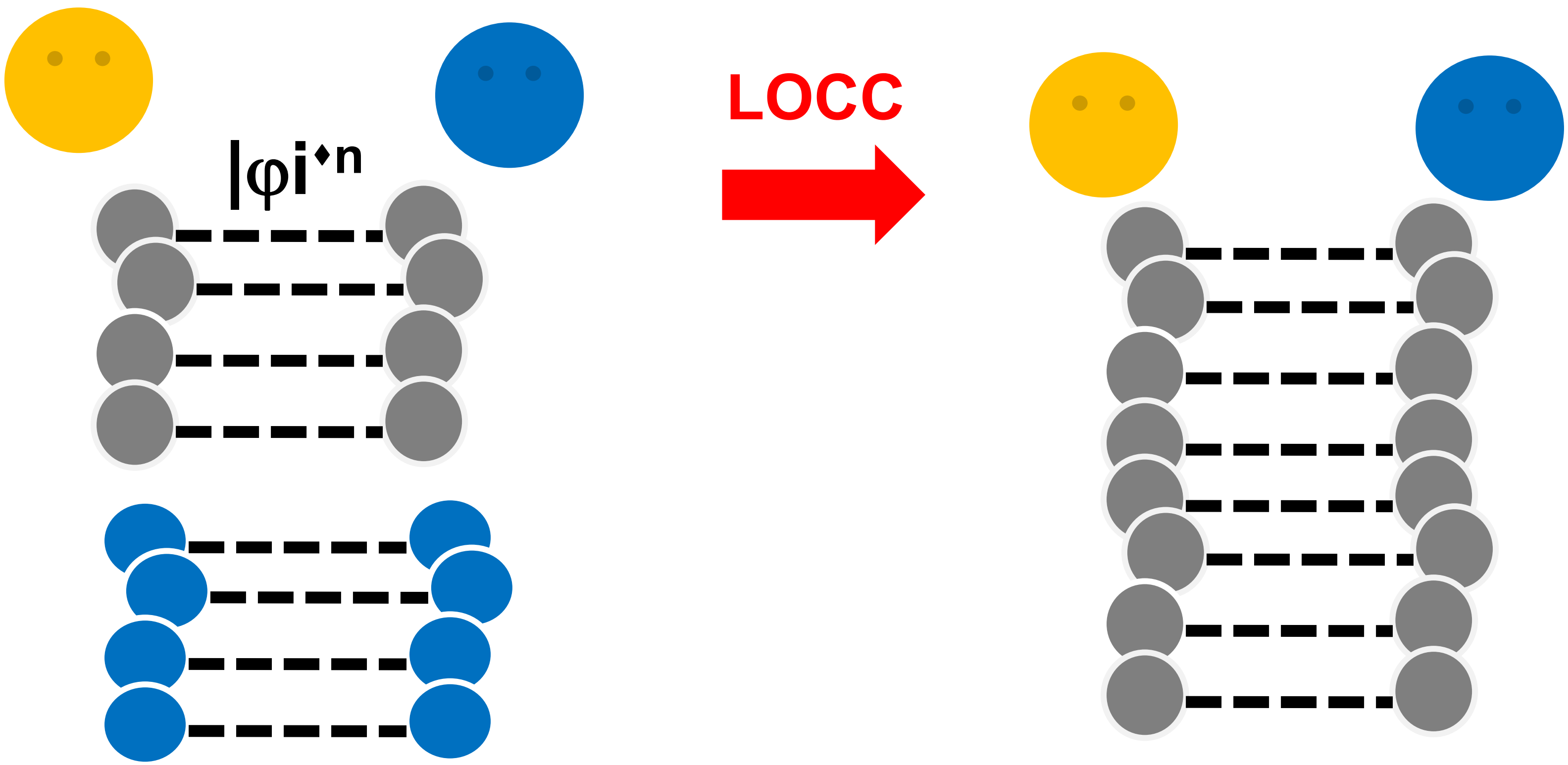
Th… nay be vi… roughly as
ch… ing a random … h on the
pa… eter space of ρ … se
en… (∝ $1/\sqrt{m}$) so fo… o,
a m… ement on ρ^m of wh…
cell th… age falls in almos…
always yi…
This measurement, …
conducted coherently, will
therefore scarcely disturb the
global state.

C Bennet, QIP 2003 (Dec. 2002)

$F = 1 - O(n^{-1/2})$

Not so gentle

An application: Local copying



$r\ n$Ec ($\rho^{n,\ r\ n}$) – n E(|φi)+o(n)

$\rho^{n,\ r\ n} ≑ |\varphi i^{⬩\ r\ n}$

F($\rho^{n,\ r\ n}$,|φi⬩ $^{r\ n}$) is close to global optimal if n>>1

Challenges in application to local copying

1. Ec ($\rho^{n,m}$) : how to characterize?

since this is not tensored-product state,

we need Te-Sun Han's **INFORMATION SPECTRUM(=smooth Reny entropy, as it had turned out, [Datta, Renato et.al 2007], [M 2007])**

2. In general, Ec (ρ) ebits is not enough to teleport half of ρ. In addition, in our case, ρ is unknown

$\Rightarrow$ **anonymous entanglement dilution for symmetric states**

Information Spectrum approach to non-i.i.d entanglement theory

Information Spectrum, history

- Han and Verdu [1991]
- Pushed forward by Han, Nagaoka
  (UEC, Tokyo)

  Prof. Te-Sun Han : Information Theory

  H. Nagaoka : Information Geometry,
  Qunatum information

Quantum version:

Nagaoka in late 90's (I first heard about it in 1998),
first appearance in public: EQIS 2002

had pushed forward by Nagaoka, Ogawa, (-2000) Hayashi,

but had not attracted much attentions.

Recently, Renner, Koenig (smooth Reny),

A. Winter (smooth version of Reny, communication complexity)

Bowen, Datta

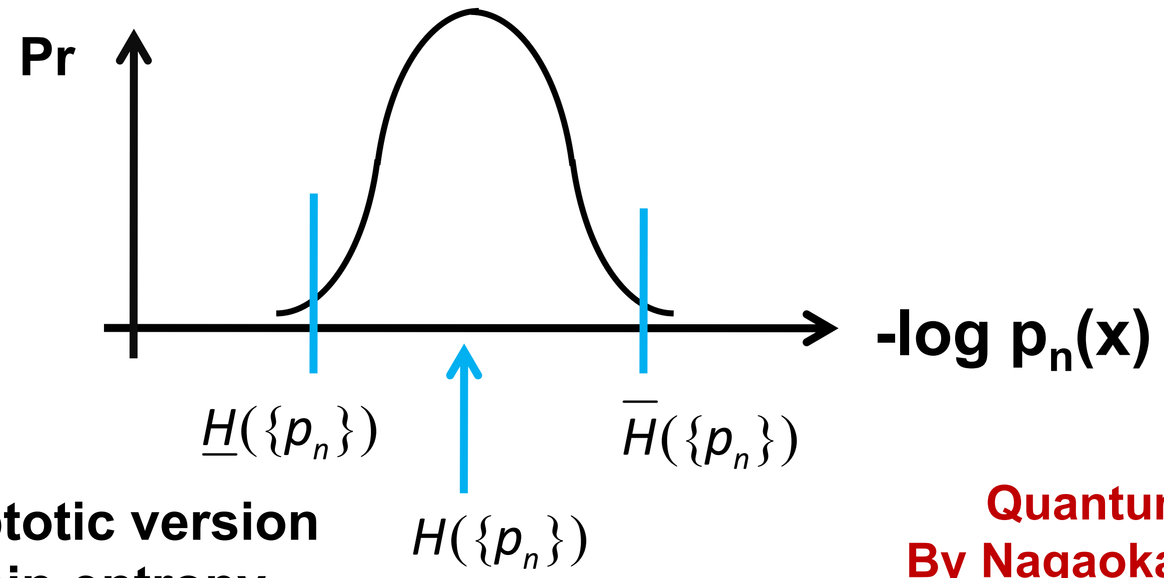
Information spectrum Te-sun Han [91]

x∼p(x)

consider -log p(x) as a random variable

ex. p(0)=a, p(1)=p(2)=b

Pr{ -log p(x)=-log a}=a , Pr{ -log p(x)=-log b}=2b



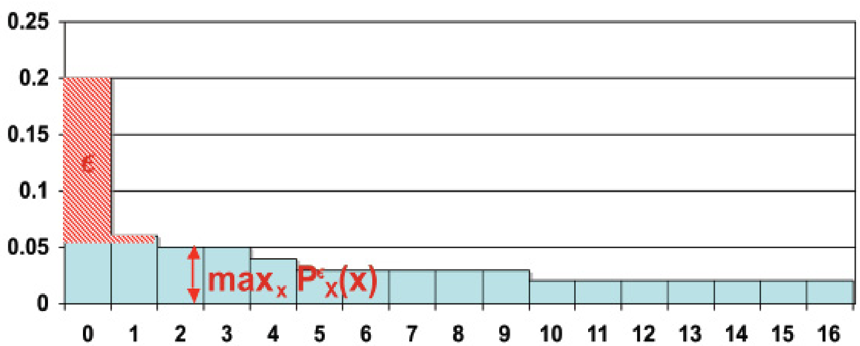
Asymptotic version of min-entropy

Quantum version
By Nagaoka (EQIS 2002)
Hypothesis test,
Entanglement

Smooth Reny entropy [Renner, Koenig 2005]

Renner, QIP 2008 (Dec, 2007)

Smoothing



Definition: Smooth entropy of P_X

$H^\epsilon(X) := \max_{X'} H_{min}(X')$

maximum taken over all $P_{X'}$ with $|| P_X - P_{X'} || \leq \epsilon$

Application: Cryptography

Fact: Smooth Reny = Spectrum

Entanglement cost of a general state sequence [Datta 07, M 07]

Theorem 2 *Given a sequence* $\{\rho^n\}_{n=1}^{\infty}$ *of bipartite quantum states, we have*

$$E_c\left(\{\rho^n\}_{n=1}^{\infty}\right) = \inf_{\{q_i^n, |\phi_i^n\rangle\}} \text{p-}\overline{\lim_{n\to\infty}} \frac{-1}{n} \log p_j^{n,i}$$

where $\text{p}-\overline{\lim}_{n\to\infty}$ *is with respect to* $\left\{q_i^n p_j^{n,i}\right\}_{n=1}^{\infty}$, *and infimum is taken over all the sequences of pure state ensembles* $\{q_i^n, |\phi_i^n\rangle\}$ *with* $\sum_i q_i^n |\phi_i^n\rangle\langle\phi_i^n| = \rho^n$.

$\text{p-}\overline{\lim}_{n\to\infty} X^n$, the minimum of x with

$$\lim_{n\to\infty} P^n\{i\,;\ X^n \le x\} = 1$$

Ec and Ed of a symmetric state [M 07]

ρ^n : supported on a symmetric subspace of $\mathrm{H}^{\otimes n}$

p_x^n (x=1…dim H): eigenvalue of ρ^n

$$E_c(\{\rho^n\}) = p-\overline{\lim_{n\to\infty}} \frac{-1}{n} \log p_x^n$$

$$E_d(\{\rho^n\}) = p-\underline{\lim_{n\to\infty}} \frac{-1}{n} \log p_x^n$$

$\mathrm{p}-\overline{\lim}_{n\to\infty} X^n$, the minimum of x with

$$\lim_{n\to\infty} P^n\{i\ ;\ X^n \leq x\} = 1$$

$\mathrm{p}-\underline{\lim}_{n\to\infty} X^n$, the maximum of x with

$$\lim_{n\to\infty} P^n\{i\ ;\ X^n \geq x\} = 1.$$

Entanglement of optimal clone

- Given n copies of $|\phi\rangle$. $E(|\phi\rangle)$: entropy measure
- Want to inflate to $r \times n$ copies. (r:const)
- What will be the entanglement of this approximate clone?

Suppose we know $|\phi\rangle$'s Schmidt basis, except for the phases.

$\rho^{n,rn}$: optimal n→rn clone

not close to $|\phi\rangle^{\otimes rn}$ at all.

Same in entanglement as $|\phi\rangle^{\otimes rn}$

$$Ec\,(\rho^{n,rn}) = E(|\phi\rangle^{\otimes r}) = r\,E(|\phi\rangle)$$

END

- By Self-Teleportation,

  LOCC estimate = Global estimate

  Cloning =Global cloning

By Self-Teleportation + universal concentration,

Local copying = Global cloning

3bodies ? Mixed states? Zero-rate QC?

An optimal adaptive strategy

1 Invest $\rho_\theta^{\otimes\sqrt{n}}$ to obtain $\widehat{\theta}_{aux}^{n}$

2 $\widetilde{M}_n$: achives $\min \mathbf{Tr} G \left(J_\theta{}^{M} \right)^{-1}$ at $\theta = \hat{\theta}_{aux}^{n}$

apply $\widetilde{M}_n$ to ρ_θ for $n - \sqrt{n}$ times

An optimal strategy

1 Invest $\rho_\theta^{\otimes \sqrt{n}}$ to obtain $\widehat{\theta}_{aux}^{n}$

2 $\rho_\theta^{\otimes n-\sqrt{n}} = \left(\rho_\theta^{\otimes m}\right)^{\otimes n'}$

$\widetilde{M}_n^m$: achives $C_\theta{}^m(G)$ at $\theta = \hat{\theta}_{aux}^{n}$

apply $\widetilde{M}_n{}^m$ to $\rho_\theta^{\otimes m}$ for n' times

The bound is almost achieved if m>>1